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Glue in the pomeron from nonlinear $k_\perp$-factorization

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We derive the nonlinear $k_\perp$-factorization for the spectrum of jets in high-mass diffractive deep inelastic scattering as a function of three hard scales – the virtuality of the photon Q^2, the transverse momentum of the jet and the saturation scale Q_A. In contrast to all other hard reactions studied so far, we encounter a clash between the two definitions of the glue in the pomeron – from the inclusive spectrum of leading quarks and the small-β evolution of the diffractive cross section. This clash casts a further shadow on customary applications of the familiar collinear factorization to a pQCD analysis of diffractive deep inelastic scattering.

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The high-mass, or leading $\log\frac{1}{\beta}$ ($\mathrm{LL}\frac{1}{\beta}$) evolution of Diffractive Deep Inelastic Scattering (DDIS) is subtle and remains under scrutiny [1 – 13], for the discussion in connection with the planned electron-ion collider eRHIC see [14]). When cast in the Good-Walker-Miettinen-Pumplin form [15], and upon the proper renormalization of wave functions, the inclusive coherent forward DDIS resembles a sum of differential cross sections of elastic scattering of multiparton Fock states of the photon [1, 2]. As such, DDIS is an inherently nonlinear functional of the color dipole scattering amplitude to be contrasted to the linear one in inclusive DIS [2, 16]. A customary treatment of DDIS as DIS off an isolated pomeron, viewed as a hadronic state with the well defined flux in the target, an indiscriminate collinear factorization analysis of DDIS final states and conclusions on glue in the pomeron from such an analysis ([17] and references therein) – all must be taken with a grain of salt, especially in a regime dominated by multipomeron exchanges, as e.g. nuclear targets with a large saturation scale. Here we present an analysis of glue in the pomeron as inferred from the $\mathrm{LL}\frac{1}{\beta}$ evolution of, and jets in, high-mass DDIS which strengthens these reservations.

In the pQCD approach to DDIS, $\gamma^* p \to X + p'$ and $\gamma^* A \to X + A'$, high-mass DDIS starts with $X = q\bar{q}g$. Our basic points derive from the comparison of nonlinear $k_\perp$-factorization for jets and $\mathrm{LL}\frac{1}{\beta}$ evolution properties of the $q\bar{q}g$ final states at very small DDIS Bjorken variable $\beta = Q^2/(Q^2 + M^2) \ll 1$, where M is the invariant mass of the diffractive state. As far as the $\mathrm{LL}\frac{1}{\beta}$ evolution properties of DDIS are concerned, one may insist on the linear color dipole representation ([2], see also the subsequent [8, 9, 13]) similar to that for inclusive DIS [16]. In 1993, such a color dipole form was derived within the double leading logarithm approximation (DLLA) and reinterpreted as a proof of the familiar DLLA evolution of DDIS [2]. Evidently, DLLA is inadequate for DDIS off nuclei (and nucleons) in the most interesting regime of saturation. Furthermore, a utility of the emerging operational definition of the glue in the pomeron, especially its $k_\perp$-factorization and unitarity properties, remain unspecified. Indeed, the fundamental point about inclusive DIS is that one and the same unintegrated glue provides the linear $k_\perp$-factorization description of both the $\mathrm{LL}\frac{1}{x}$ evolution and the leading quark spectra ([18] and references therein). Recently, this property has been shown to extend to hadron-nucleus collisions, too [19]. In this communication we show that for high-mass DDIS the same property is not borne out by a rigorous nonlinear $k_\perp$-factorization for the final states and the evolution of the DDIS cross section. Besides the identification of this clash, we report a full analytic description of the spectra of leading jets in both the photon and pomeron fragmentation regions of DDIS as a function of three hard scales – the virtuality of the photon Q^2, the transverse momentum of the jet and the saturation scale Q_A – which extends earlier limited considerations [6, 8, 11].

The first discussion of the $q\bar{q}g$ excitation goes back to the derivation of the color dipole BFKL equation [2, 20]. In the impact parameter space the excitation amplitude equals $\mathcal{A}(\mathbf{b}, q\bar{q}g) = [\mathsf{S}_{q\bar{q}g}(\mathbf{b};\boldsymbol{\rho},\mathbf{r}) - \mathsf{S}_{q\bar{q}}(\mathbf{b};\mathbf{r})]\Psi_{q\bar{q}g}(\boldsymbol{\rho},\mathbf{r})$, where $\Psi_{q\bar{q}g}(\boldsymbol{\rho},\mathbf{r})$ is the lightcone wave function of the $q\bar{q}g$ state, $\mathbf{r}$ and $\boldsymbol{\rho}$ are the $q\bar{q}$ and qg dipoles, $\mathsf{S}_{q\bar{q}g}(\mathbf{b};\boldsymbol{\rho},\mathbf{r})$ and $\mathsf{S}_{q\bar{q}}(\mathbf{b};\mathbf{r})$ are the S-matrices of the $q\bar{q}g$ and $q\bar{q}$ Fock states at the impact parameter $\mathbf{b}$ (we follow the notations of [2, 18, 21, 22]). Inclusive forward DDIS off nucleons (N) evolves as

$$16\pi \left.\frac{d\sigma_N^D}{dt}\right|_{t=0} = \langle q\bar{q}|\sigma_{q\bar{q}}^2|q\bar{q}\rangle + \langle q\bar{q}g|\sigma_{q\bar{q}g}^2 - \sigma_{q\bar{q}}^2|q\bar{q}g\rangle + \dots \tag{1}$$

