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Scattering induced spin orientation and spin currents in gyrotropic structures

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It is shown that additional contributions both to current-induced spin orientation and to the spin Hall effect arise in quantum wells due to gyrotropy of the structures. Microscopically, they are related to basic properties of gyrotropic systems, namely, linear in the wave vector terms in the matrix element of electron scattering and in the energy spectrum. Calculation shows that in high-mobility structures the contribution to the spin Hall current considered here can exceed the term originated from the Mott skew scattering.

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Spin-orbit coupling in low-dimensional semiconductor structures is attracting a great deal of attention since it underlies effects of manipulating spins of charge carriers by electrical means. Spin orientation of free carriers by electric current [1 – 4] and the spin Hall effect, where a charge current drives a transverse spin current [5 – 7], are among the widely discussed phenomena in the thriving field of semiconductor spintronics. Microscopically, both effects are caused by spin-orbit interaction and can be related either to spin-dependent scattering (extrinsic contributions) or solely to spin splitting of the band structure (intrinsic terms). Comparative roles of the extrinsic and intrinsic mechanisms in spin transport of charge carriers in bulk and low-dimensional semiconductors are, at present, the subject of experimental and theoretical discussion (see Refs. [8, 9] for review).

So far, scattering-induced spin effects in transport of conduction electrons have been attributed mainly to the Mott skew scattering [8 – 11]. The Mott term in the matrix element of scattering can be written as $\lambda \boldsymbol{\sigma} \cdot [\mathbf{k} \times \mathbf{k}']$, where λ is a parameter, $\boldsymbol{\sigma}$ is the vector of the Pauli matrices, $\mathbf{k}$ and $\mathbf{k}'$ are the initial and scattered wave vectors, respectively. This term is quadratic in the wave vector and present in any, even centrosymmetric, structure. In semiconductor quantum wells (QWs) the Mott term is not dominant contribution to the spin-dependent part of electron scattering since gyrotropic symmetry of QW structures allows for linear in the wave vector coupling of spin states. An example of such a $\mathbf{k}$-linear spin-orbit coupling is the Rashba or the Dresselhaus spin-orbit splitting of the electron subbands induced by Structure and Bulk Inversion Asymmetry (SIA and BIA, respectively). Similarly, a spin-dependent term linear in the wave vector appears in the amplitude of electron scattering by static defects or phonons (see Refs. [12, 13] and references therein). Taking into consideration this contribution and neglecting the Mott term, the matrix element of electron scattering can be presented as

$$V_{\mathbf{k}'\mathbf{k}} = V_0 + \sum_{\alpha\beta} V_{\alpha\beta}\, \sigma_\alpha (k_\beta + k'_\beta) \,, \qquad (1)$$

where V_0 is the matrix element of conventional spin-conserving scattering and $V_{\alpha\beta}$ are parameters determined by space distribution and structure details of the scatterers. In the case of elastic scattering from short-range static defects, that is assumed below, the parameters $V_{\alpha\beta}$ and V_0 are independent of the wave vectors $\mathbf{k}$ and $\mathbf{k}'$. Linear in the wave vector term seems to be the dominant spin-dependent contribution to the matrix element of electron scattering in QWs. Unlike the Mott contribution, it can be obtained in first order of the $\mathbf{k}\cdot\mathbf{p}$ perturbation theory [12].

In this letter we analyze spin transport of two-dimensional (2D) electrons in the presence of $\mathbf{k}$-linear terms in the scattering amplitude. We show that these terms together with the spin-orbit spectrum splitting give rise to additional contributions to both the current-induced spin orientation of free carriers and the spin Hall effect. The contribution to the spin orientation of 2D electrons is comparable in magnitude to that related to current-induced carrier redistribution between the spin-split subbands [14]. As regards the spin Hall current, the proposed contribution can exceed the term originated from the Mott skew scattering in QW structures with high mobility.

