

Vacuum energy: quantum hydrodynamics vs quantum gravity

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We compare quantum hydrodynamics and quantum gravity. They share many common features. In particular, both have quadratic divergences, and both lead to the problem of the vacuum energy, which in the quantum gravity transforms to the cosmological constant problem. We show that in quantum liquids the vacuum energy density is not determined by the quantum zero-point energy of the phonon modes. The energy density of the vacuum is much smaller and is determined by the classical macroscopic parameters of the liquid including the radius of the liquid droplet. In the same manner the cosmological constant is not determined by the zero-point energy of quantum fields. It is much smaller and is determined by the classical macroscopic parameters of the Universe dynamics: the Hubble radius, the Newton constant and the energy density of matter. The same may hold for the Higgs mass problem: the quadratically divergent quantum correction to the Higgs potential mass term is also cancelled by the microscopic (trans-Planckian) degrees of freedom due to thermodynamic stability of the whole quantum vacuum.

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1. Introduction. The problem of quantum hydrodynamics is at least 65 years old (see quantization of the macroscopic dynamics of liquid in the first Landau paper on superfluidity of ^4He [1]). It is almost as old as the problem of quantum gravity [2]. Quantum hydrodynamics and quantum gravity share many common features (e.g. both have quadratic divergences) and probably they will have the common destiny. The main message from quantum hydrodynamics to quantum gravity is that most probable the quantum gravity cannot be constructed, because quantum hydrodynamics cannot be constructed.

Of course, one can quantize sound waves in hydrodynamics to obtain quanta of sound waves – phonons. Similarly one can quantize gravitational waves in general relativity to obtain gravitons. But one should not use the low-energy quantization for calculation of the radiative corrections which contain Feynman diagrams with integration over high momenta. In particular, the effective field theory is not appropriate for the calculation of the vacuum energy in terms of the zero-point energy of quantum fields. The latter leads to the cosmological constant problem in gravity [3, 4], and to the similar paradox for the vacuum energy in quantum hydrodynamics: in both cases the vacuum energy estimated using the effective theory is by many orders of magnitude too big. We

know how this paradox is solved in quantum liquids, and we may expect that the same general arguments based on the thermodynamic stability of the ground state of the quantum liquid are applicable to the quantum vacuum.

There is another big discrepancy between theory and experiment, which is called the hierarchy problem in the Standard Model [5]. It is believed that the mass of the Higgs boson is on the order of or somewhat larger than the mass of the gauge boson: $m_{\text{H}}^2 \sim M_{\text{Z}}^2$. For example, in analogy with the effect of Cooper pairing in superconductivity, m_{H} can be equal to $2m_t$, where m_t is the mass for the top quark [6, 7]. However the radiative correction to the Higgs mass is quadratically diverging, and is determined by the ultraviolet cut-off. If one chooses the natural GUT or Planck scale as the cutoff energy, one obtains that the Higgs boson mass must be extremely large: $m_{\text{H}}^2 \sim \pm 10^{26} M_{\text{Z}}^2$ and $m_{\text{H}}^2 \sim \pm 10^{34} M_{\text{Z}}^2$ correspondingly (the sign of the radiative correction is determined by the relevant fermionic and bosonic content of the theory and the cut-off scheme). We argue, that this discrepancy is related to the problem of the vacuum energy, and thus the same thermodynamic arguments which have been used for the cosmological constant problem can be applicable to the hierarchy problem.

2. Classical hydrodynamics of quantum liquid. Let us consider the hydrodynamics of the isotropic superfluid liquid at $T = 0$ (such as bosonic superfluid ^4He and fermionic superfluid $^3\text{He-B}$ [8]). Though the superfluid liquid is essentially quantum, its macroscopic

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low-frequency dynamics is classical and is represented by classical hydrodynamics. It is background independent, i.e. it does not depend on details of the underlying microscopic physics, and is the same for fermionic and bosonic liquids. The equations of the non-relativistic superfluid hydrodynamics (in the absence of quantized vortices) are the Hamilton equations for the canonically conjugated fields:

$$\partial_t \rho = \frac{\delta H}{\delta \phi}, \quad \partial_t \phi = -\frac{\delta H}{\delta \rho}. \quad (1)$$