For heavy nuclei (A) the integration over the momentum transfer can be carried out explicitly and we consider the coherent DDIS cross section per unit area in the impact parameter space:

$$\frac{d\sigma_A^D}{d^2\mathbf{b}} = \langle q\bar{q}||1-\mathsf{S}_{q\bar{q}}|^2|q\bar{q}\rangle + \\ + \langle q\bar{q}g||1-\mathsf{S}_{q\bar{q}g}|^2 - |1-\mathsf{S}_{q\bar{q}}|^2|q\bar{q}g\rangle + \dots \tag{2}$$

Apart from the excitation of the physical $q\bar{q}g$ state, the $q\bar{q}g$ contribution to (1), (2) describes also the $\mathrm{LL}\frac{1}{\beta}$ evolution of the $q\bar{q}$ excitation [8]. For both targets, the transverse momenta of jets in the $q\bar{q}g$ final state add to zero: $\mathbf{p}_q + \mathbf{p}_{\bar{q}} + \mathbf{p}_g = 0$. Hereafter $\mathbf{p} \equiv \mathbf{p}_q$, $\mathbf{q} \equiv \mathbf{p}_g$, the rapidity gap variable $x_{\mathbf{I\!P}} = x/\beta$ is a fraction of nucleon's momentum carried by the pomeron, z is the Feynman variable – a fraction of the photon's lightcone momentum carried by the quark.

We derive nonlinear $k_\perp$-factorization directly for the observed DDIS cross section without the separation of the ill-defined flux of pomerons which is not borne out by the pQCD treatment of DDIS [5]. One of the building blocks is the lightcone wave function of the $q\bar{q}$ state of the photon and qg Fock state of the quark. For transverse photons, in terms of the QED splitting function $P_{q\gamma}(z) = 2N_c e_f^2 \alpha_{em}[z^2 + (1-z)^2]$ one has $|\Psi_{q\bar{q}}(z,\mathbf{p}) - \Psi_{q\bar{q}}(z,\mathbf{p}-\boldsymbol{\kappa})|^2 = P_{q\gamma}(z)\left|\boldsymbol{\psi}(\varepsilon^2,\mathbf{p}) - \boldsymbol{\psi}(\varepsilon^2,\mathbf{p}-\boldsymbol{\kappa})\right|^2$, where $\boldsymbol{\psi}(\varepsilon^2,\mathbf{p}) = \mathbf{p}/(\varepsilon^2+\mathbf{p}^2)$ and $\varepsilon^2 = z(1-z)Q^2 + m_f^2$ (we focus on light flavors, a simple extension to longitudinal photons and heavy flavors will be reported elsewhere). In high-mass DDIS gluons are soft, $z_g \ll 1$, and $\Psi_{qg}(z_g,\mathbf{p})$ does not depend on the virtuality of the parent quark, $|\Psi_{qg}(z_g,\mathbf{p}) - \Psi_{qg}(z_g,\mathbf{p}-\boldsymbol{\kappa})|^2 = 4\alpha_S C_F \left|\boldsymbol{\psi}(\mu^2,\mathbf{p}) - \boldsymbol{\psi}(\mu^2,\mathbf{p}-\boldsymbol{\kappa})\right|^2/z_g$, where the (optional) infrared parameter μ models the finite propagation radius of perturbative gluons [2, 20]. The technique of Refs. [18, 21, 22] gives the nonlinear $k_\perp$-factorization master formulas (for nuclei we cite the leading term of the large-N_c perturbation theory, where N_c is the number of colors)

$$\frac{\partial}{\partial \log\frac{1}{\beta}} \left.\frac{d\sigma_N^D}{dz d^2\mathbf{p} d^2\mathbf{q} dt}\right|_{t=0} = \\ = \frac{1}{16\pi(2\pi)^4}\cdot\left(\frac{C_A}{2C_F}\right)^2 \cdot 4\alpha_S C_F P_{q\gamma}(z) \times \\ \times \left|\int d^2\boldsymbol{\kappa} f(x_{\mathbf{I\!P}},\boldsymbol{\kappa}) H_{ij}^N(\boldsymbol{\kappa},\mathbf{q},\mathbf{p})\right|^2, \tag{3}$$

$$\frac{\partial}{\partial \log\frac{1}{\beta}} \frac{d\sigma_A^D}{dz d^2\mathbf{b} d^2\mathbf{p} d^2\mathbf{q}} = \frac{1}{(2\pi)^4} 4\alpha_S C_F P_{q\gamma}(z) \times \\ \times \left|\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 \Phi(\mathbf{b},x_{\mathbf{I\!P}},\boldsymbol{\kappa}_1)\Phi(\mathbf{b},x_{\mathbf{I\!P}},\boldsymbol{\kappa}_2) \times \right. \\ \left. \times H_{ij}^A(\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2,\mathbf{q},\mathbf{p})\right|^2, \tag{4}$$

