Pis'ma v ZhETF, vol. 83, iss. 5, pp. 227 – 231 

# Quantum phase transitions and backbending in even-even $N \sim 90$ nuclei

*J. Kvasil, R. G. Nazmitdinov*[+*1)]

*Institute of Particle and Nuclear Physics, Charles University, CZ-18000 Praha 8, Czech Republic*

[+]*Departament de Física, Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain*

[*]*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia*

Submitted 24 October 2005
Resubmitted 12 December 2005

We extend the classical Landau theory for rotating nuclei and show that the backbending in $^{162}Yb$, that comes about as a result of the two-quasiparticle alignment, is identified with the second order phase transition. We found that the backbending in $^{156}$Dy, caused by the instability of $\gamma$-vibrations in the rotating frame, corresponds to the first order phase transition. We suggest the empirical rule to determine the type of the phase transition in rotating nuclei undergoing a backbending.

PACS: 05.70.Fh, 21.10.Re, 21.60.Jz, 27.70.+q

Quantum phase transitions, that occur at zero temperature as a function of some nonthermal control parameter, attract a considerable attention in recent years. Study of such transitions has been initiated in condensed matter physics in order to understand properties of low-dimensional systems [1] and quantum behavior in variety of materials at critical points [2]. In nuclear structure, quantum phase transitions are associated with transitions between different shapes or geometric configurations of a chain of nuclei in the ground state (cf. [3, 4]). Spectroscopic data clearly indicate on the transition from spherical to deformed shapes if one starts from a nucleus, that has all occupied shells in framework of the nuclear shell model, towards to the one with a half filled next shell [5]. Most studies of the quantum phase transitions in nuclei, based on the interacting boson model (IBM) [3, 6, 7], confirm the validity of the classical Landau theory [8] for description of shape-phase transitions in the ground state.

Although the IBM can be easily extended to a thermodynamical limit $N \to \infty$, which is well suitable for the study of phase transitions, the analysis is rather oversimplified. In fact, single-particle degrees of freedom, quite important in finite systems, are not properly taken into account in the model. A general trend found for the ground shape transitions is less affected by this degree of freedom. However, it may be crucial for the study of quantum phase transitions in rotating nuclei, where statical and dynamical properties are coupled. For example, with an increase of a rotational frequency, a rotational alignment of angular momenta of a nucleon pair, occupying a high-j intruder orbital near the Fermi surface, along the axis of collective rotation takes place. The alignment breaks a singlet Cooper pairing in the pair and decreases the superfluidity of a rotating nucleus. The effect of rotation is similar to the effect of magnetic field on a superconductor, as was noticed long ago [9]. In general, the alignment is considered as a main driving force that leads to a sudden increase of a nuclear kinematical moment of inertia $\Im^{(1)} = I/\Omega$ of the lowest (yrast) level sequence as a function of a rotational frequency $\Omega$, so-called, the backbending phenomenon (see Ref.[5] and experimental results for $^{156}$Dy and $^{162}$Yb in Fig.1). While one observes a similar picture for the backbending in the considered nuclei (see Fig.1a,b), a different response of a nuclear field upon the rotation becomes more evident with aid of the experimental dynamical moment of inertia $\Im^{(2)} = dI/d\Omega \approx 4/\Delta E_\gamma$ as a function of the angular frequency (see Fig.1c,d). Indeed, the dynamical moment of inertia, due to obvious relation $\Im^{(2)} = \Im^{(1)} + \Omega d\Im^{(1)}/d\Omega$, is very sensitive to structural changes of a nuclear field. At the transition point $\Im^{(2)}$ wildly fluctuates with a huge amplitude in $^{156}$Dy, whereas these fluctuation are quite mild in $^{162}$Yb. Continuing the analogy between the backbending and the behavior of superconductors in a magnetic field, the dynamical moment of inertia is similar to a susceptibility of a sample. One of the purposes of the present Letter is to interpret the experimental results in $^{156}$Dy and $^{162}$Yb (see Fig.1) from perspective of the Landau theory of phase transitions, taking care upon the single-particle degrees of freedom. We will show that the interplay be-