1. Microscopic model. Microscopically, mechanism of spin Hall current generation and spin orientation of 2D electrons by electric current due to $\mathbf{k}$-linear terms in the scattering amplitude is as follows. Application of an electric field $\mathbf{E}$ in the QW plane results in a directed

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flow of the carriers. Due to spin-dependent asymmetry of scattering, electrons driven by the electric field are scattered in preferred directions depending on their spin states. It leads to generation of a spin-dependent electron distribution, where particles with a certain wave vector **k** carry a certain spin orientation. The explicit form of the distribution depends on the origin of the spin-dependent scattering. Since **k**-linear terms in the scattering amplitude are caused by inversion asymmetry of QWs, one can distinguish, similarly to the spectrum splitting, the SIA and BIA contributions to the scattering amplitude. The corresponding distributions of the spin density in **k** space are shown in Fig.1 for electrons confined in (001)-grown QW and subjected to the in-plane electric field along $[1\bar{1}0]$ and [110] axes. De-

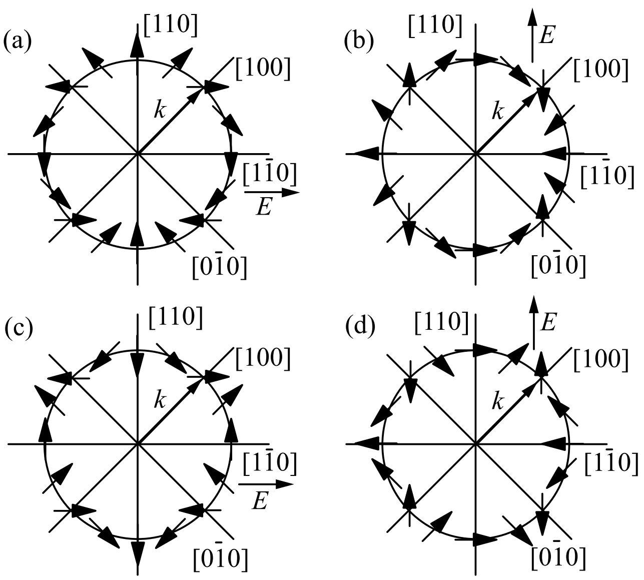

Fig. 1. Distribution of electron spin caused by **k**-linear terms in the scattering amplitude for various directions of the electric field. Figs. (a),(b) and (c),(d) correspond to the SIA and BIA **k**-linear terms in the scattering amplitude in (001)-grown QWs, respectively

tailed calculation of the presented distributions is given in the next section. Here we only note that the angular dependencies of the spin distributions are described by quadratic harmonics. Average spin polarization remains zero and, in contrast to the Mott scattering, electrons with the opposite wave vectors **k** and $-$**k** carry the same spin. Therefore, the spin-dependent scattering of the form (1) does not lead itself to a current-induced spin orientation nor to the spin Hall effect.

A net spin orientation of the electron gas and a spin current appear as a result of the subsequent spin dynamics of carriers. The spin dynamics of the conduction electrons is known to be governed by spin-orbit coupling that may be considered as an effective magnetic field acting on electron spins. In this effective magnetic field spins of the carriers, initially directed according to the spin-dependent scattering processes, precess. Such a spin dynamics is illustrated in Fig.2. To be specific

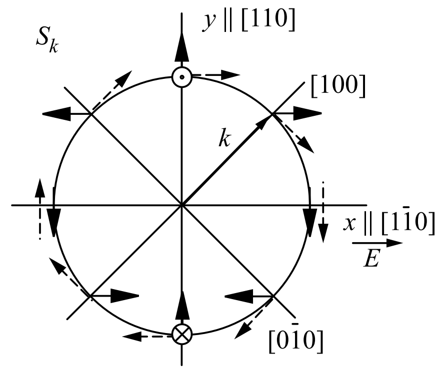

Fig.2. Microscopic mechanism of the current-induced spin orientation and the spin Hall effect. Spin-dependent asymmetry of electron scattering followed by spin precession in the effective magnetic field leads to (i) generation of a spin current and (ii) net spin orientation of carriers