where

$$H_{ij}^N(\boldsymbol{\kappa},\mathbf{q},\mathbf{p}) = \\ = \psi_i(\mu^2,\mathbf{q})\Big\{[\psi_j(\varepsilon^2,\mathbf{p}+\mathbf{q}) - \psi_j(\varepsilon^2,\mathbf{p})] + \\ + [\psi_j(\varepsilon^2,\mathbf{p}-\boldsymbol{\kappa}+\mathbf{q}) - \psi_j(\varepsilon^2,\mathbf{p}-\boldsymbol{\kappa})]\Big\} - \\ - \{\mathbf{q}\to\mathbf{q}+\boldsymbol{\kappa}\}, \tag{5}$$

$$H_{ij}^A(\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2,\mathbf{q},\mathbf{p}) = \psi_i(\mu^2,\mathbf{q})[\psi_j(\varepsilon^2,\mathbf{p}-\boldsymbol{\kappa}_1+\mathbf{q}) - \\ - \psi_j(\varepsilon^2,\mathbf{p}-\boldsymbol{\kappa}_1)] - \{\mathbf{q}\to\mathbf{q}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2\}, \tag{6}$$

the free-nucleon glue $f(x_{\mathbf{I\!P}},\boldsymbol{\kappa})$ is related to the $q\bar{q}$ color dipole cross section by $\sigma(x,\mathbf{r}) = \int d^2\boldsymbol{\kappa} f(x,\boldsymbol{\kappa})[1 - \exp(i\boldsymbol{\kappa}\cdot\mathbf{r})]$, $\Phi(\mathbf{b},x_{\mathbf{I\!P}},\boldsymbol{\kappa}) = \mathsf{S}[\mathbf{b};\sigma_0(x_{\mathbf{I\!P}})]\delta^{(2)}(\boldsymbol{\kappa}) + \phi(\mathbf{b},x_{\mathbf{I\!P}},\boldsymbol{\kappa})$ is defined in terms of the nuclear S-matrix for the $q\bar{q}$ dipole [18, 21, 22], $\mathsf{S}[\mathbf{b};\sigma_0] = \exp[-\frac{1}{2}\sigma_0 T(\mathbf{b})]$, $T(\mathbf{b})$ is the optical thickness of the nucleus and $\sigma_0(x_{\mathbf{I\!P}}) = \int d^2\boldsymbol{\kappa} f(x_{\mathbf{I\!P}},\boldsymbol{\kappa})$. For heavy nuclei a useful analytic approximation is [18] $\Phi(\mathbf{b},x_{\mathbf{I\!P}},\boldsymbol{\kappa}) \approx Q_A^2(\mathbf{b},x_{\mathbf{I\!P}})/\pi(Q_A^2(\mathbf{b},x_{\mathbf{I\!P}})+\mathbf{q}^2)^2$, where $Q_A^2(\mathbf{b},x) \approx \frac{4\pi^2}{N_c}\alpha_S(Q_A^2)G_N(x,Q_A^2)T(\mathbf{b})$ is the saturation scale [18] and $G_N(x,Q^2)$ is the integrated glue of the free nucleon. For $\mathbf{q}^2 \gg Q_A^2$ we shall often encounter

$$\int^{\mathbf{q}^2} d^2\boldsymbol{\kappa}\, \boldsymbol{\kappa}^2 \Phi(\mathbf{b},x_{\mathbf{I\!P}},\boldsymbol{\kappa}) \approx \\ \approx \frac{1}{2}Q_A^2(\mathbf{b},x_{\mathbf{I\!P}})\frac{\alpha_S(\mathbf{q}^2)G_N(x,\mathbf{q}^2)}{\alpha_S(Q_A^2)G_N(x,Q_A^2)}. \tag{7}$$

The jet-integrated DDIS cross section can be cast in the color dipole form,

$$\frac{\partial}{\partial\log\frac{1}{\beta}} \left.\frac{d\sigma_N^D}{dt}\right|_{t=0} = \langle q\bar{q}|\sigma_N^{\mathbf{I\!P}}(x_{\mathbf{I\!P}},\beta,\mathbf{r})|q\bar{q}\rangle, \tag{8}$$

which defines the $\mathrm{LL}\frac{1}{\beta}$ unintegrated glue in the pomeron via $\sigma_N^{\mathbf{I\!P}}(x_{\mathbf{I\!P}},\beta,\mathbf{r}) = \int d^2\boldsymbol{\kappa} f^{\mathbf{I\!P}}(x_{\mathbf{I\!P}},\beta,\boldsymbol{\kappa})\cdot[1-\exp(i\boldsymbol{\kappa}\cdot\mathbf{r})]$ (a somewhat odd dimension, $[\sigma_N^{\mathbf{I\!P}}] = [\mathrm{mb}\cdot\mathrm{GeV}^{-2}]$ is unimportant):