1)e-mail: rashid@thsun1.jinr.ru

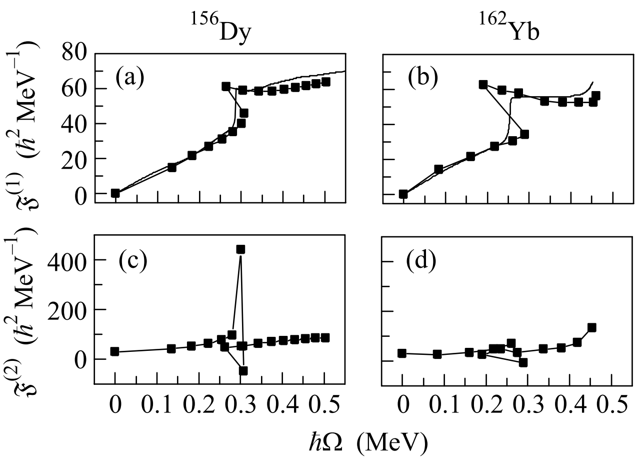


Fig.1. Rotational behavior of the experimental, kinematical $\Im^{(1)} = I/\Omega$ and dynamical $\Im^{(2)} \approx 4/\Delta E_\gamma$ moments of inertia. Here, $\hbar\Omega = E_\gamma/2$, $E_\gamma$ is the $\gamma$-transition energy between two neighboring states that differ on two units of the angular momentum $I$ and $\Delta E_\gamma$ is the difference between two consecutive $\gamma$-transitions. The experimental data denoted by black squares are taken from [10]. The experimental rotational frequency at the transition point is $\hbar\Omega_c \approx 0.27, 0.32$ MeV for $^{162}$Yb and $^{156}$Dy, respectively. The results of calculations for $\Im^{(1)}$ on plates (a) and (b) are connected by a solid line

tween the alignment and quantum fluctuations of the order parameter determines the type of the shape-phase transition at the backbending.

The phase transition is usually detected by means of an order parameter as a function of a control parameter. In many cases, an order parameter is obtained with aid of a model consideration. In particular, in rotating nuclei one can suggest a few order parameters like deformation parameters of a nuclear effective potential, $\beta$ and $\gamma$, that characterize the geometrical configuration (cf. [3, 4]), – as a function of the rotational frequency, i.e., the control parameter. To analyze the experimental data of above nuclei, we use a cranked Hamiltonian

$$H_\Omega = H - \sum_{\tau=N,P} \lambda_\tau \widehat{N}_\tau - \Omega \widehat{J}_x + H_{\rm int}. \qquad (1)$$

The term $H = H_N + H_{\rm add}$ contains the Nilsson Hamiltonian $H_N$ and the additional term that restores the local Galilean invariance of the Nilsson potential in the rotating frame. The Nilsson potential naturally incorporates the deformation parameters of a nuclear shape [5]. The interaction includes separable monopole pairing, double stretched quadrupole-quadrupole (QQ) and monopole-monopole terms. The details about the model Hamiltonian (1) can be found in Refs.[11, 12].

We solve the Hartree-Bogoliubov (HB) equations for the Hamiltonian (1) self-consistently. However, in the vicinity of the backbending, the solution of nonlinear HB equations becomes highly unstable. In order to avoid unwanted singularities for certain values of $\Omega$, we followed the phenomenological prescription [13] for the definition of the pairing gap parameter (see details in Refs. [11, 12]). Parameters of the Nilsson potential were taken from Ref. [14]. In our calculations we include all shells up to $N = 9$. Near the transition point we extended our configuration space up to $N = 10$ shells. The difference between results from the former and the latter cases was small and all presented results are obtained with $N = 0 - 9$ shells. In contrast to standard calculations with the Nilsson potential, based on a "single stretched" coordinate method (cf. [5]), we take into account the $\Delta N = 2$ mixing exactly, which improves the accuracy of the mean field calculations. The consistency between the mean field and the residual interactions of the Hamiltonian (1) was achieved by varying the strength constants of the pairing and $QQ$ interactions in the random phase approximation (RPA). It results in the separation collective excitations from those that are related to the symmetries broken by the mean field. Among them are the conservation of particle numbers and space symmetries. The details about the RPA approach will be presented in forthcoming publication. Here, we only use the results of the RPA analysis for the lowest non-spurious quadrupole mode (phonon) in $^{156}$Dy and $^{162}$Yb, since it is related to the shape-phase transition.