Here ρ the mass density; ϕ is the velocity potential (in the absence of quantized vortices the superfluid velocity is potential: $\mathbf{v} = \nabla \phi$); the Hamiltonian is the energy functional of the liquid expressed in terms of ρ and ϕ :

$$H = \int d^3r \left[\frac{1}{2} \rho (\nabla \phi)^2 + \bar{\epsilon}(\rho) \right], \quad (2)$$

where $\bar{\epsilon}(\rho) = \epsilon(\rho) - \mu \rho$; $\epsilon(\rho)$ is the energy density of the liquid expressed in terms of the liquid density; and μ is chemical potential – the Lagrange multiplier which takes into account the mass conservation $\int d^3r \rho = \text{const}$. At fixed chemical potential μ , this functional has minimum at $\mathbf{v} = 0$ and $\rho = \rho_0(\mu)$, where the equilibrium density ρ_0 is determined by equation $d\bar{\epsilon}/d\rho = 0$ (or $d\epsilon/d\rho = \mu$). This is the ground state of the liquid, with the relevant energy density $\bar{\epsilon}(\rho_0)$ and the relevant total energy

$$E_0 = V \bar{\epsilon}(\rho_0). \quad (3)$$

Note again that this consideration is completely classical and operates with quantities ρ_0 and $\bar{\epsilon}(\rho_0)$, which are the classical output of the quantum system: the superfluid ^4He and superfluid $^3\text{He-B}$ are systems of strongly correlated, strongly interacting and highly entangled helium atoms governed by the laws of quantum mechanics. The reason for classicality is the macroscopic character of the collective motion.

3. Quantized hydrodynamics. What happens if we try to construct the quantum hydrodynamics, i.e. to quantize the hydrodynamic motion of the liquid determined by Eqs.(1)? Using hydrodynamic variables only we are unable to reconstruct the whole microscopic Hamiltonian for the interacting atoms. This is because from the big realm of the complicated quantum motion of the ^4He atoms we have chosen only the hydrodynamic modes whose wavelengths are much bigger than the inter-atomic spacing a , which plays the role of the Planck length: $ka \ll 1$. That is why, what we can do at best is to quantize the sound modes. But even in this case there is a danger of the double counting, because starting from the quantum system we have obtained the

classical behavior of soft variables ρ and ϕ , and now we are trying to quantize them again. In particular, the energy E_0 in Eq.(3) is the whole energy of the quantum liquid, and it already includes from the very beginning the energy of those degrees of freedom which are described in terms of phonons. Let us see how this double counting typically occurs.

The conventional quantization procedure for sound waves in the background of the state with $\rho = \rho_0$ and $\phi = 0$ is the introduction of the commutation relations for the canonically conjugated variables:

$$\left[\hat{\phi}_{\mathbf{k}}, \hat{\rho}_{\mathbf{k}'} \right] = i \hbar \delta_{\mathbf{k}\mathbf{k}'}, \quad (4)$$

where

$$\hat{\rho}(\mathbf{r}) = \rho_0 + \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} (\hat{\rho}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + \text{c.c.}), \quad (5)$$

$$\hat{\phi}(\mathbf{r}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} (\hat{\phi}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + \text{c.c.}). \quad (6)$$

Introducing these perturbations to Eq. (3) one obtains the quantum Hamiltonian as the sum of the ground state energy and the Hamiltonians for quantum oscillators:

$$\begin{aligned} \hat{H} &= E_0 + \frac{1}{2} \sum_{\mathbf{k}} \left(\rho_0 k^2 |\hat{\phi}_{\mathbf{k}}|^2 + \frac{c^2}{\rho_0} |\hat{\rho}_{\mathbf{k}}|^2 \right) = \\ &= E_0 + \frac{1}{2} \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \left(a_{\mathbf{k}} a_{\mathbf{k}}^\dagger + a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \right). \end{aligned} \quad (7)$$

where $\omega_{\mathbf{k}} = ck$; and the speed of sound c is given by $c^2 = \rho_0 d^2 \epsilon / d\rho^2 |_{\rho_0}$.