$$f_N^{\mathbf{IP}}(x_{\mathbf{IP}},\beta,\mathbf{p}) = \frac{1}{16\pi}\cdot\frac{C_A}{C_F}\cdot\frac{C_A\alpha_S}{2\pi^2}\times$$
$$\times\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2\Big\{f(x_{\mathbf{IP}},\boldsymbol{\kappa}_1)f(x_{\mathbf{IP}},\boldsymbol{\kappa}_2)\times$$
$$\times\Big[4K(\mathbf{p},\mathbf{p}+\boldsymbol{\kappa}_1)-2K(\mathbf{p},\mathbf{p}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)+$$
$$+4K(\mathbf{p}+\boldsymbol{\kappa}_1,\mathbf{p}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)-2K(\mathbf{p}+\boldsymbol{\kappa}_1,\mathbf{p}+\boldsymbol{\kappa}_2)\Big]-$$
$$-f(x_{\mathbf{IP}},\boldsymbol{\kappa}_1)f(x_{\mathbf{IP}},\mathbf{p}-\boldsymbol{\kappa}_1)\times$$
$$\times\Big[2K(\boldsymbol{\kappa}_2,\boldsymbol{\kappa}_2+\boldsymbol{\kappa}_1)-K(\boldsymbol{\kappa}_2,\mathbf{p}+\boldsymbol{\kappa}_2)\Big]-$$
$$-2f(x_{\mathbf{IP}},\mathbf{p})f(x_{\mathbf{IP}},\boldsymbol{\kappa}_1)\Big[K(\boldsymbol{\kappa}_2,\boldsymbol{\kappa}_2+\mathbf{p})+$$
$$+K(\boldsymbol{\kappa}_2,\boldsymbol{\kappa}_2-\boldsymbol{\kappa}_1)-K(\boldsymbol{\kappa}_2,\mathbf{p}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)\Big]\Big\}=$$
$$=\frac{1}{16\pi}\frac{C_A}{2C_F}\mathcal{K}_{BFKL}\otimes f_D(x_{\mathbf{IP}},\mathbf{p})+\frac{1}{16\pi}\cdot\frac{C_A}{C_F}\mathcal{K}_0\times$$
$$\times\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2\Big\{f(x_{\mathbf{IP}},\boldsymbol{\kappa}_1)f(x_{\mathbf{IP}},\boldsymbol{\kappa}_2)\times$$
$$\times\Big[2K(\mathbf{p}+\boldsymbol{\kappa}_1,\mathbf{p}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)-K(\mathbf{p}+\boldsymbol{\kappa}_1,\mathbf{p}+\boldsymbol{\kappa}_2)\Big]-$$
$$-f(x_{\mathbf{IP}},\boldsymbol{\kappa}_1)f(x_{\mathbf{IP}},\mathbf{p}-\boldsymbol{\kappa}_1)K(\boldsymbol{\kappa}_2,\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)-$$
$$-f(x_{\mathbf{IP}},\mathbf{p})f(x_{\mathbf{IP}},\boldsymbol{\kappa}_1)\times$$
$$\times\Big[K(\boldsymbol{\kappa}_2,\boldsymbol{\kappa}_2+\boldsymbol{\kappa}_1)-K(\boldsymbol{\kappa}_2,\mathbf{p}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)\Big]\Big\},\qquad(9)$$

where $K(\mathbf{p},\mathbf{q})=|\boldsymbol{\psi}(\mu^2,\mathbf{p})-\boldsymbol{\psi}(\mu^2,\mathbf{q})|^2$. The first form of this nonlinear $k_\perp$-factorization result looks as a fusion of two pomerons, described by $f(x_{\mathbf{IP}},\boldsymbol{\kappa}_{1,2})$, into the third one described by $f_N^{\mathbf{IP}}(x_{\mathbf{IP}},\beta,\mathbf{p})$. In the second form we singled out the linear BFKL evolving component with $f_D(x_{\mathbf{IP}},\mathbf{p}) = \sigma_0(x_{\mathbf{IP}})[2f(x_{\mathbf{IP}},\mathbf{p}) - f^{(2)}(x_{\mathbf{IP}},\mathbf{p})]$ which describes $\sigma^2(x_{\mathbf{IP}},\mathbf{r}) = \int d^2\boldsymbol{\kappa} f_D(x_{\mathbf{IP}},\mathbf{p})[1-\exp(i\boldsymbol{\kappa}\cdot\mathbf{r})]$ [7], where $f^{(2)}(x_{\mathbf{IP}},\mathbf{p}) = \frac{1}{\sigma_0(x_{\mathbf{IP}})}\Big(f\otimes f\Big)(x_{\mathbf{IP}},\mathbf{p})$ and $\mathcal{K}_0 = C_A\alpha_S/2\pi^2$. The interpretation of $f_D(x_{\mathbf{IP}},\mathbf{p})$ as a gluon density must be taken with a big grain of salt as it is not positive valued, see the discussion of the (anti)shadowing properties of $f^{(j)}(x,\mathbf{p})$ in Ref. [7]. The related LL$\frac{1}{\beta}$ result for high-mass DDIS off heavy nuclei reads (here $\Phi^{(2)}=\Phi\otimes\Phi$)

$$\Phi_A^{\mathbf{IP}}(\mathbf{b},x_{\mathbf{IP}},\beta,\mathbf{p})=\mathcal{K}_0\Big\{\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 d^2\boldsymbol{\kappa}_3\times$$
$$\times\Big[2\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_1)\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_2)\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_3)\times$$
$$\times K(\mathbf{p}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2,\mathbf{p}-\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_3)-$$
$$-2\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_1)\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_2)\Phi(\mathbf{b},x_{\mathbf{IP}},\mathbf{p}-\boldsymbol{\kappa}_1)\times$$
$$\times K(\boldsymbol{\kappa}_3,\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2+\boldsymbol{\kappa}_3)\Big]+\Phi^{(2)}(\mathbf{b},x_{\mathbf{IP}},\mathbf{p})\times$$
$$\times\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2\Phi^{(2)}(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_2)K(\boldsymbol{\kappa}_1,\mathbf{p}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)-$$
$$-\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2\Phi^{(2)}(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_1)\times$$
$$\times\Phi^{(2)}(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_2)K(\mathbf{p}-\boldsymbol{\kappa}_1,\mathbf{p}-\boldsymbol{\kappa}_2)\Big\}.\qquad(10)$$

