Possible reason for the disruption of the dissipationless state under conditions of the quantum Hall effect

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The disruption of the dissipationless regime is attributed to the narrowness of the energy band of itinerant electron states. An instability of a steady-state current occurs as a result.

The disruption of the conditions for the quantum Hall effect has been the subject of numerous experimental and theoretical studies. This disruption was discovered right after the discovery of the quantum Hall effect itself.[1,2] An attempt has been made to explain the situation on the basis that the electron drift velocity may exceed the velocity of sound in sufficiently strong electric fields, and that a strong additional dissipation could arise from the emission of phonons[3,4] (these explanations were subsequently developed further and added to in Ref. 5). The various studies of the disruption of the quantum Hall effect are reviewed in Ref. 6. However, we still do not have a clear, uncontested physical picture of this phenomenon. In particular, the sound theory leads to critical currents which are too high. Bliek *et al.*[7] have reported observing discrete values of the voltage. Some discrete quantized values of the voltage have also been observed recently[8,9] upon a gradual change in the magnetic field at a sufficiently high current. These values were accompanied by a hysteresis.

A distribution of 2D charges and potential which ensure that there are no current sources (div $\bar{j}=0$) was discussed in some recent theoretical papers.[10] Some electric field distributions calculated numerically for the Corbino geometry turned out to be extremely nonuniform, especially near the conductivity minimum. However, the area under the sharp peak in the electric field is vanishingly small in comparison with the remainder of the voltage drop; such a distribution thus could not by itself be the reason for the disruption of the quantum Hall effect. However, as was shown in that paper, it is important to note that the electric fields are generated primarily by electrons in a localized state, while the conductivity results from a small number of itinerant electrons.

In this paper we attempt to give a qualitative physical explanation of the disruption of the dissipationless state in the quantum Hall effect. We also describe certain implications of this explanation for the current-voltage characteristics. We consider the very simple geometry of a Corbino disk. Our basic starting point is the idea that itinerant states exist in an extremely small fraction of the entire number of states, in a narrow energy band. Theoretical considerations based on a renormalization-group approach lead to a vanishing width for this mobility band as the temperature approaches zero.[11] A qualitative study carried out for the case of a smooth random potential[12] yields the following estimate for this width:

$$\Delta \sim \frac{l_H^2}{\lambda^2} Ur \tag{1}$$

(l_H is the magnetic length, λ is the correlation length, and U_r is the mean square value of the random potential). This estimate is small in the limit $l_H/\lambda \to 0$. At the moment there is no reliable experimental estimate of Δ.

The itinerant states themselves are associated only with elastic scattering by the random potential, and a conductivity or diffusion through these states occurs even if inelastic processes are vanishingly slow.

If the mobility band has a nonzero width, an itinerant electron can undergo elastic displacements only over a distance $l = \Delta/eE$, where E is the applied electric field. In low electric fields this distance is extremely large, and inelastic processes are capable of reducing the energy of the electron. The electron is therefore always inside the mobility band. The conductivity of the electron or the current is essentially independent of the rates of inelastic processes.

To evaluate the critical electric fields we need to compare the time scale for inelastic processes, τ, with the time (τ_0) taken to traverse a distance l. We assume that the value of Δ and the conductivity in the mobility band are both independent of the temperature. We assume, in accordance with the experimental conditions, that the occupation corresponds to the middle of the Hall plateaus, so the observed conductivity is related to the activation of charge carriers into the mobility band. It is thus completely justified to take a one-electron approach to calculate the conductivity.

The time taken by an electron to traverse a distance l is determined either by diffusion, in which case we would have $\tau_0 = l^2/D$, where D is the diffusion coefficient of the itinerant electrons, or by the electron drift velocity $v_d = \sigma E/en$, where n is the density of itinerant electrons. In the latter case we would have $\tau_0 = l/v_d$. We need to take the smaller of these two times and equate it to the time scale for inelastic processes, τ. For the critical electric fields E_c we find two expressions

$$(E_c^D)^2 = \frac{\Delta^2}{l^2(\tilde{D}_{\tilde{\tau}})}, \quad (E_c^{\sigma})^2 = \frac{\Delta n}{\sigma \tau}.$$

The effective diffusion coefficient $\tilde{D}$ can be defined in terms of the partial diffusion coefficients $D(\epsilon)$ of itinerant electrons with a given energy ϵ:

$$\tilde{D} = \int_\Delta D(\epsilon) \frac{\partial f}{\partial \mu} \frac{\partial v}{\partial \epsilon} \, d\epsilon \left(\frac{\partial n}{\partial \mu} \right)^{-1},$$

where

$$n = \int_\Delta f(\epsilon) \frac{\partial v}{\partial \epsilon} \, d\epsilon$$

is the number of itinerant electrons. Here μ is the chemical potential, $\partial v/\partial \epsilon$ is the density of states, and f is the distribution function. The conductivity σ, on the other hand, is determined by the total current over the itinerant electrons:

$$\sigma = \int_{\Delta} D(\epsilon) \frac{\partial f}{\partial \mu} \frac{\partial v}{\partial \epsilon} d\epsilon.$$