There is nothing bad with this quantization, if we are constrained by condition $ka \ll 1$. However, if we start to apply this consideration to the diverging quantities such as the vacuum energy, we are in trouble.

4. Vacuum energy in quantum hydrodynamics. The vacuum energy – the ground state of the Hamiltonian (7) – contains the zero-point energies of quantum oscillators:

$$\langle \text{vac} | \hat{H} | \text{vac} \rangle = E_0 + \frac{1}{2} \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}}. \quad (8)$$

However, the vacuum expectation value of \hat{H} must be equal to E_0 by definition. This is because E_0 is not the “bare” energy, but is the total relevant energy of the liquid, which includes all quantum degrees of freedom of the liquid. Thus the naive application of the zero-point energy leads to the paradoxical conclusion that

$$\frac{1}{2} \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} = 0. \quad (9)$$

This contradicts to our intuition that the zero-point fluctuations give for the vacuum energy the estimate $\bar{\epsilon}_{zp} \sim \hbar c k_{uv}^4$, where $k_{uv} \sim 1/a$ is the ultraviolet cut-off. However, the equation (9) simply means that the zero-point energy of phonons had already been included into the original E_0 together with all other modes, i.e. by writing Eq.(9) we simply prevent the double counting for the vacuum energy. Thus the correct form of the Hamiltonian for phonons must be

$$\hat{H} = E_0 + \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}}. \quad (10)$$

One may ask what is the role of the quantum fluctuations of the hydrodynamic field. Do they provide the main or substantial part of E_0 ? To see this let us consider the vacuum energy of superfluid liquid in case when an external pressure P is applied to the liquid. According to the Gibbs-Duhem relation which is valid for the equilibrium states one has:

$$P = TS + \mu \rho_0 - \epsilon(\rho_0). \quad (11)$$

At $T = 0$ one obtains that the vacuum energy is regulated by external pressure

$$\bar{\epsilon} = -P. \quad (12)$$

For the positive external pressure $P > 0$, one obtains the negative energy density of the vacuum, $\bar{\epsilon} < 0$, which certainly cannot be obtained by summation of the positive zero-point energies of phonons. Thus the contribution of zero-point energies of phonons to E_0 gives no idea on the total value of E_0 , since it may even give the wrong sign of the vacuum energy.

Furthermore, for the superfluid helium liquid isolated from the environment, the pressure $P = 0$, and thus the vacuum energy density and vacuum energy are zero: $\bar{\epsilon}(\rho_0) = 0$, $E_0 = 0$. For the finite system – the helium droplet – the vacuum energy density becomes non-zero due to the capillary pressure:

$$\bar{\epsilon} = -P = -\frac{2\sigma}{R}. \quad (13)$$

It is expressed through the classical parameters of the liquid droplet: its radius R and surface tension σ . When we compare this physical result with the naive estimation which only takes into account the contribution of the phonon zero-point energy $\bar{\epsilon}_{zp} \sim \hbar c k_{uv}^4$, one finds that their ratio is determined by the ratio of the quantum microscopic to the classical macroscopic scales: $\bar{\epsilon}_{true}/\bar{\epsilon}_{zp} \sim a/R$. For the macroscopic bodies the discrepancy is big.

5. Application to quantum vacuum. This demonstrates that the vacuum energy is determined by

the macroscopic thermodynamic laws and is not related to the diverging contribution of the zero-point motion of phonons. The result $E_0 = 0$ for the self-sustained homogeneous systems shows that in this system the large positive contribution $\bar{\epsilon}_{zp} \sim \hbar c k_{uv}^4$ of phonons is completely compensated without any fine-tuning by the other quantum degrees of freedom, i.e. by the microscopic (atomic \equiv trans-Planckian) degrees of freedom which cannot be described in terms of the effective (hydrodynamic) field [9].