Whether the LL$\frac{1}{\beta}$ inspired operational definitions (9) and (10) as unintegrated glue in the pomeron are viable or not, can only be decided upon inspecting their utility for the description of diffractive final states. To this end we recall a remarkable linear $k_\perp$-factorization for the forward $q\bar{q}$ dijets in DIS off free nucleons

$$\frac{2(2\pi)^2 d\sigma_N(\gamma^*\to q\bar{q})}{dz d^2\mathbf{p} d^2\boldsymbol{\Delta}}=$$
$$=f(x,\boldsymbol{\Delta})P_{q\gamma}(z)\left|\boldsymbol{\psi}(\varepsilon^2,\mathbf{p})-\boldsymbol{\psi}(\varepsilon^2,\mathbf{p}-\boldsymbol{\Delta})\right|^2.\qquad(11)$$

Here the distribution of the experimentally measurable dijet acoplanarity momentum, $\boldsymbol{\Delta}=\mathbf{p}_q+\mathbf{p}_{\bar{q}}$, is described by precisely the same unintegrated glue $f(x,\boldsymbol{\Delta})$ which describes the LL$\frac{1}{x}$ evolution of DIS. In diffractive $q\bar{q}g$ final states $\boldsymbol{\Delta}=-\mathbf{q}$. Following Ref. [18] we observe that one can cast the DDIS cross section (8) in the Fourier form,

$$\langle q\bar{q}|\sigma_N^{\mathbf{IP}}(x_{\mathbf{IP}},\beta,\mathbf{r})|q\bar{q}\rangle=\int_0^1 dz P_{q\gamma}(z)\times$$
$$\times\int d^2\mathbf{p} d^2\mathbf{q} f_N^{\mathbf{IP}}(x_{\mathbf{IP}},\beta,\mathbf{q})\left|\boldsymbol{\psi}(\varepsilon^2,\mathbf{p})-\boldsymbol{\psi}(\varepsilon^2,\mathbf{p}-\mathbf{q})\right|^2.\ (12)$$

Undoing the $z,\mathbf{p}$ and $\mathbf{q}$ integrations in (12), and treating $\mathbf{p}$ as the quark jet momentum, i.e., enforcing a certain unitarity interpretation on the mathematical Fourier representation (12), one would obtain precisely the form (11) for the diffractive dijet spectrum. However, such a reinterpretation does not match the dijet cross section given by Eqs. (3), (5). The source of this clash is that (12) derives from the integral form of (3) upon shifts of the integration variables $\mathbf{p},\mathbf{q}$, and after such shifts their meaning of the observed jet momentum is lost. Such shifts signal the distortions of dipoles by multi-pomeron exchanges which is familiar from our nonlinear $k_\perp$-factorization results for the single-jet and dijet spectra [18, 21, 22] – in DDIS such a nonlinearity and the found mismatch between the two definitions of the glue in the pomeron persist already for the free-nucleon target. We conclude that the DDIS dijet spectrum, given by the correct unitarity cut (3), is not linear $k_\perp$-factorizable in terms of the pomeron glue (9) operationally defined by the LL$\frac{1}{\beta}$ color dipole representation for DDIS.

In inclusive DIS, the leading quark spectrum is linear $k_\perp$-factorizable even for nuclear targets:

$$\frac{(2\pi)^2 d\sigma_A(\gamma^*\to q\bar{q})}{dz d^2\mathbf{p} d^2\mathbf{b}} = = P_{q\gamma}(z)\int d^2\mathbf{q}\,\phi(\mathbf{b},x,\mathbf{q})\left|\boldsymbol{\psi}(\varepsilon^2,\mathbf{p})-\boldsymbol{\psi}(\varepsilon^2,\mathbf{p}-\mathbf{q})\right|^2. \quad (13)$$

Repeating the considerations around (12), we would obtain for DDIS dijets the representation (13) in terms of $\phi_A^{\mathbf{IP}}(\mathbf{b},x_{\mathbf{IP}},\beta,\mathbf{q})$, which must be compared to (4) integrated over the gluon momentum $\mathbf{q}$. Upon such a comparison, we reiterate the already made point: the spectrum of leading quarks in DDIS off nuclei is not linear $k_\perp$-factorizable in terms of the (nonlinear) LL$\frac{1}{\beta}$ evolution-defined glue in the nuclear pomeron, the failure of the latter to describe the diffractive finial states shows it is not an observable with the appropriate unitarity cut properties.