The results for rotational dependence of equilibrium deformation parameters $\beta$ and $\gamma$ exhibit a transition to the triaxiality $\gamma \neq 0$ at $I = 8\hbar \to I = 10\hbar$ in $^{162}$Yb and $I = 14\hbar \to I = 16\hbar$ in $^{156}$Dy (see Fig.2). We found

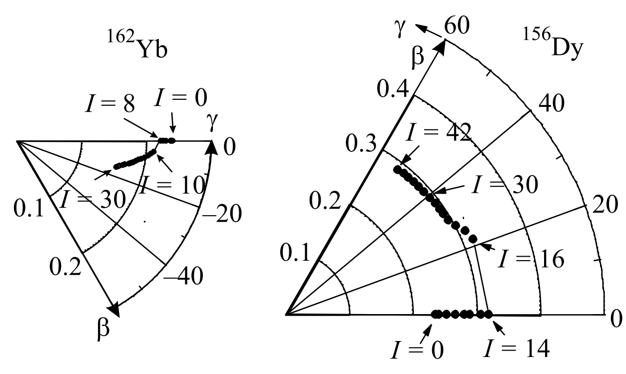


Fig.2. Equilibrium deformations in $\beta - \gamma$ plane as a function of the angular momentum $I = \langle \widehat{J}_x \rangle - 1/2$ (in units of $\hbar$)

a stable, single minimum $E_\Omega(\beta,\gamma) = \langle H_\Omega \rangle$ in $^{162}$Yb that slowly moves on the potential energy surface $(\beta,\gamma)$ from the axially symmetric shape to the triaxial one with the increase of the rotational frequency. In contrast, in $^{156}$Dy at the vicinity of the transition point, we have obtained a coexistence of the axially symmetric ($\gamma = 0$) and non-axial ($\gamma \neq 0$) configurations. Slightly above the transition point, the configuration suddenly changes from the axially symmetric into the triaxial one.

While the rotational evolution of the mean field energy $E_\Omega(\beta, \gamma)$ is relatively smooth at the transition point $\hbar\Omega_c \approx 0.25$ MeV in $^{162}$Yb, there is an abrupt change of this energy at the transition point $\hbar\Omega_c \approx 0.3$ MeV for $^{156}$Dy (see Fig.3). The agreement between experimental and calculated values for the kinematical moment of inertia (Fig.1), the critical rotational frequency at the transition point (Fig.3) is quite remarkable. We emphasize that the additional term (see Ref. [12]), restoring the local Galilean invariance, plays a crucial role to achieve such an agreement.

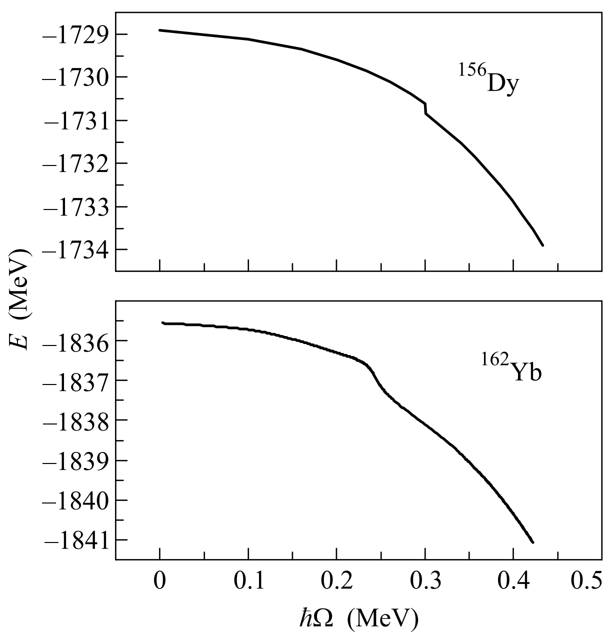


Fig.3. The mean field energy minimum $E_\Omega(\beta, \gamma) = \langle H_\Omega \rangle$ as a function of the rotational frequency $\hbar\Omega$ for $^{156}$Dy (upper panel) and $^{162}$Yb (lower panel)