This compensation which occurs in the homogeneous vacuum does not prohibit the Casimir effect in the systems with boundaries. If the energy difference between two vacua comes solely from the long-wavelength physics, it is within responsibility of the phonon (photon) modes and can be calculated using their zero-point energies.

One can immediately apply this lesson to the quantum gravity: the vacuum energy density ϵ_{vac} (or cosmological constant) is not renormalized by the zero-point energies of quantum fluctuations of the low-energy modes. It is meaningless to represent the vacuum energy as the sum over zero-point oscillations $\frac{1}{2} \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}}$, and to estimate the vacuum energy density as $\epsilon_{zp} \sim \hbar c k_{uv}^4$. The vacuum energy (and thus the cosmological constant) is the final classical output of the whole quantum vacuum with all its degrees of freedom, sub-Planckian and trans-Planckian. It is regulated by macroscopic physics, and obeys the macroscopic thermodynamic laws. The thermodynamic Gibbs-Duhem relation (analog of Eq.(12)) must be satisfied for the equilibrium vacuum, and it does follow from the cosmological term in Einstein action:

$$\Lambda = \epsilon_{vac} = -P_{vac}. \quad (14)$$

For the vacuum isolated from the “environment” (free vacuum), the pressure is zero and thus the vacuum energy density is zero too, $\epsilon_{vac} = -P_{vac} = 0$ [9, 10]. This means that the natural value of the cosmological constant in the equilibrium homogeneous time-independent free vacuum is $\Lambda = 0$ rather than the contribution $\Lambda \sim \hbar c k_{uv}^4$ of the zero point energies of the effective quantum fields, which is completely compensated.

In the case of the developing Universe polluted by matter, the vacuum energy is disturbed, and the compensation is not complete. But again the natural value of Λ is determined not by the quantum zero-point energy, but (as in quantum liquid in Eq.(13)) by the classical macroscopic parameters of the Universe dynamics: the Hubble radius R of the Universe, the Newton constant G and the energy density of matter ρ_{mat} . This implies that, depending on the details of the process, one has $\Lambda \sim \rho_{mat}$, or $\Lambda \sim 1/GR^2$ (or Λ is given by some combi-

nation of these factors). Both estimates are comparable to the measured value of Λ , and are much smaller than the naive estimation of the zero-point energy of quantum fields: $\Lambda_{\text{true}}/\Lambda_{\text{zp}} \sim a^2/R^2 \sim 10^{-120}$. As in quantum hydrodynamics this contains the ratio of the quantum microscopic scale $a = 1/k_{\text{uv}} = \hbar c/E_{\text{Planck}}$ to the classical macroscopic scale R .

6. Higgs mass problem. As distinct from quantum gravity, the Standard Model of electroweak and strong interactions is the low-energy effective theory which can exist in the quantum form. The condensed matter experience demonstrates that this effective theory can be emergent. In condensed matter systems with point nodes in the energy spectrum of fermions, the effective gauge fields and chiral fermions gradually emerge in the low-energy corner. The point nodes are protected by topology in momentum space and thus are generic [9], that is why the effective theory of the Standard-Model type is generic. Though this effective theory is the final output of the underlying quantum system, it can be quantized again, in spite of the ultraviolet divergences. The reasons for that is that above the symmetry breaking electroweak scale, i.e. at $k^2 \gg M_Z^2$, the divergences are logarithmic, of the type $\ln(k^2/M_Z^2)$. Since the logarithm is concentrated mostly in the sub-Planckian region $k_{\text{uv}} \gg k^2 \gg M_Z^2$, it is within the jurisdiction of the low-energy effective theory.

At $k^2 \sim M_Z^2 \sim 10^{-34} E_{\text{Planck}}^2$ the symmetry breaking occurs and the non-zero vev of the Higgs field develops. One may expect that at these extremely low energies as compared to the Planck or GUT scale, the effective theory must work well. Instead the ultraviolet problem arises. In particular, if one tries to calculate the quantum radiative corrections to the mass of the Higgs boson, the quadratic divergence occurs, which is outside the jurisdiction of the Standard Model. What can one say on this problem using the condensed matter experience?