Now we turn to the experimentally measurable single-jet spectra. We go directly to the most interesting case of nuclear targets with large saturation scale, $Q_A^2(\mathbf{b},x_{\mathbf{IP}})\gg\mu^2$, and illustrate our technique with hard leading quark jets, $\mathbf{p}^2+\varepsilon^2\gg Q_A^2(\mathbf{b},x_{\mathbf{IP}})$, a detailed treatment for the free-nucleon target will be reported elsewhere. This spectrum is dominated by the contribution from $\boldsymbol{\kappa}_i^2,\mathbf{q}^2\lesssim Q_A^2(\mathbf{b},x_{\mathbf{IP}})$, and one can expand

$$H_{ij}^A(\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2,\mathbf{q},\mathbf{p}) = \frac{1}{\mathbf{p}^2+\varepsilon^2}B_{jk}\times \times\Big[\frac{\mathbf{q}_i\mathbf{q}_k}{\mathbf{q}^2}-\frac{(\mathbf{q}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)_i(\mathbf{q}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)_k}{(\mathbf{q}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)^2}\Big], \quad (14)$$

with $B_{jk}=\delta_{jk}-2\mathbf{p}_j\mathbf{p}_k(\mathbf{p}^2+\varepsilon^2)^{-1}$. Now notice that, upon the azimuthal integrations

$$\langle H_{ij}^A(\mathbf{p},\mathbf{q})\rangle\equiv\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_1)\times \times\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_2)H_{ij}^A(\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2,\mathbf{k},\mathbf{p}) = = B_{jk}\left(\frac{\mathbf{q}_i\mathbf{q}_k}{\mathbf{q}^2}-\frac{1}{2}\delta_{ik}\right)\int d^2\boldsymbol{\kappa}\Phi^{(2)}(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa})\times \times\left[\theta(\boldsymbol{\kappa}^2-\mathbf{q}^2)+\frac{\boldsymbol{\kappa}^2}{\mathbf{q}^2}\theta(\mathbf{q}^2-\boldsymbol{\kappa}^2)\right] = = B_{jk}\left(\frac{\mathbf{q}_i\mathbf{q}_k}{\mathbf{q}^2}-\frac{1}{2}\delta_{ik}\right)C(2Q_A^2(\mathbf{b},x_{\mathbf{IP}}),\mathbf{q}^2), \quad (15)$$

where

$$C(Q_A^2,\mathbf{q}^2)\approx \approx\left[\frac{Q_A^2}{Q_A^2+\mathbf{q}^2}+\frac{Q_A^2}{\mathbf{q}^2}\left(\log\frac{Q_A^2+\mathbf{q}^2}{Q_A^2}-\frac{\mathbf{q}^2}{Q_A^2+\mathbf{q}^2}\right)\right] \quad (16)$$

vanishes at $\mathbf{q}^2\gtrsim Q_A^2$ and, as it was anticipated, $\int d^2\mathbf{q}C^2(Q_A^2,\mathbf{q}^2)$ converges at $\mathbf{q}^2\sim Q_A^2$.

Now we notice that to DLLA the integrand of (13) for inclusive DIS equals

$$\phi_A(\mathbf{b},x,\mathbf{q})(\mathbf{q}_iB_{ik})^2\frac{1}{(\mathbf{p}^2+\varepsilon^2)^2}\approx \approx\phi_A(\mathbf{b},x,\mathbf{q})\frac{\mathbf{q}^2(\mathbf{p}^2+\varepsilon^2)^2-4\varepsilon^2(\mathbf{p}\cdot\mathbf{q})^2}{(\mathbf{p}^2+\varepsilon^2)^4} \quad (17)$$

and one can try to make a contact with DDIS in which the similar rôle is played by

$$\langle H_{ij}^A(\mathbf{p},\mathbf{q})\rangle^2 = \frac{\mathbf{p}^4+\varepsilon^4}{2(\mathbf{p}^2+\varepsilon^2)^4}C^2(2Q_A^2,\mathbf{q}^2). \quad (18)$$

The two integrands, (17) and (18), have different dependence on $(\mathbf{p}\cdot\mathbf{q})$. With this reservation, a comparison of the two suggests

$$\phi_A^{\mathbf{IP}}(\mathbf{b},x_{\mathbf{IP}},\beta,\mathbf{q})\sim\frac{1}{\mathbf{q}^2}C^2(2Q_A^2,\mathbf{q}^2), \quad (19)$$

which has a pure higher twist behavior $\phi_A^{\mathbf{IP}}(\mathbf{b},x_{\mathbf{IP}},\beta,\mathbf{q})\sim 1/\mathbf{q}^6$ for $\mathbf{q}^2\gtrsim Q_A^2$, to be contrasted with $\phi(\mathbf{b},x,\mathbf{q})\sim 1/\mathbf{q}^4$ in inclusive DIS.

Omitting the technicalities of the derivation for minijets, $\mathbf{p}^2\lesssim Q_A^2$, we cite our principal result

$$\frac{\partial}{\partial\log\frac{1}{\beta}}\frac{d\sigma_A^D}{dzd^2\mathbf{b}d^2\mathbf{p}}\sim\frac{Q_A^2(\mathbf{b},x_{\mathbf{IP}})}{[Q_A^2(\mathbf{b},x_{\mathbf{IP}})+\varepsilon^2+\mathbf{p}^2]^2}, \quad (20)$$