To elucidate the different character of the shape transition from axially symmetric to the triaxial shape, we consider potential landscape sections in the vicinity of the shape transition. It is frequently argued that the potential landscape has to be akin that of classical critical phase transitions (cf. [3, 4, 6]). Since we analyze a shape transition from the axially symmetric shape ($\gamma = 0$) to the triaxial one ($\gamma \neq 0$), we choose the deformation parameter $\gamma$ as the order parameter that reflects the broken axial symmetry. Such a choice is well justified, since the deformation parameter $\beta$ preserves its value before and after the shape transition in both nuclei: $\beta_t \approx 0.2$ for $^{162}$Yb and $\beta_t \approx 0.31$ for $^{156}$Dy. Thus, we consider a mean field value of the cranking Hamiltonian, $E_\Omega(\gamma; \beta_t) \equiv \langle H_\Omega \rangle$, for different values of $\Omega$ (our state variable) and $\gamma$ (order parameter) at fixed value of $\beta_t$.

For $^{156}$Dy we observe the emergence of the order parameter $\gamma$ above the critical value $\Omega_c$ of the control parameter $\Omega$ (see a top panel in Fig.4). Below and above the transition point there is a unique phase whose properties are continuously connected to one of the coexistent phases at the transition point. The order parameter changes discontinuously as the nucleus passes through the critical point from axially symmetric shape to the triaxial one. This transition posses typical features of the classical first order phase transition.

The RPA analysis of $\gamma$-vibrational (lowest) excitations of the positive signature in the vicinity of the shape transition demonstrates a collective nature of this soft mode. Although two-quasiparticle states align their angular momenta along the axis $x$ (collective rotation), the axial symmetry persists till the transition point (see also discussion in Ref. [11]). The mode blocks a transition to the triaxial shape. However, at the transition point, this mode is anomalously low in the rotating frame. To see the connection of this fact to the phase transition let us consider an axially deformed system, defined by the Hamiltonian $\tilde{H}$ in the laboratory frame, that rotates about a symmetry axis z with a rotational frequency $\Omega$. The angular momentum is a good quantum number and, consequently, $[\hat{J}_z, O_K^\dagger] = K O_K^\dagger$. Here, the phonon $O_K^\dagger$ describes the vibrational state with $K$ being the value of the angular momentum carried by the phonons $O_K^\dagger$ along the symmetry axis, $z$ axis. Thus, one obtains

$$[H_\Omega, O_K^\dagger] = [\tilde{H} - \Omega\hat{J}_z, O_K^\dagger] = (\tilde{\omega}_K - K\Omega) O_K^\dagger \equiv \omega_K O_K^\dagger. \quad (2)$$

Here, $\tilde{\omega}_K$ is the phonon energy of the mode K in the laboratory frame at $\Omega = 0$. This equation implies that at the rotational frequency $\Omega_{cr} = \tilde{\omega}_K / K$ one of the RPA frequencies $\omega_K$ vanishes in the rotating frame [15 − 17]. At this point of bifurcation we could expect *a spontaneously symmetry breaking effect of the rotating mean field due to the appearance of the Goldstone boson related to the multipole-multipole forces with quantum number* $K$. For an axially deformed system, one obtains the breaking of the axial symmetry, since the lowest critical frequency corresponds to $\gamma$-vibrations with $K = 2$ [15, 17]. The rotation around the x axis couples, however, all modes and, therefore, affects the actual value of the critical rotational frequency. Thus, we have anomalously low value of $\gamma$-vibrations (fluctuations of the order parameter $\gamma$) coupled to the other modes, which should be respon-