we consider that the electric field **E** is directed along the $[1\bar{1}0]$ axis and both the effective magnetic field and the **k**-linear terms in the scattering amplitude are induced by structure inversion asymmetry. The effective magnetic field caused by spin-orbit coupling in QWs is known to be an odd function of the wave vector **k**. In the case of SIA it is directed perpendicular to **k** as shown in Fig.2 by dashed arrows. Therefore, electrons moving along or opposite to the $x||[1\bar{1}0]$ axis carry spins oriented parallel (or antiparallel) to the effective magnetic field, while the particles moving along or opposite to the $y||[110]$ axis carry spins oriented perpendicular to the effective field. As a result of the spin precession, the spin component $S_z > 0$ ($S_z < 0$) appears for the carriers with the positive (negative) wave vector k_y. This state corresponds to the spin Hall effect where an electric field drives a transverse spin current, i.e. oppositely directed flows of carriers with the opposite spins. Moreover, the spin precession in the effective magnetic field leads to appearance of an average spin orientation of carriers in the QW plane, opposite to the y axis. Indeed, electrons with the spins directed opposite to the y axis are affected by collinear magnetic field and retain the spin orientation, while the carriers with the spins directed along the y axis partially lose the polarization due to the spin precession. The rate of the spin generation is determined by the average angle of spin rotation in the effective magnetic field, similarly to the Hanle effect.

2. Theory. Theory of the scattering-induced spin orientation and the spin Hall effect is developed here by using the spin-density-matrix technique. Dynamics of the density matrix $\rho_{\mathbf{k}}$ of electrons subjected to an in-plane electric field $\mathbf{E}$ is given by the kinetic equation

$$\frac{\partial \rho_{\mathbf{k}}}{\partial t} + e\mathbf{E}\frac{\partial \rho_{\mathbf{k}}}{\partial \hbar\mathbf{k}} + \frac{i}{\hbar}[H_{so}, \rho_{\mathbf{k}}] = \mathrm{St}\rho_{\mathbf{k}} \,. \quad (2)$$

Here e is the electron charge, H_{so} is the Hamiltonian of spin-orbit coupling that describes spin precession in an effective magnetic field

$$H_{so} = \sum_{\alpha\beta} \gamma_{\alpha\beta}\, \sigma_\alpha k_\beta = \frac{\hbar}{2}\, \mathbf{\Omega}_{\mathbf{k}} \cdot \boldsymbol{\sigma} \,, \quad (3)$$

$\gamma_{\alpha\beta}$ are the structure parameters, $\mathbf{\Omega}_{\mathbf{k}}$ is the Larmor frequency corresponding to the effective field, and $\mathrm{St}\rho_{\mathbf{k}}$ is the collision integral. In the present letter we restrict ourselves to the case of low temperatures when transport properties are determined by the Fermi surface and energy mixing of carriers is suppressed. For elastic scattering from static defects uniformly distributed in the QW plane the collision integral has the form

$$\mathrm{St}\rho_{\mathbf{k}} = \frac{\pi}{\hbar} N_d \sum_{\mathbf{k}'} (2V_{\mathbf{k}\mathbf{k}'}\rho_{\mathbf{k}'}V_{\mathbf{k}'\mathbf{k}} - V_{\mathbf{k}\mathbf{k}'}V_{\mathbf{k}'\mathbf{k}}\rho_{\mathbf{k}} - \\ - \rho_{\mathbf{k}}V_{\mathbf{k}\mathbf{k}'}V_{\mathbf{k}'\mathbf{k}})\, \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}) \,, \quad (4)$$

where N_d is the sheet density of defects. Note, that spin-orbit splitting of the energy spectrum is neglected in the collision integral (4), since the splitting is much smaller than the electron kinetic energy. Corrections to the energy due to the spin-orbit splitting as well as similar corrections to the electron velocity, which can be crucial for intrinsic spin effects, are unimportant here.

