Phenomenological analysis of magnetic pairing in a spin quantum fluid of a high-temperature superconductor

I. E. Dzyaloshinskiĭ
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

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In a spin quantum fluid there are two types of interactions of spin fluctuations with charged holes or particles. The first type of interaction, which is responsible for frustration of the antiferromagnetic state, is effective only in the singlet state. The sign of the second type, which describes the effect of the fluctuations on the magnetization of the charge carriers, depends on the particular features (which have so far not been clarified) of the properties of the spin quantum fluid.

Nearly since the discovery of high-temperature superconductors, the magnetic pairing mechanism has gained the greatest popularity among theorists. The development of the theory began with the work of Anderson who studied the case of extremely strong repulsion between electrons. It was continued by Yakovenko and Dzyaloshinskiĭ[1] and Schrieffer *et al.*,[2] who analyzed the attraction due to the exchange of magnetic fluctuations in the opposite weak-coupling limit and who developed the concept of a "spin bag." The interaction of current carriers with the fluctuations of the magnetic system is now being subjected to increasingly closer scrutiny, but is not yet fully understood.[3-6] Furthermore, although now there is reasonable agreement that in most of the high-T_c superconductors containing copper the copper is always in the d^9 state and may be assumed to be a "neutral" atom with a spin 1/2, while the current carriers are holes which are trapped by oxygen, the discovery of an electronic superconductor by Tokura *et al.*[7] destroyed the universality of this model. It is therefore legitimate to discuss the phenomenological model which is insensitive to an approach that draws a distinction between the electron and hole superconductivity at the elementary level.

A very simple phenomenological model of the magnetic subsystem of a high-T_c superconductor, i.e., the Heisenberg exchange between the copper spins, is a theory in which the orientation is described by an antiferromagnetic unit vector **n** (the so-called nonlinear σ model). In the limiting quantum case of zero temperature the action is given by

$$\frac{1}{2}J\int d^2x\, d\tau\{(\nabla\mathbf{n})^2 + \frac{1}{w^2}\dot{\mathbf{n}}^2\}\ . \qquad (1)$$

Here τ is the virtual time of quantum statistics, J is the exchange integral, and w is the spin-wave velocity. After changing the time scale $z = w\tau$, quantum action (1) converts to energy of a classical three-dimensional magnet

$$\frac{1}{2T_{eff}}\int d^3x\,(\nabla_3\mathbf{n})^2, \qquad T_{eff} = \frac{w}{J}\ . \qquad (2)$$

Theory (1), (2) was comprehensively developed by Chakravarty *et al.*[8]

We can write two expressions for the interaction of fluctuations of the spin system with the particle or hole operators (ψ,ψ^+) which are consistent with the accuracy of theory (1), (2), i.e., which do not contain higher than second-order derivatives. The first coupling

$$g_s \int d^2x d\tau\, \psi^+ \psi (\nabla \mathbf{n})^2 \tag{3}$$

is in fact responsible for the exchange renormalization J (or renormalization of the effective temperature $T_{\rm eff}$) upon the introduction of the charge carriers. At $g_s < 0$ an increase in the carrier concentration leads to a decrease in J and w (frustration) and to a warming of a classical $3D$ magnet (2). Beginning with Anderson, many investigators proposed elementary mechanisms for frustration (see Refs. 9 and 10). One of these mechanisms, e.g., that of Ref. 9, is based on the fact that the charge carriers are holes. The Anderson mechanism is indifferent to the charge sign: the frustration in it is attributed to the trapping of an electron or a hole by copper $(d^9 \to d^{10}, d^8)$.

The discussion below is restricted to the case of total frustration (high $T_{\rm eff}$). It is also assumed that the magnetic system is in the spin quantum-fluid state.[1)] This assumption is reasonable because so far there is no experimental evidence of a clearly manifested coexistence of antiferromagnetism and superconductivity.