To describe the electroweak transition, the action for the gauge fields and quarks and leptons is supplemented by the action for the Higgs field

$$S_{\text{Higgs}} = \int d^4x \left(\frac{1}{4} \lambda (\phi^2 - \phi_0^2)^2 + (c^2 \nabla \phi)^2 - (\partial_t \phi)^2 \right), \quad (15)$$

and the terms describing interaction of the Higgs field with fermions and gauge fields. The Higgs field ϕ here is a weak doublet. The Hamilton function for the Higgs field is:

$$H_{\text{Higgs}} = E_0 + \int d^3x \left(\frac{1}{4} \lambda (\phi^2 - \phi_0^2)^2 + (c^2 \nabla \phi)^2 + (\partial_t \phi)^2 \right). \quad (16)$$

The Higgs field is massive with mass

$$m_{\text{H}}^2 = \lambda \phi_0^2. \quad (17)$$

From the condensed matter point of view, the mass m_{H}^2 and the vacuum energy E_0 are the final classical output of the whole underlying quantum vacuum. We know that the energy of the whole vacuum must be zero according to the thermodynamic stability of the free vacuum: $E_0 = H(\phi = \phi_0) = 0$. If one does not take into account the possible infrared anomalies, the equation (16) with $E_0 = 0$ is the general form satisfying the stability condition at $T = 0$ in the absence of the external environment. In the effective theory, the information from the underlying physics is thus lost, and the type of the effective theory only depends on the symmetry and topology of the system.

Two parameters of the effective theory – λ and the equilibrium value ϕ_0 of the Higgs field – can be considered as phenomenological. Though these parameters are determined by the microscopic (Planck) physics, their values may essentially differ from the microscopic scales. Examples are provided by superconductivity of metals and superfluidity of Fermi liquids, whose energy scale is exponentially small compared to the corresponding microscopic energy scale E_F (the Fermi energy): $\lambda \phi_0^2 \sim \sim E_F^2 \exp(-1/g)$. Since the effective coupling g is typically small, $g \ll 1$, this leads to the macroscopic energy and length scales for the effective Ginzburg-Landau theory of superconductivity. In Rhodium metal, which has the lowest transition temperature observed in metals, $T_c \sim 0.3\text{mK}$ [11], one has $\lambda \phi_0^2 \sim 10^{-14} E_F^2$. In the fermionic superfluid liquid ^3He , the coupling g is small due to the many-body effects, and one has $\lambda \phi_0^2 \sim 10^{-6} E_F^2$; the superfluidity of the ^3He atoms in dilute ^3He - ^4He mixture has not yet been found, which suggests that in this system $\lambda \phi_0^2 < 10^{-8} E_F^2$.

Let us see what happens if we take into account the zero-point energy of quantum fields, including the zero-point energy of the Higgs field ϕ . In condensed matter this means that we quantize the system again, but now instead of the whole system of atoms we are only dealing with the collective bosonic and fermionic fields which enter the effective theory.

Introducing the quantum operators for the Higgs field

$$\hat{\phi} = \phi_0 + \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \left(\hat{\phi}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} + \text{c.c.} \right), \quad (18)$$

one obtains the Hamiltonian for the Higgs bosons:

$$\hat{H}_{\text{Higgs}} = E_0 + \frac{1}{2} \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \left(a_{\mathbf{k}} a_{\mathbf{k}}^\dagger + a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \right). \quad (19)$$

The vacuum energy contains now the zero-point energy of ϕ -field, and for completeness one must also add the zero-point energy of other bosonic fields of the Standard Model – the gauge fields – and the negative energy of the Dirac vacuum of fermions (quarks and leptons):

$$E_{zp} = \frac{1}{2} \sum_{\mathbf{k}b} \hbar\omega_{\mathbf{k}b} - \sum_{\mathbf{k}f} \hbar\omega_{\mathbf{k}f}. \quad (20)$$

However, from the condensed matter experience, it follows that we must forget about the zero-point energy of the effective fields. They have already been included into the vacuum energy E_0 together with the energies of microscopic degrees of freedom. Moreover, the total vacuum energy must be zero in equilibrium, which means that the trans-Planckian physics fully compensates the zero-point energy contribution of the sub-Planckian modes without any fine-tuning.