The density matrix can be presented as follows

$$\rho_{\mathbf{k}} = (f_0 + \delta f_{\mathbf{k}})I + \mathbf{S}_{\mathbf{k}} \cdot \boldsymbol{\sigma} \,, \quad (5)$$

where f_0 is the function of the equilibrium carrier distribution, I is the 2×2 unit matrix, $\delta f_{\mathbf{k}}$ and $\mathbf{S}_{\mathbf{k}}$ are the electric field-induced corrections to the diagonal and spin components of the density matrix. We assume that the electric field oscillates at the frequency ω, $\mathbf{E} \propto \exp(-\mathrm{i}\omega t)$. Then, in linear in the electric field regime, the terms $\delta f_{\mathbf{k}}$ and $\mathbf{S}_{\mathbf{k}}$ have the same time dependence. To first order in the parameters $V_{\alpha\beta}$ the charge transport is independent of the spin part of scattering amplitude and the diagonal correction to the equilibrium density matrix has the form

$$\delta f_{\mathbf{k}} = -\frac{e\tau\, \mathbf{E}\cdot\mathbf{v}}{1 - \mathrm{i}\omega\tau}\frac{df_0}{d\varepsilon} \,, \quad (6)$$

where $\mathbf{v} = \hbar\mathbf{k}/m^*$ is the velocity, m^* is the electron effective mass, $\tau = \hbar^3/(V_0^2 N_d m^*)$ is the momentum isotropization time, and ε is the electron kinetic energy. Note, that $\delta f_{\mathbf{k}}$ is the correction that describes the conventional (Drude) ac conductivity of the electron gas.

Equation for the spin component of the density matrix $\mathbf{S}_{\mathbf{k}}$ can be derived from Eq. (2). To first order in spin-dependent part of the matrix element of scattering V_s it takes the form

$$-\mathrm{i}\omega\mathbf{S}_{\mathbf{k}} + [\mathbf{S}_{\mathbf{k}} \times \mathbf{\Omega}_{\mathbf{k}}] = -\frac{\mathbf{S}_{\mathbf{k}} - \bar{\mathbf{S}}_{\mathbf{k}}}{\tau} + \mathbf{g}_{\mathbf{k}} \,, \quad (7)$$

where $\bar{\mathbf{S}}_{\mathbf{k}}$ is $\mathbf{S}_{\mathbf{k}}$ averaged over directions of the wave vector $\mathbf{k}$, $\mathbf{g}_{\mathbf{k}}$ is the spin generation rate into the state with the wave vector $\mathbf{k}$,

$$\mathbf{g}_{\mathbf{k}} = \frac{2\pi}{\hbar} N_d \sum_{\mathbf{k}} \mathrm{Tr}[\boldsymbol{\sigma} V_0 V_s](\delta f_{\mathbf{k}'} - \delta f_{\mathbf{k}})\, \delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'}) \,. \quad (8)$$

The left-hand side of Eq. (7) describes spin dynamics of the carriers during their free motion between consecutive collisions with structure defects, while the right-hand side of the equation stands for the scattering-induced spin redistribution among electron states with different wave vectors. Particularly, the first term on the right-hand side of Eq. (7) describes isotropization of the spin density and the second term corresponds to the spin density generation due to carrier drift in the electric field in the presence of spin-dependent scattering. We note, that spin-flip processes are neglected in Eq. (8) since they appear only in second order in V_s. For linear in the wave vector terms in the matrix element of scattering given by Eq. (1), components of the spin generation rate have the form

$$g_{\mathbf{k},\alpha} = \frac{2e\hbar/m^*}{1 - \mathrm{i}\omega\tau}\frac{df_0}{d\varepsilon} \sum_{\beta\mu} \frac{V_{\alpha\beta}}{V_0}\left(k_\beta k_\mu - \frac{k^2}{2}\delta_{\beta\mu}\right) E_\mu \,. \quad (9)$$

Dependence of the spin generation $\mathbf{g}_{\mathbf{k}}$ on the wave vector is determined by both the direction of the applied electric field $\mathbf{E}$ and the explicit form of the coefficients $V_{\alpha\beta}$. The latter is governed by the QW symmetry and can be varied. Symmetry analysis shows that in (001)-grown QWs there are two nonzero components of the pseudotensor $V_{\alpha\beta}$, namely V_{xy} and V_{yx}, which can be expressed via the SIA and BIA contributions as follows: $V_{xy} = V_{\mathrm{BIA}} + V_{\mathrm{SIA}}$, $V_{yx} = V_{\mathrm{BIA}} - V_{\mathrm{SIA}}$. Figure 1 demonstrates scattering-induced distributions of the spin polarization $\mathbf{g}_{\mathbf{k}}$ in $\mathbf{k}$ space for the electric field directed along the x and y axes. Figures (a),(b) and (c),(d) correspond to the cases where the $\mathbf{k}$-linear terms in the matrix element of scattering are caused by SIA ($V_{xy} = -V_{yx}$) and BIA ($V_{xy} = V_{yx}$), respectively.