The second coupling involves the angular momentum of the $\mathbf{n}$ field: $(\mathbf{n}\times\dot{\mathbf{n}})$ and the magnetic moment of the carriers:

$$g_t \int d^2x d\tau\, \psi^+ \vec{\sigma} \psi \cdot (\mathbf{n}\times\dot{\mathbf{n}})\,. \tag{4}$$

In contrast with Eq. (3), Eq. (4) vanishes in the classical limit when the field $\mathbf{n}$ no longer depends on the time τ. In the nonfrustrated antiferromagnetic state Eq. (4) is the usual coupling of electrons with the spin waves: only $\mathbf{n}$ in (4) in this case should be replaced by its equilibrium value $\mathbf{n}_0$.

In the case of small charges g_s and g_t, the effective coupling between the carriers can be determined from diagrams a and b in Fig. 1, where the dashed line represents the correlation function

$$S_{kl} = \langle n_k(x,\tau) n_l(0,0)\rangle\,,$$

and the hatched circle denotes the total amplitude of the scattering of the fluctuations in the spin quantum fluid. It is easy to verify that because of the tensor structure of the

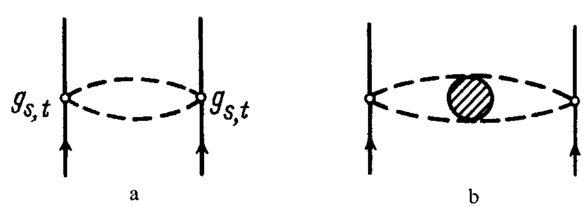

FIG. 1.

triplet vertex in (4), and because of the exchange nature of the spin quantum fluid: $S_{kl} \sim \delta_{kl}$, the diagrams in Fig. 1 have no interference terms which are proportional to $g_s g_t$. The coupling generated by the singlet charge (3) has a constant negative sign:

$$\sim - g_s^2 \langle (\nabla \mathbf{n})_1^2 (\nabla \mathbf{n})_2^2 \rangle .$$

In the case of triplet coupling the situation has not been clarified. The sign of even the simple diagram in Fig. 1a cannot be determined. This diagram corresponds to the coupling

$$g_t^2 \vec{\sigma}_1 \vec{\sigma}_2 \{ \left(\frac{dS}{d\tau}\right)^2 - S \frac{d^2 S}{d\tau^2} \} , \tag{5}$$

where $S(x,\tau) = \langle \mathbf{n}(x,\tau)\mathbf{n}(0,0) \rangle$. There is still a dearth of experimental data (see, e.g., Ref. 12) on the behavior of $S(x,\tau)$ at small times and short ranges. Furthermore, the assumption that the triplet forces have an appreciable temporal variance is of no help. We know the scaling expression in the limit $\tau \to \infty$:

$$S \sim \exp \left(- \frac{\sqrt{x^2 + y^2 + w^2 \tau^2}}{R_c} \right) .$$

After substituting it in (5), however, we have zero, so that all depends on the unknown coefficient of the exponential function.

Fortunately, the uncertainty of the sign of the triplet coupling is apparently not so important in real high-T_c superconductors. We know,[12] for example, that the fluctuations in La_2CuO_4 become classical fluctuations at temperatures $T \gtrsim 10$ K, which are one-fourth the temperature of the superconducting transition. If the transition in the other magnetic high-T_c superconductors occurs in the region of classical fluctuations, $\dot{\mathbf{n}} = 0$, the triplet coupling in them will also be suppressed. The singlet charge g_s can, in principle, be eliminated from the dependence of the $2D$ exchange integral on the carrier concentration:

$$g_s \sim dJ/dn_{e,h} .$$

The experimental study of Birgenau and Shirane,[12] however, gives us either a concentration dependence of the antiferromagnetic transition point, T_N, which is not directly related to J, or the spin-wave velocity, $w \sim \sqrt{J}$, which varies with the concentration only slightly, making it impossible to extract g_s. The situation here is even more complicated in that one theory,[9] in contrast, predicts a very large value of dJ/dn_h for La_2CuO_4.

[1)]The theory with the Hopf invariant (the θ term: see, e.g., Ref. 11) is avoided intentionally. Here the spin quantum fluid is a "trivial," singlet, van Vleck paramagnet with a finite gap for the triplet excitations and constant susceptibility, $\chi \sim 1/T_{\text{eff}}$.

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