We thus conclude

$$\left(\frac{E_c^D}{E_c^\sigma} \right)^2 = \Delta \frac{l}{n} \frac{\partial n}{\partial \mu} = \frac{\Delta}{T}. \tag{2}$$

At high temperatures, $T > \Delta$, we must therefore choose the diffusion time, while at low temperatures, $T < \Delta$, we must choose the drift time. We restrict the discussion here to the case of high temperatures, $T \gg \Delta$.

We make use of the fact that the partial conductivity in the mobility band satisfies $D(\epsilon)(\partial v/\partial \epsilon) = \sigma(\epsilon) \sim e^2/h$ according to experimental data and theoretical predictions. We also use the estimate

$$n \approx \Delta \left(\frac{\partial v}{\partial \epsilon} \right)_{E_0} f(\epsilon_0).$$

We find

$$(E_c^D)^2 = \frac{\Delta}{T} \frac{\Delta n}{\sigma \tau} = \frac{h}{e^2} \frac{\Delta^2}{\tau} \left(\frac{\partial v}{\partial \epsilon} \right)_{\epsilon_0}, \tag{3}$$

where ϵ_0 is the average energy over the mobility band. The numerical value of E_c is unreliable because there are no experimental data on the quantities in (3).

As the electric field is raised further, $E > E_c$, inelastic processes become the governing factor for the magnitude of the current. Over a time τ an electron is displaced a distance on the order of l, so the current satisfies

$$j \sim en \frac{l}{\tau} = n \frac{\Delta}{E\tau}.$$

We see that the current falls off with increasing electric field. This mechanism for the onset of a negative differential conductivity was studied in Ref. 13 for narrow bands in semiconductors. The negative differential conductivity in the 3D case leads to an instability, to the formation of domains with different values of the electric field, and to the Gunn effect.[14]

To study the stability in the case of a negative differential conductivity, we need to jointly solve the Poisson and continuity equations, assuming that the current $j(E)$ depends on the electric field alone[14]

$$\frac{\partial n'}{\partial t} + \text{div}_2 \frac{\partial j}{\partial E} E' = 0, \quad \text{div}_3 E' = 4\pi n' \delta(z),$$

where the primes mean the deviations of the quantities from their steady-state values, and the subscripts 3 and 2 mean the 3D and 2D divergences, respectively (the latter in the plane of 2D layer, i.e., the $z=0$ plane). Here also, $\delta(z)$ is the Dirac δ-function.

We assume that the instability develops rapidly, and that small wavelengths are important for perturbations. We then find the following result for the instability growth rate:

$$\gamma = -|k| 2\pi \frac{\partial j}{\partial E}.$$

We see that primarily short-wave perturbations grow in the 2D case, in contrast with the 3D case, in which the growth rate is independent of k.[14] It is an extremely complicated matter to find the nonlinear regime which arises as a result of this instability. To find it, we would need to take account of the changes in the distributions of localized charges and of the electric fields which these charges generate, not only the density of itinerant charges, as we did in studying the stability.

However, we can propose a family of steady-state potential distributions which lead to this value of the current and which are stable in the sense of the local current-voltage characteristic. Let us see what happens if we superimpose a regular step potential on the random potential. We work from the semiclassical picture of the drift of electrons in a smooth random potential,[12] in which the itinerant states are concentrated near the percolation threshold. In a region with a constant value of the regular potential, these electrons diffuse in precisely the same manner as they would in the absence of a regular potential. However, they cannot overcome the step, because they are unable to undergo a transition into the mobility band at the other value of the regular potential in an elastic process. An exception to this rule is the case in which the jump has the value at which this mobility band corresponds to the mobility band of another Landau level. This situation corresponds to the case in which the electron energies in the regular potential are shifted by an integer number $\hbar\omega_c$, if we assume that the position of the mobility band corresponds within the value of Δ to the Landau levels. In the region between potential jumps there are weak electric fields, which support a stable current flow. A potential distribution of this sort must be set up by localized electrons, which are not involved in the conductivity. Such distributions would be locally stable in the sense of the current-voltage characteristic, and they would be essentially the same as in the case without steps.

Further research is required to determine the condition under which such distributions are realized and the lifetime of these distributions.

We can say in conclusion that the picture drawn here makes use of only the fact that the electrons propagate in an elastic fashion. This picture makes it possible to explain in principle the quantization of the voltage between the source and the sink in the Corbino geometry and also, apparently, in a different geometry, without the introduction of any additional hypotheses.

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