Phase diagram of a Josephson junction with a "periodic" dissipation

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The effect of the quantum shot noise ("periodic" dissipation) on the properties of the Josephson junction is studied. At $\sigma \leqslant 1$, where σ is the exponent showing the degree to which the "nonlocal"-dissipation kernel decreases, $\sim \tau^{-(1+\sigma)}$, the system is found to undergo a phase transition. The characteristics of this phase transition are found. A phase diagram, which differs from the phase diagram in the case of a Gaussian quantum noise, is constructed.

Analysis of the effect of dissipation on the quantum tunneling in Josephson junctions on the basis of the phenomenological theory (with an imaginary time τ)[1] showed that the dissipative part S_D of the effective action of the model, S_{eff}, depends on the dissipation mechanism.[2,3] In this theory the partition function of the junction (as the temperature $T \to 0$) has the form ($\hbar = 1$)

$$Z = \int D\varphi \, e^{-S_{\text{eff}}[\varphi]}, \qquad S_{\text{eff}} = S_K + S_D + S_I,$$

$$S_K + S_D = \frac{1}{2}\int d\tau \left[m\dot{\varphi}^2 + \frac{\eta}{(2\pi)^2} \int d\tau' \alpha(\tau - \tau') f\left(\frac{\varphi(\tau) - \varphi(\tau')}{2} \right) \right], \tag{1}$$

$$S_I = \int d\tau V(\varphi), \quad V(\varphi) = -g\cos\varphi,$$

where φ is the phase difference at the junction, $m = C/(2e)^2$, $\eta = \pi/2e^2 R_{\text{eff}}$ is the effective friction factor, $g = T_c/2e$, I_c, C, and R_{eff} are the critical current, capacitance, and the effective resistance of the junction, $\alpha(\tau) = \tau^{-2}$ in the case of ohmic resistance, and $\alpha(\tau) = \tau^{-(1+\sigma)}$ ($\sigma \gtrless 1$) if the friction depends on the frequency. The form of the function $f(\varphi)$ depends on the dissipation mechanism: 1) if the dissipation is determined by the quasiparticle tunneling current (and by the small shunt), we will have

$$f(\varphi) = 2(1 - \cos\varphi) = 4\sin^2\frac{\varphi}{2}, \qquad R_{\text{eff}}^{-1} = R_N^{-1} + R_{\text{sh}}^{-1} \tag{2}$$

where R_N is the normal resistance, and R_{sh} is the shunt resistance (the "periodic" dissipation); 2) if the dissipation is determined by the large shunt (the Gaussian quantum noise), we will have

$$f(\varphi) = \varphi^2, \qquad R_{\text{eff}} = R_{\text{sh}}. \tag{3}$$

In each case the function $f(\varphi)$ can be treated as an asymptotic expression

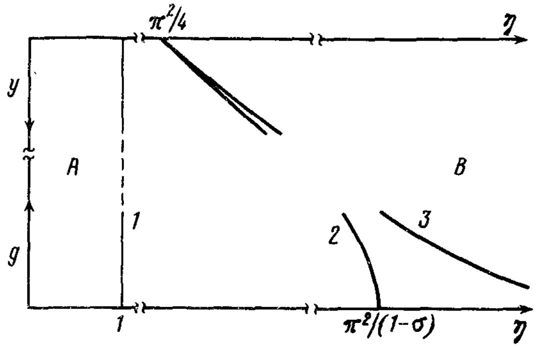

FIG. 1.

$$f_\gamma(\varphi) = \frac{2}{\gamma^2}[1-\cos\gamma\varphi] = \frac{4}{\gamma^2}\sin^2\frac{\gamma\varphi}{2} \quad (4)$$

for $\gamma = 1$ and $\gamma \ll 1$, respectively. We can remove γ from $\cos\gamma\varphi$ by redefining the phase. In this case $2\pi \to 2\pi\gamma$, $m \to \overline{m} = m\gamma^{-2}$, and $\eta \to \overline{\eta} = \eta\gamma^{-2}$.

For a contact with a Gaussian noise it was shown in Ref. 4 that as $T \to 0$, the system may have two phases, depending on the value of η: if $\eta > 1$, the phase difference of the junction is fixed (phase B in Fig. 1) and if $\eta < 1$, the phases of the contact are not correlated (phase A in Fig. 1). It was recently shown in Ref. 5 that for contacts with a dissipation the phase diagram must have a different form (Fig. 1) (for $\sigma = 1$). We present here the results of a study of a general case, $\gamma \neq 1$ and $\sigma \gtrless 1$, we show that at $\sigma \leqslant 1$ the system undergoes a phase transition, and we find the explicit form of the phase boundary which at $\sigma = 1$ confirms the diagram from Ref. 5, which was obtained on the basis of qualitative considerations.