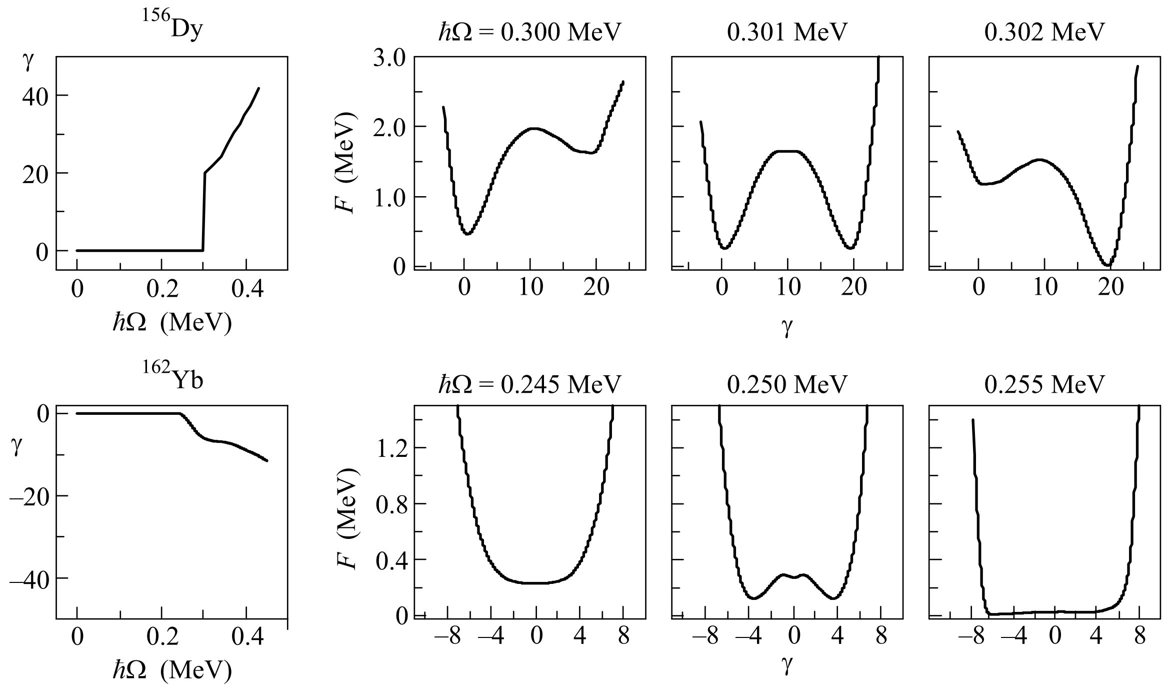


Fig.4. The rotational dependence of the order parameter $\gamma$ (in degrees) and the energy surfaces sections $F(\Omega,\gamma) = E_\Omega(\gamma,\beta_t) - E_{\min}$ for $^{156}$Dy (top) and $^{162}$Yb (bottom) before and after the transition point. The energy is given relative to the value $E_{\min} = E_\Omega(\beta_t,\gamma)$ at $\hbar\Omega = 0.255$, 0.302 MeV for $^{162}$Yb, $^{156}$Dy, respectively

sible for this transition of the first order. Indeed, the results are in a good agreement with the experimental data: a half of an experimental value for $\gamma$-vibrations $\hbar\tilde{\omega}_{K=2}/2 \equiv \hbar\Omega_{cr} = 0.345\,\text{MeV}$ is close to the collective rotational frequency $\hbar\Omega_c = 0.301\,\text{MeV}$ at which the shape transition occurs. A drastic change of the mean field configuration leads to large fluctuations of the dynamical moment of inertia at the transition point, since $\Im^{(2)} = -d^2E_\Omega/d^2\Omega$. A full treatment of $\Im^{(2)}$ can be done only in the RPA (cf. [11]), which is beyond the scope of the paper and will be presented elsewhere. The observed phenomenon resembles very much the structural phase transition discussed within the anharmonic Landau-type model in solid state physics [18]. In the latter case a generic first-order Landau model explains the jump in the order parameter, when a standart soft-mode theory of the continuos transitions is not applicable (see also Ref. [2]).