Let us suppose that we are not aware on this thermodynamic principle of perfect compensation. Then we must seriously consider the zero-point energy of bosonic and fermionic quantum fields (massless and massive) and estimate its contribution to radiative correction to the Higgs mass. Due to interaction of the Higgs field with gauge bosons and fermions, all the fermions and also W - and Z -bosons become massive:

$$\omega_{\mathbf{k}f}^2 = m_f^2 + c^2 k^2, \quad \omega_{\mathbf{k}b}^2 = m_b^2 + c^2 k^2, \quad (21)$$

and masses of fermions and bosons depend on the Higgs field

$$m_{f,b}^2 = \lambda_{f,b} \phi_0^2, \quad (22)$$

(for the Higgs field itself the corresponding $\lambda_H = \lambda$ in Eq.(17); while the photon remains massless, $\lambda_A = 0$). This dependence leads to the ϕ_0^2 term in the zero-point energy, which must be identified with the additional mass term for the Higgs field coming from the quantum effects:

$$\delta(m_H^2) = \frac{1}{2} \frac{d^2 E_{zp}}{d\phi_0^2} \approx \frac{1}{2} \sum_{\mathbf{k}b} \frac{\lambda_b}{\omega_{\mathbf{k}b}} - \sum_{\mathbf{k}f} \frac{\lambda_f}{\omega_{\mathbf{k}f}}. \quad (23)$$

The sum in Eq.(23) diverges quadratically. Neglecting fermion masses except for that of the heaviest fermion – the top quark with mass m_t – one obtains for the radiative correction to the Higgs mass:

$$\delta(m_H^2) \sim \lambda k_{uv}^2 \frac{m_H^2 + 2M_Z^2 + 4M_W^2 - 12m_t^2}{m_H^2}. \quad (24)$$

With $M_W \sim M_Z \sim m_t \sim m_H \sim 10^2 - 10^3 \text{ GeV}$, $\lambda \sim 1$ and $k_{uv} \sim 10^{16} - 10^{19} \text{ GeV}$, the quantum correction $\delta(m_H^2)$ is by many orders of magnitude bigger

than m_H^2 itself; this is the hierarchy problem in the Standard Model (see review paper [5]). From the condensed-matter point of view, the origin of this huge discrepancy is the same as in the cosmological constant problem: it is the diverging zero-point energy of quantum fields. The general form of the zero point energy in Eq.(20) is

$$E_{zp} = a_4 k_{uv}^4 + a_2 k_{uv}^2 m_H^2 + a_0 m_H^4 \ln(k_{uv}^2/m_H^2), \quad (25)$$

where $|a_4|$, $|a_2|$ and $|a_0|$ are of order unity. The main contribution of zero-point energy to the vacuum energy and cosmological constant diverges quartically, while its contribution to the Higgs mass diverges quadratically. The term with a_4 gives too large value for the vacuum energy, which in systems with gravity transforms to the large cosmological constant $\Lambda \sim k_{uv}^4$ leading to the main cosmological constant problem [3, 4]. The term with a_2 leads to the hierarchy problem in the Standard Model, since it gives $\delta(m_H^2) = (1/2)d^2 E_{zp}/d\phi_0^2 \sim k_{uv}^2$ according to Eq.(17). Both problems come from the same estimation of the vacuum energy as the zero point energy.

The exact calculation and also the thermodynamic analysis demonstrate that in equilibrium the total vacuum energy is zero, $E_{vac} = 0$, if the system is isolated from the environment. The zero-point energy of quantum fields, which is only a part of the whole energy of the quantum vacuum, is thus fully compensated by the microscopic degrees of freedom. This suggests the solution to the cosmological constant problem, and also makes doubtful the use of the zero-point energy for the estimation of the mass of Higgs boson, since the quadratically diverging term in the vacuum energy is also absorbed by microscopic physics to nullify the energy density of the equilibrium vacuum. The same thermodynamic principle which leads to the full compensation of the k_{uv}^4 term, leads to the full compensation of the k_{uv}^2 term. Thus the condensed matter suggests that the thermodynamic principle of the stability of the free vacuum provides the solution of both the cosmological constant problem and the hierarchy problem.