I. Weak-potential region. The dimensional parameters m and g give two characteristic scales. The high-frequency scale m/η and the low-frequency scale g/η which plays the role of the correlation length ξ. Assuming $g\tau_0 \ll 1$, $\overline{\eta} \gg 1$, and $\tau_0 \sim (m/\eta)$, we renormalize (1) by the low-temperature renormalization-group method[6,7] and in the lower nonvanishing orders we find the renormalization-group equations

$$\begin{aligned} d_l\overline{\alpha} &= (1-\overline{\sigma})\overline{\alpha} - \overline{\alpha}^2, \qquad \overline{\alpha} = 4\pi^2/\overline{\eta}, \\ d_l y &= y(1-\gamma^{-2}\overline{\alpha}), \qquad y = g\tau, \quad l = \ln\tau/\tau_0, \\ d_l v &= -v(1-\overline{\alpha}), \quad v = m/\tau, \quad \tau = \tau_0 e^l - \text{instantaneous cutoff.} \end{aligned} \quad (5)$$

We can infer from (5) that v is an unimportant variable for the phase transition and that the equation for $\overline{\alpha}$ does not contain y and is the same as that for a 1D XY magnetic material without anisotropy.[7] The behavior of the system depends on the value of σ. At $\sigma > 1$ Eq. (5) has only a trivial immobile point $\overline{\alpha} = 0$ and hence there is no phase transition. At $\sigma < 1$ a nontrivial immobile point appears

$$\overline{\alpha}^* = (1-\sigma) \quad (6)$$

which means that at $\bar{\alpha} = \bar{\alpha}^*$ the system undergoes a second-order phase transition with power-law singularities of the physical quantities at the phase-transition point[8]:

$$\langle \cos\varphi \rangle \sim (\bar{\alpha}^* - \bar{\alpha})^{\beta}, \quad \langle \cos\varphi(\tau)\cos\varphi(\tau') \rangle \sim (\tau - \tau')^{1-\eta}.$$

The critical indices in this case are

$$\eta = 2 - \sigma, \quad \beta = 1/2 .$$

The behavior of the phase boundary in this case is described by curve 2 in Fig. 1. At $\sigma = 1$ the equation for $\bar{\alpha}$ does not have a term linear in $\bar{\alpha}$, which gives rise to a second-order phase transition with logarithmic corrections for the physical values.[8] We can see this result from the solution of (5)

$$\bar{\alpha}(l) = \bar{\alpha}/(1 - \bar{\alpha} l),$$
$$y(l) \simeq y_0 e^{l} (1 - \bar{\alpha} l)^{-\gamma^{-2}} , \tag{7}$$

in which we must substitute the correlation scale length $l_{\rm cor}$ which is determined from the condition

$$l_{\rm cor} = \ln \xi / \tau_0 = \ln (g(l_{\rm cor})\, \bar{\alpha}(l_{\rm cor}))^{-1} \tag{8}$$

or within logarithmic accuracy $l_{\rm cor} = \ln 1/g\bar{\alpha}$. All these expressions can be used in the phase with a small $\bar{\alpha}$ until $l_{\rm cor} < l^* = 1/\bar{\alpha}$, where l^* is a scale corresponding to the pole in (7). The condition $l_{\rm cor} \simeq l^*$ determines the phase-transition line (curve 3 in Fig. 1). Within logarithmic accuracy we have

$$\bar{\alpha}^* = 1/\ln (g\bar{\alpha}^*)^{-1} \simeq 1/\ln g^{-1} + O(\ln \ln g^{-1}) . \tag{9}$$

At $\gamma \ll 1$ the phase-transition line approaches, as follows from (5), the phase-transition line of the system with a Gaussian dissipation. Equations (7)–(9) can be used in the region $\exp(-\gamma^2/\bar{\alpha})/\bar{\alpha} \ll g \ll \bar{\alpha}$. Since the phase fluctuations are small in this region $\langle \varphi^2 \rangle \sim \alpha \ln 1/g\bar{\alpha} \ll \gamma^2$, the phase fluctuates (in phase B) near one of the minima of φ_i which has been singled out by the anisotropy. The results which we obtained are therefore independent of the phase-space topology φ. But strong fluctuations caused by the tunneling transitions between the minima can occur in the presence of several degenerate minima. These fluctuations will be discussed in the next section.

