structure of this sort inside the "gap" has indeed been observed in several experiments.\textsuperscript{5,6}

The value found for the ratio $\Delta(0)/T_c$ from tunneling measurements may thus be considerably higher than the standard BCS values. In real high-$T_c$ systems the presence of a large number of layers, with various values of the hopping integrals for hopping between these layers, disrupts the gapless nature of the superconductivity,\textsuperscript{3} but it does not qualitatively change the results derived here. In a numerical analysis of a model with five different layers in the unit cell, Tachiki et al.\textsuperscript{7} found indications of a fine structure. Because of the complexity of their model,\textsuperscript{7} found indications of a fine structure. Because of the complexity of their model,\textsuperscript{7} however, it is not possible to draw any conclusions about the changes in the characteristics of this system with an increase in the coupling between layers.

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\textsuperscript{1}D. M. Ginzberg, in Physical Properties of High Temperature Superconductors (ed. D. M. Ginzberg) [Russian translation], Mir, Moscow, 1990.

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Continuous topological defects on the $^3$He $A-B$ interface

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The microscopic structure of topological defects on the $^3$He $A-B$ interface is considered. An explicit description of a certain class of such defects is presented. Nonequivalence of positive and negative topological charges is demonstrated.

Recently a topological classification of the defects on the $^3$He $A-B$ interface was proposed.\textsuperscript{1–3} Here we consider possible microscopic structure of some of the defects. Characteristic for our solution is a nonvanishing and everywhere-continuous distribution of the order parameter. These properties seem to contradict the topological nature
of the defects because the defects with nonzero topological charges have as it is well known, a singular “hard” core. Inside the hard core region of the order parameter no longer belongs to the vacuum manifold of a given phase and may vanish. We will show, however, that in some cases this singularity can be eliminated by changing the shape of the interface involving creation of handles.\textsuperscript{1)}

The bulks of the $A$ and $B$ phases are described by distributions of the order parameter which has a form $A_{ai}^A = \Delta_A d_{\alpha} (e_{1i} + ie_{2i})$ in the $A$ phase and $A_{ai}^B = \Delta_B \exp (i\Phi) R_{ai}$ in the $B$ phase. As a boundary condition we require that the vector $\tilde{l} = \tilde{e}_1 \times \tilde{e}_2$ in the $A$ phase near the interface should be parallel to it.\textsuperscript{4,5} Other constraints should be added in order to make the boundary condition complete. They specify for each value of the order parameter $A_{ai}^A$ in the $A$ phase a set of permissible values of the order parameter $A_{ai}^B$ in the $B$ phase on the opposite side of the interface, and vice versa. In other words, a pair $(A_{ai}^A, A_{ai}^B)$ satisfies the boundary condition if it can be obtained from the pair $(A_{ai}^{0A}, A_{ai}^{0B})$, where $A_{ai}^{0A} = \Delta_A \tilde{x}_{\alpha} (\tilde{x}_1 - i\tilde{x}_2)$, $A_{ai}^{0B} = \Delta_B \delta_{ai}$, by the action of an element of the symmetry group $G = U(1) \times SO(2)^L SO(3)^S$. Here $x$ is normal to the interface, $U(1)$ is the gauge group, $SO(2)^L$ denotes the group of space rotations around $x$, and $SO(3)^S$ is the group of all spin rotations.

The result of the topological analysis can be summarized as follows:\textsuperscript{2,3} a pointlike singularity of the interface is characterized by a triplet $(m_\Phi, m_l, m_R)$, where $m_\Phi, m_l \in \mathbb{Z}$ are winding numbers for the phase $\Phi$ of the order parameter (both in the $A$ and $B$ phases) and for the vector $\tilde{l}$ (in the $A$ phase); the index $m_R \in \mathbb{Z}_2$ stands for disclinations in the field of $R$ matrix in the $B$ phase. Here we study two types of defects (Figs. 1a and 1b):

a) pointlike singularities localized at the interface (boojums), for which $m_l$ is even; $m_\Phi = m_r = 0$.

b) singular lines (vortices and disclinations) of the $B$ phase which terminate in the pointlike defect of the interface; in this case $m_\Phi + m_l$ is even

A possible microscopic picture of the defects a), b) is shown in Figs. 1c and 1d.

![FIG. 1. Schematic diagram of (a) boojums and (b) singular lines in the $B$ phase which terminates at the interface; (c) microscopic structure in the case with $g = 2$, (d) microscopic structure in the case with $g = 1$.](image-url)
The $A$--$B$ interface is bent to form a connected surface $C$ which separates the bulks of the $A$ and $B$ phases. This changing of the shape of the interface is energetically preferable if there exists a continuous distribution of the order parameter in the bulk compatible with the boundary conditions on $C$. One then has the structure which macroscopically looks like the appropriate boojum or vortex but has no singularities in the microscopic order-parameter distribution.

