Chiral effects in a system of 2D magnetoexcitons

Yu. A. Bychkov
L. D. Landau Institute of Theoretical Physics, Russian Academy of Sciences, 142432, Chernogolovka, Moscow Oblast'

(Submitted 12 December 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **55**, No. 3, 177–179 (10 February 1992)

The chiral field which arises in a 2D silicon metal–insulator–semiconductor structure because of the presence of magnetoexcitons has some nontrivial nonuniform metastable states. This chiral field is equivalent to a three-component planar magnetic material.

Chiral fields, i.e., fields which take on values in a nonlinear space, can have some nontrivial topological invariants.[1] In particular, Belavin and Polyakov[2] have studied nonuniform metastable states of an isotropic 2D ferromagnet (see also Ref. 3). There is accordingly the question of whether there exists a real physical system in which states of this sort could be observed experimentally. In the present letter we show that in a silicon-based 2D metal–insulator–semiconductor structure with a (110) surface orientation it is possible to arrange conditions which result in the appearance of an effective chiral field which is equivalent to a three-component planar magnetic material. This system thus provides an opportunity to experimentally study some topologically nontrivial, nonuniform metastable states.

The properties of a 2D electron system based on silicon were studied in Refs. 4–6. For a (110) surface orientation, the electrons have two degenerate valleys by virtue of time reversal, when the partial lifting of degeneracy is taken into account. Let us examine the properties of a system of interaction particles in a strong magnetic field in the case in which the characteristic Coulomb energy $e^2/\epsilon l_H$ [$l_H = (ch/eH)^{1/2}$ is the magnetic length, and ϵ is the dielectric constant] is very small in comparison with the

 0021-3640/92/030170-04$03.00

distance between Landau levels. In this approximation, the interaction does not alter the index of the electron Landau level (so we will omit this index everywhere below). The state of a free electron in a magnetic field for this system is characterized by a momentum $p_y (\equiv p)$ and a valley index $n (=1,2)$ in the Landau gauge for the vector potential. The electron-electron interaction plays a decisive role in shaping the ground state of the system. In this case, the particular distribution of electrons between valleys is important. A transition of one electron from one filled valley to another (vacant) valley results in the formation of an exciton—a magnetoexciton in the case at hand. Since there are two valleys, we can according to the results of Refs. 4–6, introduce isospin operators

$$\hat{S}_i = \frac{1}{2} \sum_{n,m;p} \hat{a}^+_{p,n} (\sigma_i)_{nm} \hat{a}_{p,m}, \tag{1}$$

where the operators $\hat{a}^+_{p,n}$ ($\hat{a}_{p,n}$) create (annihilate) electrons with a momentum p in valley $n(=1,2)$, and σ_i are the Pauli matrices. A very important result is that the operators $\hat{S}_i$ satisfy the usual commutation relations for infinitesimal-rotation operators.[4]

In low-temperature thermodynamics, only those rotations of the operators $\hat{S}_i$ which are slightly nonuniform are important. They can be described through the introduction of operators[6] ($l_H = 1$)

$$\hat{S}_i(\vec{k}) = \frac{1}{2} \sum_{n,m;p} e^{ik_x(p+k_y/2)} \hat{a}^+_{p,n} (\sigma_i)_{nm} \hat{a}_{p+k_y,m}. \tag{2}$$

At small values of the momentum k, the quantities $\hat{S}_i(\vec{k})$ become macroscopic if there is a spontaneous isospin field at $k=0$. The analysis by Bychkov and Iordanskiĭ[6] has shown that slightly nonuniform fields lead to the following effective Hamiltonian, which corresponds to a three-component planar magnetic material:

$$H_{int} = \frac{1}{2} \int d^2x \left[\lambda S_z^2(\vec{x}) + J \frac{\partial S_i(\vec{x})}{\partial x^\alpha} \cdot \frac{\partial S_i(\vec{x})}{\partial x^\alpha} \right], \tag{3}$$

$$i = x, y, z; \quad \alpha = 1, 2.$$

Here the parameter J is $\sim e^2/\epsilon l_H$, and λ is an anisotropy parameter. The analysis in Ref. 5 shows that the condition $J \gg |\lambda| l_H^2$ holds (the anisotropy is very slight), and λ can have either sign. Since there is an anisotropy parameter λ, there can be a transition to an ordered phase at a critical temperature