On the other hand, the logarithmically diverging term in Eq.(25) is within responsibility of the effective theory, and it may play an important role when the deviations from the equilibrium are considered.

7. Discussion. There are some lessons from the condensed matter, and from quantum hydrodynamics in particular, for the relativistic quantum fields and gravity. Because of the power-law divergences, the quantum hydrodynamics cannot be constructed. One can quantize the acoustic field to obtain its quanta – phonons – and use this only at the tree level. All other diagrams are not within the responsibility of the low-energy effective theory. Similarly, the quantum gravity can be used

only at the tree level. The classical energy functional (3) in hydrodynamics, as well as the classical Einstein action for gravity, and Eq.(16) for the Higgs field represent the final classical output of the whole quantum vacuum. They cannot be renormalized by the zero-point energy of the effective quantum fields, which present only a part of all the degrees of freedom of the quantum vacuum. The quadratically and quartically divergent terms are not within the jurisdiction of the effective low-energy theory alone. They must be considered using the microscopic theory of the vacuum state, which is known in condensed matter but is not known in particle physics.

However, the condensed matter suggests, that the whole quantum vacuum, which contains the zero-point motion of the effective quantum fields as well as the trans-Planckian degrees of freedom, obeys the thermodynamic laws. The latter state that the energy of the equilibrium free vacuum must be zero. This means that the huge energy of the zero-point motion of the effective fields is cancelled without any fine tuning by the trans-Planckian degrees of freedom. The exact cancellation of the quartic terms k_{uv}^4 in the vacuum energy provides the possible solution of the cosmological constant problem. Thus, when the whole vacuum is considered, the natural value of the cosmological constant Λ is zero in the free equilibrium vacuum. In the perturbed vacuum, Λ is determined not by the quantum zero-point energy, but by the classical macroscopic parameters of the Universe dynamics: the Hubble radius R of the Universe, the Newton constant G and the energy density of matter ρ_{mat} .

Similar exact cancellation of the quadratic terms k_{uv}^2 , which are responsible for the radiative corrections to the Higgs mass, provides the possible solution of the hierarchy problem in the Standard Model. The superconductivity in metals and superfluidity in Fermi liquids demonstrate, that the analog of the Higgs mass in these systems is typically much smaller than the relevant ultraviolet energy scale.

The main condensed-matter argument against the quantum gravity is that in condensed matter, the effective metric field is the low energy phenomenon, which naturally emerges together with gauge fields and chiral fermions in the low-energy corner [9]. At high energy the metric modes can no longer be separated from all other microscopic degrees of freedom of the quantum vacuum, and thus the pure quantum gravity cannot exist. Nevertheless there were attempts to construct the quantum gravity in terms of the metric field only, see e.g. Ref. [12] where the ultraviolet fixed point for quantum gravity has been derived. This makes sense under the following conditions: the ultraviolet fixed point must

occur much below the real microscopic energy scale, i.e. in the region where the metric field is still well determined; in the infrared limit the cosmological constant must be zero to match the thermodynamic requirement. One may look for the similar ultraviolet fixed point in quantum hydrodynamics, i.e. one can try to construct such a liquid in which the fixed point occurs at the intermediate length scale much bigger than the interatomic distance, where the macroscopic hydrodynamic description is still valid.

The possibility of such intermediate energy cut-off scale in the Standard Model has been discussed in Ref. [13]. It was suggested that the intermediate scale is close to the Planck scale, and the Newton constant is determined by this scale. In this scenario, the real microscopic ("atomic") energy scale where, say, the Lorentz invariance is violated, is much bigger. This scenario has many advantages. In particular the merging of the running couplings constants of the weak, strong, and electromagnetic fields occurs naturally and does not require the unification of these gauge fields at high energy.

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