In the case of $^{162}$Yb the energy $E(\Omega;\gamma)$ (Fig.3) and the order parameter (Fig.4) are smooth functions in the vicinity of the transition point $\Omega_c$. This implies that two phases, $\gamma = 0$ and $\gamma \neq 0$, on either side of the transition point should coincide. Therefore, for $\Omega$ near the transition point $\Omega_c$ we can expand our functional $F(\Omega,\gamma) = E_\Omega(\gamma,\beta_t) - E_{\min}$ (see Fig.4) in the form

$$F(\Omega;\gamma)=F_1(\Omega)\gamma+F_2(\Omega)\gamma^2+F_3(\Omega)\gamma^3+F_4(\Omega)\gamma^4+\ldots \quad (3)$$

The conditions of the phase equilibrium, $\partial F/\partial\gamma = 0$ and $\partial^2F/\partial\gamma^2 \geq 0$, that should be fulfilled for all values of $\Omega$ and $\gamma$ (including $\gamma = 0$), yield $F_1(\Omega) = 0$. As a result, we obtain from the above conditions the inequality $2F_4(\Omega)\gamma^2 \geq F_2(\Omega)$. This inequality holds for all values of $\gamma$ (including $\gamma = 0$ at $\Omega = \Omega_c$), which leads to $F_2(\Omega = \Omega_c) \leq 0$. On the other hand, from the stability condition $\partial^2F/\partial\gamma^2 \geq 0$ at the transition point $\Omega_c$ and $\gamma = 0$, we also have $F_2(\Omega = \Omega_c) \geq 0$. The both inequalities can coincide only when $F_2(\Omega = \Omega_c) = 0$. Using the result $F_1(\Omega_c) = F_2(\Omega_c) = 0$ and the fact that all phases at the transition point should coincide, we obtain from $\partial F/\partial\gamma = 0$ that $F_3(\Omega = \Omega_c) = 0$. Assuming that $F_3 = 0$ for all $\Omega$, the minimum condition $\partial F/\partial\gamma = 0$ yields the following solution for the order parameter

$$\gamma_1 = 0\,, \quad \gamma_{2,3}^2 = -\frac{F_2(\Omega)}{2\,F_4(\Omega)} = \begin{cases} \neq 0 & \text{for } \Omega \neq \Omega_c \\ = 0 & \text{for } \Omega = \Omega_c \end{cases} \quad (4)$$

Since at the transition point $F_2(\Omega_c) = 0$, one can propose the following definition of the function $F_2(\Omega)$:

$$F_2(\Omega) \approx \frac{dF_2(\Omega)}{d\Omega}\,(\Omega - \Omega_c)\,. \quad (5)$$

Thus, we have $\gamma \sim (\Omega - \Omega_c)^\nu$ and the critical exponent $\nu = 1/2$, in accord with the classical Landau theory, where the temperature is replaced by the rotational frequency.

Our extension of the Landau-type approach for rotating nuclei is nicely confirmed by the numerical results. The polynomial fit of the energy potential surfaces for $^{162}$Yb (Fig.4) yields $F_1(\Omega) = F_3(\Omega) = 0$ for all considered values of the rotational frequencies and $F_2(\Omega_c) = 0$ at $\hbar\Omega_c = 0.250$ MeV. Moreover, in the vicinity of $\Omega_c$ we obtain $dF_2(\Omega)/\hbar d\Omega < 0$ ($F_4(\Omega) > 0$ for all $\Omega$). In an agreement with Eqs.(4),(5), we have only the phase $\gamma = 0$ for $\hbar\Omega < \hbar\Omega_c$ and the phase $\gamma \neq 0$ for $\hbar\Omega > \hbar\Omega_c$. The energy surfaces are symmetric with regard of the sign of $\gamma$ and this also supports the idea that the effective energy $F$ can be expressed as an analytic function of the order parameter $\gamma$. Thus, the backbending in $^{162}$Yb can be classified as a phase transition of the second order. The smooth behaviour of the function $F$ at the transition point implies a small amplitude of fluctuations of the dynamical moment of inertia. The RPA analysis of the lowest $\gamma$-vibrational mode in $^{162}$Yb indicates on the breakdown of the quadrupole phonon. At the vicinity of the transition point one proton and one neutron two-quasiparticle components dominate ($\sim 95\%$) in the phonon structure and the backbending is caused by the alignment of the neutron two-quasiparticle configuration.