As mention above, the net spin orientation of carriers and the spin Hall current appear as a result of the spin-dependent scattering and the subsequent precession of electron spins in the effective magnetic field. Following Eq. (7) and taking into account that the frequency $\mathbf{\Omega_k}$ is an odd function of the wave vector, while $\mathbf{g_k}$ is an even function of $\mathbf{k}$ and $\bar{\mathbf{g}}_\mathbf{k} = 0$, one can derive the equation for the spin component $\bar{\mathbf{S}}_\mathbf{k}$

$$\left\langle \tau \frac{\mathbf{\Omega_k} \times [\mathbf{\Omega_k} \times (\bar{\mathbf{S}}_\mathbf{k} + \tau \mathbf{g_k})]}{[(1-\mathrm{i}\omega\tau)^2 + (\Omega_\mathbf{k}\tau)^2]} \right\rangle + \mathrm{i}\omega \bar{\mathbf{S}}_\mathbf{k} = 0 \,, \quad (10)$$

where the angle brackets mean averaging over directions of the wave vector. Solutions of Eqs. (7) and (10) allow one to find the spin components of the density matrix and calculate the spin polarization of the electron gas as well as the spin Hall current.

3. Results and discussion. Calculation shows that in the case $\Omega_\mathbf{k}\tau \ll 1$ the average electron spin, defined as $\sum_\mathbf{k} \mathbf{S_k}/N_e$, with N_e being the carrier concentration, has the form

$$s_x(\omega) = -\frac{1}{2}\left[\frac{V_{yx}}{V_0}\frac{\gamma_{xy}}{\gamma_{yx}} + \frac{V_{xy}}{V_0}\right] \frac{K_y(\omega)}{1 - \mathrm{i}\omega T_x(1-\mathrm{i}\omega\tau)^2} \,, \quad (11)$$

$$s_y(\omega) = -\frac{1}{2}\left[\frac{V_{xy}}{V_0}\frac{\gamma_{yx}}{\gamma_{xy}} + \frac{V_{yx}}{V_0}\right] \frac{K_x(\omega)}{1 - \mathrm{i}\omega T_y(1-\mathrm{i}\omega\tau)^2} \,.$$

Here V_{xy}, V_{yx}, γ_{xy} and γ_{yx} are nonzero components of the pseudotensors $V_{\alpha\beta}$ and $\gamma_{\alpha\beta}$ in (001)-grown structures, $\mathbf{K}(\omega)$ is the average electron wave vector corresponding to the drift velocity of carriers in the electric field

$$\mathbf{K}(\omega) = \frac{e\tau/\hbar}{1-\mathrm{i}\omega\tau}\mathbf{E} \,, \quad (12)$$

and T_α ($\alpha = x, y$) are the D'yakonov-Perel' relaxation times of the spin components

$$1/T_\alpha = \tau[\langle \Omega_\mathbf{k}^2 \rangle - \langle \Omega_{\mathbf{k},\alpha}^2 \rangle] \,. \quad (13)$$

Magnitude of the spin orientation achieved by dc electric current $s(0)$ depends on the ratio γ_{xy}/γ_{yx} rather than on absolute value of the spin-orbit splitting $\hbar\Omega_\mathbf{k}$. This can be understood considering the average spin as a balance between processes of spin generation and spin relaxation. $s(0)$ is given by $T\dot{s}$, where the spin relaxation time $T \propto 1/\Omega_\mathbf{k}^2$ and the spin generation rate $\dot{s} \propto \Omega_\mathbf{k}^2$.