which interpolates between all regimes and is one of the novelties of our paper. For hard jets, $\mathbf{p}^2+\varepsilon^2\gg$ $\gg Q_A^2(\mathbf{b},x_{\mathbf{IP}})$, the expansion (14) amounts to vanishing intranuclear distortions. Indeed, in this limit the impact parameter integration gives $d\sigma_A^D\propto\int d^2\mathbf{b}Q_A^2(\mathbf{b},x_{\mathbf{IP}})\propto$ $\propto A^1 d\sigma_N^D$, while in the opposite limit of minijets, $\mathbf{p}^2+\varepsilon^2\ll Q_A^2(\mathbf{b},x_{\mathbf{IP}})$, there is an obvious strong nuclear suppression (cf. the discussion in [21]),

$$R_{A/N}=\frac{d\sigma_A^D}{Ad\sigma_N^D}\propto\left(\frac{\mathbf{p}^2+\varepsilon^2}{Q_A^2(\mathbf{b},x_{\mathbf{IP}})}\right)^2\sim A^{-2/3}. \quad (21)$$

We emphasize the importance of measuring leading quark jets at fixed Q^2 as a function of their Feynman variable z, as that would give a handle on ε^2 and the width of the plateau, $Q_A^2(\mathbf{b},x_{\mathbf{IP}})+\varepsilon^2$, and allow an accurate determination of the saturation scale Q_A^2.

Now we turn to leading jets in the pomeron fragmentation region, i.e., gluons at the boundary of the rapidity gap. Here one integrates over the quark momenta with the result

$$\int d^2\mathbf{p}\Big|\int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 \Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_1)\times \times\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_2)H^A_{ij}(\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2,\mathbf{q},\mathbf{p})\Big|^2 = = \int d^2\boldsymbol{\kappa}_1 d^2\boldsymbol{\kappa}_2 \Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_1)\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa}_2)\times \times\Big[\psi_i(\mu^2,\mathbf{q})\psi_i(\mu^2,\mathbf{q}) - 2\psi_i(Q_A^2,\mathbf{q}+\boldsymbol{\kappa}_1)\psi_i(\mu^2,\mathbf{q}) + +\psi_i(Q_A^2,\mathbf{q}+\boldsymbol{\kappa}_1)\psi_i(Q_A^2,\mathbf{q}+\boldsymbol{\kappa}_2)\Big]\times \times\Big[\Omega((\mathbf{q}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)^2)-\Omega((\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2)^2)\Big], \quad (22)$$

where we used $\int d^2\boldsymbol{\kappa}\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa})\psi(\mu^2,\mathbf{p}-\boldsymbol{\kappa}) \approx \approx \psi(Q_A^2,\mathbf{p})$ valid for the interesting case of heavy nuclei with $Q_A^2 \gg \mu^2$. The dependence on the virtuality of the photon Q^2 is concentrated in ($a^2 = \mathbf{q}^2/2\varepsilon^2$)

$$\Omega(\mathbf{q}^2) = \int d^2\mathbf{p}|\psi(\varepsilon^2,\mathbf{p}+\mathbf{q})-\psi(\varepsilon^2,\mathbf{p})|^2 = = 2\pi\left(\frac{1+2a^2}{2a\sqrt{1+a^2}}\log\frac{\sqrt{1+a^2}+a}{\sqrt{1+a^2}-a}-1\right), \quad (23)$$

For hard gluon jets, $\mathbf{q}^2 \gg Q_A^2$, the dominant contribution would come from $\boldsymbol{\kappa}_i^2 \ll \mathbf{q}^2$, when one can expand $\Omega((\mathbf{q}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)^2) \approx \Omega(\mathbf{q}^2)+2\Omega'(\mathbf{q}^2)(\mathbf{q}\cdot\boldsymbol{\kappa}_1+\mathbf{q}\cdot\boldsymbol{\kappa}_2)$. The first term of this expansion gives a rise to a contribution to (22) of the form

$$\Omega(\mathbf{q}^2)\Big\{\int d^2\boldsymbol{\kappa}\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa})\times \times\left[\psi(\mu^2,\mathbf{q})-\psi(Q_A^2,\mathbf{q}+\boldsymbol{\kappa})\right]\Big\}^2 \approx \approx \frac{\Omega(\mathbf{q}^2)}{\mathbf{q}^2}\cdot\left(\frac{Q_A^2}{Q_A^2+\mathbf{q}^2}\right)^2\times \times\left[\frac{\alpha_S(\mathbf{q}^2+Q_A^2)G_N(x,\mathbf{q}^2+Q_A^2)}{\alpha_S(Q_A^2)G_N(x,Q_A^2)}\right]^2. \quad (24)$$

Making use of Eq. (23), one can readily check that the contribution from the second term, $\Omega'(\mathbf{q}^2)(\mathbf{q}\cdot\boldsymbol{\kappa}_{1,2})$, will have a similar large-$\mathbf{q}^2$ asymptotics.