Summarizing, we have established the connection between the backbending and the quantum shape-phase transition in $^{156}$Dy, caused by the instability of gamma vibrations in the rotating frame. This is a novel mechanism that complements the standard single-particle one, related to the alignment. We propose to consider *the onset of the $\gamma$-instability at the transition point, accompanied with large fluctuations of the dynamical moment of inertia*, that exceed by few times the value of the kinematical moment of inertia, as a *manifestation of a shape-phase transition of the first order.* We extend the classical Landau theory for the description of continuos shape-phase transitions that occur at the backbending. Applying this theory to the description of the backbending in $^{162}$Yb, caused by the alignment of the two-quasiparticle configuration, we found that the shape-phase transitions carries all features of the second order phase transition. We conclude: *if the amplitude of the fluctuations of the dynamical moment of inertia at the critical point is of the same order of magnitude as the value of the kinematical moment of inertia, a backbending may be associated with a quantum shape-phase transition of the second order.*

We thank P. Cejnar and D. Sanchez for useful discussions. This work is a part of the research plan MSM 0021620834 supported by the Ministry of Education of the Czech Republic. It is also partly supported by Grant # BFM2002-03241 from DGI (Spain). R. G. N. gratefully acknowledges support from the Ramón y Cajal programme (Spain).


1. S. Sachdev, *Quantum Phase Transitions*, Cambridge University Press, Cambridge, 1999.
2. D. Belitz, T. R. Kirkpatrick, and T. Vojta, Rev. Mod. Phys. **77**, 579 (2005).
3. J. Jolie et al., Phys. Rev. Lett **89**, 182502 (2002).
4. F. Iachello, Phys. Rev. Lett. **91**, 132502 (2003); F. Iachello and N. V. Zamfir, ibid **92**, 212501 (2004).
5. S. G. Nilsson and I. Ragnarsson, *Shapes and Shells in Nuclear Structure*, Cambridge University Press, Cambridge, 1995.
6. P. Cejnar and J. Jolie, Phys. Rev. C **69**, 011301(R) (2004).
7. R. V. Jolos, Phys. At. Nuclei **67**, 931 (2004).
8. L. D. Landau and E. M. Lifshitz, *Statistical Physics, Cource of Theoretical Physics*, Pergamon Press, Oxford, 1980, Part 1, Vol. V.
9. B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. **5**, 511 (1960).
10. http://www.nndc.bnl.gov/nudat2/.
11. J. Kvasil and R. G. Nazmitdinov, Phys. Rev. C **69**, 031304(R) (2004).
12. J. Kvasil, N. Lo Iudice, R. G. Nazmitdinov et al., Phys. Rev. C **69**, 064308 (2004).
13. R. Wyss, W. Satula, W. Nazarewicz, and A. Johnson, Nucl. Phys. A **511**, 324 (1990).
14. A. K. Yain et al., Rev. Mod. Phys. **62**, 393 (1990).
15. E. R. Marshalek and R. G. Nazmitdinov, Phys. Lett. B **300**, 199 (1993).
16. E. R. Marshalek, Phys. Rev. C **54**, 159 (1996).
17. W. D. Heiss and R. G. Nazmitdinov, JETP Lett. **72**, 106 (2000); Phys. Rev. C **65**, 054304 (2002).
18. J. A. Krumhansl and R. J. Gooding, Phys. Rev. B **39**, 3047 (1989).