We consider first the case a) (boojums). The boundary surface can be compactified at infinity, where the $A$ phase is the interior of the compactified surface. We will then have a compact, two-dimensional, orientable manifold $\tilde{C}$ homeomorphic to a Riemannian surface of some genus $g$, i.e., to the two-dimensional sphere $S^2$ with $g$ handles. According to the boundary condition, the vectors $\tilde{l}$ form a tangent field on $\tilde{C}$, which is continuous everywhere except at "the infinitely distance" point $N$ which is added to the surface $C$ to make it compact: $\tilde{C} = C \cup \{N\}$. In view of this circumstance, it is necessary to find the obstructions for the existence of such a field. The answer is known as the Euler theorem: the sum of the indices of all singular points of a tangent vector field is $2 - 2g$ (Euler's characteristic of the Riemannian surface of genus $g$).

Since the index of the $\tilde{l}$ field in $N$ is equal to $2 - m_1$, we obtain $2 - m_1 = 2 - 2g$ or $m_1 = 2g$. We conclude that for $m_1 < 0$ such a structure cannot exist. We found that for $m_1 = 2$ (in this case $g = 1$, and the appropriate surface $\tilde{C}$ is a torus) there exists a continuous distribution of all other components of the order parameter which includes $\tilde{\varphi}_1$, $\tilde{\varphi}_2$, $\tilde{d}$, $R_{\alpha\ell}$ and $\Phi$ and which satisfies all the boundary conditions. It is shown schematically in Fig. 2.

For larger $m_1$ one can construct similar distributions. They contain pointlike

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**FIG. 2.** Schematic diagram of the order parameter distribution for the case $m_1 = 2$, $g = 1$. The $A$ phase fills the interior of the torus. Lines of the $\tilde{l}$ vector coincide with the parallels of the torus (see Section $C_2$). Triads $(\tilde{e}_1, \tilde{e}_2, \tilde{l})$ are uniform throughout a given cross section (see, e.g., triad $\{1,2,3\}$ in the section $C_3$); $\tilde{d}$ vectors on the surface of the torus are perpendicular to it and form a continuous funnel-like structure in the interior (section $C_1$). The matrix $R_{\alpha\ell}$ in the $B$ phase in $\delta_{\alpha\ell}$. Surfaces of a constant phase $\Phi$ of the $B$ phase look like closed domes leaning on the parallels of the torus. The disk bounded by the shortest parallel corresponds to $\Phi = \pi$. The horizontal plane surface corresponds to $\Phi = 0, 2\pi$. 

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FIG. 3. Lines of the $\tilde{l}$ vector on the surface of the funnel in the case $m_l = +1$ (left) and $m_l = -1$ (right). For the former we have $I_S = 1$, which allows for a uniform distribution of $\tilde{l}$ near $S$, while for the latter $I_S = -1$ and thus a texture of $\tilde{l}$ near $S$ arises unavoidably.

singularities in the $A$ phase (hedgehogs of the $\tilde{d}$-vector distribution). Their total topological charge is $1 - g$.

Let us consider now case b) of the vortex lines which terminate at the interface. In order to compactify the surface $C$, one has to add the point $N$ and also to glue "the neck of the funnel" by a point $S$. The previous considerations of the $\tilde{l}$-vector distribution will then apply and we find the index of the $\tilde{l}$ field in $S$ to be $I_S = m_l - 2g$. We see that only $I_S = 1$ allows for a space-uniform distribution of $\tilde{l}$ near $S$. Any other $I_S$ involves a texture with large $(\nabla l)^2$ in the vortex core. (Because for $m_l = +1$ one can set $g = 0$ and obtain $I_S = 1$, which is impossible for $m_l = -1$; see Fig. 3.) This observation implies that the vortices with $m_l = +1$ and $m_l = -1$ are not equivalent with respect to their ability to penetrate the $B$ phase.

In conclusion, I should mention that, as suggested by G. Volovik, it is possible that this nonequivalence between different ends of the $B$-phase quantized vortices was manifested in the Helsinki NMR experiments on the phase boundary under rotation.

I am grateful to G. Volovik for stimulating discussions and all communications.

1) As I was informed by G. Volovik, the development of singularities into the shape of the $A$-$B$ interface was initially suggested by E. Thuneberg.