II. Strong-potential region.[1)] The equations corresponding to $S_{\rm eff}$ have two types of classical solutions: instantons and kinks (which exist only in the case of integer values of γ^{-1}; in what follows, we restrict the discussion to the case $\gamma = 1 = \sigma$). The SG kinks $\varphi_k(\tau)$

$$\varphi_k(\tau) = 4 \arctan (e^{\tau\omega_K}), \quad S_k = 8(gm)^{1/2}, \quad \omega_k = (m/g)^{-1/2} \tag{10}$$

are the minima of $S_k + S_I$, describe the change in the phase by 2π, and are a good approximation for the solution of the complete equation for $(gm)^{1/2} > \eta \ln (gm)^{1/2}$. The instantons $\varphi_i(\tau)$

$$\varphi_{\rm inst}(\tau) = 4 \arctan (\tau \omega_{\rm inst}), \quad S_{\rm inst} = \bar{\eta}, \tag{11}$$

where $\omega_{\rm inst}^{-1}$ is the width of the instanton (arbitrary width for the present), are topologically nontrivial minima of S_D, have a topological charge, and describe the change in the phase by 4π. (The general q-instanton solutions will be considered separately.) At $\eta \gg 1$, $(gm)^{1/2}$ a good approximation of the solution of the complete equation is the deformed instanton which is comprised of two kinks ($\widetilde{S}_{\rm inst} \simeq \eta + 2S_k$)

$$\tilde{\varphi}_{\rm inst}(\tau) \simeq 4[\arctan(e^{(\tau-\Delta)\omega_k}) + \arctan e^{(\tau+\Delta)\omega_k}], \tag{12}$$

$$2\Delta \simeq \tau_0 \exp(\pi^2/2),$$

which at $e^{(S_k, S_{\rm inst})} \gg 1$ may simply be assumed to be the θ-functions. The remarkable point is that the various types of trajectories now do not interact with each other, in contrast with the case of Gaussian dissipation. The approximation of the partition function Z by the gas of such trajectories therefore leads to a factorization of the kinks into a logarithmic-gas partition function Z_k and into an ideal-instanton-gas partition function $Z_{\rm inst}$ (since they do not interact even with each other)

$$Z = Z_k Z_{\rm inst}, \qquad Z_{\rm inst} = \sum_{n=0}^{\infty} x^n/n! \int_0^{1/T} \prod_{i=1}^{n} d\tau_i = e^{x/T}$$

$$Z_k = \sum_{n=0} z^{2n} \sum_{\{e_i\}}^{2n} \int \prod_{i=1}^{2n} d\tau_i \exp\left[\frac{2\bar{\eta}}{\pi^2} \sum_{i<j} (-1)^{i+j} \ln|\tau_i - \tau_j/\omega_k\right] \equiv \sum z^{2n} Z_{2n}$$

$$x = e^{-\widetilde{S}_{\rm inst}} \Delta (\widetilde{S}_{\rm inst})^{1/2} C\left(\frac{\omega_0}{\omega_k}\right), \quad z = e^{-S_k} \omega_k \left(\frac{2S_k}{\pi}\right)^{1/2}, \quad C(\omega) \text{ is a function}$$

where $\Sigma_{\{e_i\}}^{2n}$ is the sum over the possible neutral configurations of $2n$ kinks. In the case of a compact phase (i.e., in the case where φ amd $\varphi + 2\pi$ are identified), we have $\Sigma_{\{e_i\}}^{2n} = 2$, since there are only two sign-changing configurations. We renormalize the corresponding ln-gas and at $\bar{\eta}^* = \pi^2$ we have a phase transition.[9] The phase diagram, shown for this case in Fig. 1, is the same as that in Ref. 5. In the case of a noncompact phase, $\Sigma_{\{e_i\}}^{2n}$ contains the sum of all possible neutral configurations of the $2n$ kinks and is equal to $(2n)!/(n!)^2$. As a result, Z_k can no longer be renormalized exactly: In each Z_{2n} the renormalization of $\bar{\eta}$ has an n-dependent coefficient, $K_n = 2(2n+1)/n+1$. But in the limit $n \to \infty$, $K_n \to 4$, and since the behavior of Z_{2n} with large values of n is responsible for the phase transition, we expect that the limiting renormalization-group equations, which are the same as the renormalization-group equations for the compact case, will describe correctly the critical behavior of Z_k. Although there is a phase transition in the kink system, and the hopping to the nearest minima is forbidden in the B phase, the ideal instantons are responsible for the hopping between the minima of the same parity with an amplitude x, also in agreement with the results of Ref. 5.

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