For soft gluons, $\mathbf{q}^2 \ll \boldsymbol{\kappa}_{1,2}^2 \sim Q_A^2$, there is an apparent small-$\mathbf{q}$ singularity of $\psi(\mu^2,\mathbf{q})$. Upon the expansion, $\Omega((\mathbf{q}+\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)^2)-\Omega((\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2)^2) \approx \approx \Omega'(\boldsymbol{\kappa}_1^2+\boldsymbol{\kappa}_2^2)(\mathbf{q}^2+2(\mathbf{q}\cdot\boldsymbol{\kappa}_1+\mathbf{q}\cdot\boldsymbol{\kappa}_2)+4\boldsymbol{\kappa}_1\cdot\boldsymbol{\kappa}_2)$, one would find that all the contributions are finite at $\mathbf{q}^2 \to 0$. For instance, in the DIS regime of $\varepsilon^2 \gg Q_A^2$, the contribution from the term $\propto \boldsymbol{\kappa}_1\cdot\boldsymbol{\kappa}_2$ will be proportional to $\Big\{\int d^2\boldsymbol{\kappa}\boldsymbol{\kappa}_j\Phi(\mathbf{b},x_{\mathbf{IP}},\boldsymbol{\kappa})\left[\psi_i(\mu^2,\mathbf{q})-\psi_i(Q_A^2,\mathbf{q}+\boldsymbol{\kappa})\right]\Big\}^2$. The explicit forms of $\Omega(\mathbf{q}^2)$ and $\Phi(\mathbf{b},x,\boldsymbol{\kappa})$ allow a comprehensive study of the interplay of three scales – $\mathbf{q}^2, Q_A^2$ and ε^2 – and we obtain a simple formula, which interpolates from real photoproduction to DIS and from soft to hard gluons:

$$\frac{\partial}{\partial\log\frac{1}{\beta}}\frac{d\sigma_A^D}{d^2\mathbf{b}d^2\mathbf{q}} \sim \frac{1}{\pi^3}\alpha_S\alpha_{em}e_f^2C_FN_c\times \times\left(\frac{Q_A^2}{Q_A^2+\mathbf{q}^2}\right)^2\left[\frac{\alpha_S(\mathbf{q}^2+Q_A^2)G_N(x,\mathbf{q}^2+Q_A^2)}{\alpha_S(Q_A^2)G_N(x,Q_A^2)}\right]^2\times \times\frac{1}{Q^2}\log\frac{\mathbf{q}^2+Q^2+Q_A^2}{\mathbf{q}^2+Q_A^2}. \quad (25)$$

As far as the dependence on three scales is concerned, Eq. (25) would hold for the free-nucleon target too. Notice, that compared to inclusive DIS, the scaling violation in (25) is short of one power of $\log Q^2$, as it was shown already in [2, 8]. The gluon-jet spectrum has (i) a plateau in the soft region, $\mathbf{q}^2 \lesssim Q_A^2$, (ii) the $1/\mathbf{q}^4$ behavior for intermediate momenta $Q_A^2 \lesssim \mathbf{q}^2 \lesssim Q^2$ (for free nucleons this result is known for quite a time [6, 8]) and (iii) the $1/\mathbf{q}^6$ asymptotics for $\mathbf{q}^2 \gtrsim Q^2$. The latter regime can be viewed as the diffraction dissociation, $q \to qg$, of the isolated quark from the $q\bar{q}$ Fock state of the photon, in the dipole space it corresponds to rare $q\bar{q}g$ configurations with $\boldsymbol{\rho}^2 \ll \mathbf{r}^2$, while the bulk of DDIS comes from $\boldsymbol{\rho}^2 \gtrsim \mathbf{r}^2$ [2, 5]. The results for nuclear mass number dependence are instructive: in hard regions (ii) and (iii) the impact parameter integration gives $d\sigma_A^D \propto \int d^2\mathbf{b}\, Q_A^4(\mathbf{b},x_{\mathbf{IP}}) \sim A^{4/3}$, while in the plateau region $d\sigma_A^D \propto \int d^2\mathbf{b}\, Q_A^{-2}(\mathbf{b},x_{\mathbf{IP}}) \sim A^{1/3}$ for $Q^2 \ll Q_A^2$ and $d\sigma_A^D \propto \int d^2\mathbf{b} \sim A^{2/3}$ for $Q^2 \gtrsim Q_A^2$. The width of the plateau has only marginal dependence on Q^2, the quantity $\mathbf{q}^2 d\sigma_A^D$ will take its maximum value at $\mathbf{q}^2 \approx Q_A^2$ independent of Q^2. Prior to our analytic formula (25), the Q_A and Q dependence of the spectrum of gluons was studied numerically [11] within a model which corresponds to a Gaussian parameterization for $\phi(\mathbf{b},x,\boldsymbol{\kappa})$. The numerical trends observed in [11] are consistent with our Eq. (25).

We come to a summary. We derived the nonlinear $k_\perp$-factorization formulas, and reported full analytic results, for the inclusive spectrum of leading quarks and gluons in the photon and pomeron fragmentation regions of high-mass DDIS, respectively. We have also derived the nonlinear $k_\perp$-factorization representation for the unintegrated glue in the pomeron as defined by the LL$\frac{1}{\beta}$ evolution of DDIS cross section in the color dipole representation. Our main conclusion is that, in the general case, the LL$\frac{1}{\beta}$ evolution-defined glue does not have the unitarity properties appropriate for a description of the jet spectra and DDIS final states. This mismatch entails a breaking of the familiar collinear factorization treatment of final states and is especially acute for nuclear targets with large saturation scale, but it would become relevant also to DDIS off free-nucleon targets at very small $x_{\mathbb{P}}$, when the saturation effects become substantial for DIS off free nucleons too. The issue of numerical corrections to the DLLA for DDIS off free nucleons needs further scrutiny in view of the improved statistics at HERA.

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