The spin current is characterized by a pseudotensor $\hat{\mathbf{J}}$ with the components J_β^α describing the flow in the β direction of spins oriented along the α axis. In terms of the kinetic theory such a component of the spin current is contributed by a non-equilibrium correction $\propto \sigma_\alpha k_\beta$ to the electron spin density matrix and given by

$$J_\beta^\alpha = \sum_\mathbf{k} \mathrm{Tr}\left[\frac{\sigma_\alpha}{2} v_\beta \rho_\mathbf{k}\right] = \sum_\mathbf{k} S_{\mathbf{k},\alpha} v_\beta \,. \quad (14)$$

Calculation shows that the components of the scattering-induced spin Hall current in (001)-grown QWs have the form

$$J_x^z(\omega) = = \mathrm{i}\frac{\tau k_F^2}{2\hbar}\left[\frac{V_{yx}}{V_0}\gamma_{xy} + \frac{V_{xy}}{V_0}\gamma_{yx}\right] \frac{\omega T_x\, j_y(\omega)/e}{1 - \mathrm{i}\omega T_x(1-\mathrm{i}\omega\tau)^2} \,, \quad (15)$$

$$J_y^z(\omega) = = -\mathrm{i}\frac{\tau k_F^2}{2\hbar}\left[\frac{V_{yx}}{V_0}\gamma_{xy} + \frac{V_{xy}}{V_0}\gamma_{yx}\right] \frac{\omega T_y\, j_x(\omega)/e}{1 - \mathrm{i}\omega T_y(1-\mathrm{i}\omega\tau)^2} \,,$$

where k_F is the Fermi wave vector and $\mathbf{j}(\omega) = eN_e\hbar\mathbf{K}(\omega)/m^*$ is the charge current.

Equations (11) and (15) describe the current-induced spin orientation and the spin Hall current for the case $\Omega_\mathbf{k}\tau \ll 1$. In high-mobility 2D structures this inequality may be violated. At arbitrary $\Omega_\mathbf{k}\tau$ solutions of Eqs. (7) and (10) have complicated form but become simpler if the absolute value of the Larmor frequency $\Omega_\mathbf{k}$ is independent of the wave vector direction. This is fulfilled when the spin-orbit splitting is caused only by BIA or by SIA, $\gamma_{xy}/\gamma_{yx} = \pm 1$, respectively. In this particular case the spin orientation and the spin Hall current are given by Eqs. (11) and (15), where the denominators $1 - \mathrm{i}\omega T_\alpha(1-\mathrm{i}\omega\tau)^2$ are replaced by $1 - \mathrm{i}\omega T[(1-\mathrm{i}\omega\tau)^2 + 2\tau/T]$, with the time T ($T_x = T_y$) being determined by Eq. (13).

Frequency dependencies of the current-induced spin polarization $|s_y(\omega)|$ and the spin Hall current $|eJ_y^z(\omega)/j_x(0)|$ are plotted in Fig.3 for different parameters $\Omega_{k_F}\tau$. It is assumed that the electric field is directed along the x axis and the $\mathbf{k}$-linear terms both in the scattering amplitude and in the spectrum splitting are related to structure inversion asymmetry. Magnitude of the spin orientation reaches maximum in the dc limit and decreases with increasing the field frequency ω, see Fig.3a. In systems where $\Omega_k\tau \ll 1$, the spin relaxation time is much longer than the momentum relaxation time, $T \gg \tau$, and the frequency dependence of the spin orientation $s_\alpha(\omega)$ is given by $s_\alpha(0)/(1-\mathrm{i}\omega T_\alpha)$. Such a behavior is natural because the spin orientation represents a carrier redistribution between the spin states and the spin dynamics is governed by the spin relaxation time.

In contrast to the current-induced spin orientation, the spin Hall current reveals nonmonotonic frequency dependence. It vanishes at zero frequency, increases linearly with the frequency at low ω, reaches maximum and decreases when ω exceeds $1/\tau$. The cancellation of the spin Hall effect in the dc limit as well as the linear growth at small ω is a direct consequence of the particular rela-

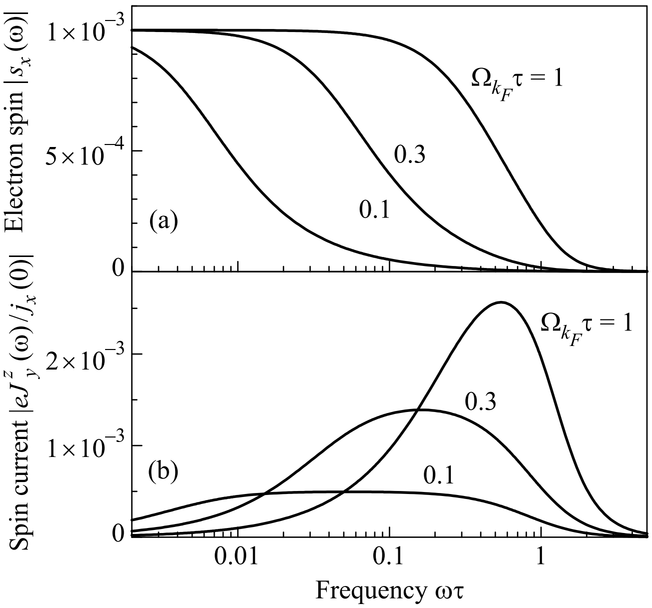

Fig.3. Frequency dependenc of (a) the spin orientation $|s_y(\omega)|$ and (b) the spin current $|eJ_y^z(\omega)/j_x(0)|$ for different $\Omega_{k_F}\tau$ and $V_{xy}/V_0 = 10^{-8}$ cm, $k_F = 10^6$ cm^{-1}, $K(0) = 10^5$ cm^{-1}

tion between spin dynamics and spin fluxes in systems with spin-orbit splitting of the band structure linear in the wave vector [15]. The relation between components of the spin and the spin current can be easily obtained from Eq. (7). Summing Eq. (7) over $\mathbf{k}$ and taking into account that $\Omega_{\mathbf{k}}$ is a linear function of the wave vector, in particular, $\Omega_{\mathbf{k},x} \propto k_y$ and $\Omega_{\mathbf{k},y} \propto k_x$ in (001)-grown structures, one obtains the relation $J_\alpha^z(\omega) \propto \omega s_\alpha(\omega)$. At small frequencies ω the spin orientation is a finite value, implying the linear growth of the spin Hall current with the frequency. In the range $1/T \ll \omega \ll 1/\tau$ the spin orientation drops as ω^{-1} and the frequency dependence of the spin Hall current has a plateau. After that, in the high-frequency limit, $\omega \gg 1/\tau$, the spin orientation and the spin Hall current decrease as ω^{-4} and ω^{-3}, respectively. We note that the absorption of the high-frequency electric field by free carriers can also lead to pure spin photocurrents [13, 16, 17] and spin orientation of the carriers [12].

Finally, we present estimations for the considered effects. Following Eq. (11) the current-induced spin orientation at zero frequency can be estimated as 10^{-3} for $V_{xy}/V_0 \sim 10^{-8}$ cm, appropriate to GaAs-based structures, and the drift wave vector $K(0) \sim 10^5$ cm^{-1}. This value corresponds to the spin orientation caused by other mechanism, namely, current-induced electron redistribution between the spin-split subbands. For this particular mechanism the spin orientation can be estimated as $\gamma_{xy}K(0)/E_F$ that also gives 10^{-3} for $\gamma_{xy} \sim 10^{-7}$ meV·cm and the Fermi energy $E_F \sim$ ~ 10 meV [14]. Further increase of the Fermi energy (or temperature for the Boltzmann statistics) leads to predominance of the scattering-related mechanism. As regards the spin Hall effect, the ratio of the spin current (15) to the contribution caused by the Mott skew scattering can be estimated as $V_{xy}/(\lambda k_F)\,\Omega_{k_F}\tau$. In high mobility structures, where $\Omega_{k_F}\tau \sim 1$, the spin Hall current caused by $\mathbf{k}$-linear terms in the scattering amplitude exceeds by an order of magnitude the contribution related to the skew scattering.

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