$$T_c = 4\pi J \left(\ln \frac{J}{|\lambda| l_H^2} \right)^{-1}.$$

If the very slight anisotropy of the system is ignored, we find that there exists a chiral field $\vec{n} = \vec{S}/|\vec{S}|$. Attempts to find the value of $S_0 = |\vec{S}|$ run into major difficulties, but according to Ref. 5 we have $S_0 \leqslant 1/2$. As a result, we find the Hamiltonian

$$H_{int} = \frac{JS_0^2}{2} \int d^2x \frac{\partial n_i(\vec{x})}{\partial x^\alpha} \cdot \frac{\partial n_i(\vec{x})}{\partial x^\alpha}, \tag{4}$$

to which all the conclusions reached in Ref. 2 apply (see also Ref. 3). We will simply summarize these conclusions here, referring the interested reader to the cited papers for the details. Hamiltonian (4) reaches an extremum on fields $\vec{n}(\vec{x})$ which satisfy the equation

$$\Delta\vec{n} = \vec{n}(\vec{n}\Delta\vec{n}) \tag{5}$$

and the condition $\vec{n}(\vec{x}) \to \vec{n}_0(|\vec{x}| \to \infty)$. The meaning here is that the plane $\vec{x}$ is equivalent to the sphere S^2, and the chiral field $\vec{n}(\vec{x})$ performs the mapping $S^2 \to S^2$. The overall space of fields $\vec{n}(\vec{x})$ is thus broken up into sectors, each characterized by an integer q, which is the degree of the mapping[1,3] and which is given by

$$q = \frac{1}{8\pi} \int \epsilon_{\mu\nu} \vec{n} \left(\frac{\partial \vec{n}}{\partial x^\mu} \times \frac{\partial \vec{n}}{\partial x^\nu} \right) d^2x. \tag{6}$$

A result of fundamental importance is that Hamiltonian (4) satisfies the condition

$$H_{int} \geq 4\pi J S_0^2 |q|. \tag{7}$$

In other words, condition (7) states that there exists a lower value of the energy in each homotopic class characterized by an integer q, which is the degree of the mapping.

Some comments are in order here. If the Hamiltonian is completely isotropic ($\lambda = 0$), we obtain a paradoxical result: There is no interaction of the magnetoexcitons in the ground state (see Ref. 5 and the papers cited there). It is thus crucial to study effects associated with the anisotropy parameter λ. Of particular interest in this regard is silicon with a (110) surface, for which there are two degenerate valleys. This system was first studied in Ref. 4 in an effort to explain dissipative phenomena in the quantum Hall effect above a critical electron velocity. Those dissipative phenomena stemmed from the emission of Goldstone 'valley waves" associated with long-wave oscillations of isospin vector (1). Rasolt *et al.*[4] have also attempted to evaluate the anisotropy parameter λ. Bychkov and Iordanskiĭ[5] found the contributions to this parameter from various mechanisms. The problem of valley-density waves was also taken up by Tesanovic and Halperin.[7] In the present letter we have studied metastable states for a system of magnetoexcitons in silicon—states associated with a nonuniform distribution of electrons between valleys. According to Rasolt *et al.*,[4] the operator representing the interaction of electrons with impurities contains an operator $\hat{S}_+ = \hat{S}_x + i\hat{S}_y$; i.e., the states under consideration may arise as modes localized near impurities. We need to stress that these states have a charge.[5] So far, there has been no direct observation of valley-density waves or of a critical temperature.

[1]B. A. Dubrovin, S. P. Novikov, and A. T. Fomenko, *Contemporary Geometry*, Vol. 2, Springer-Verlag, New York, 1986.

[2]A. A. Belavin and A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 503 (1975) [JETP Lett. **22**, 245 (1975)].

[3]R. Rajaraman, *Solitons and Instantons: An Introduction to Solitons and Instantons in Quantum Field Theory*, Elsevier, North-Holland, 1982.
[4]M. Rasolt, B. I. Haperin, and D. Vanderbilt, Phys. Rev. Lett. **57**, 126 (1986).
[5]Yu. A. Bychkov and S. V. Iordanskiĭ, Zh. Eksp. Teor. Fiz. **93**, 1049 (1987) [Sov. Phys. JETP **66**, 591 (1987)].
[6]Yu. A. Bychkov and S. V. Iordanskiĭ, Fiz. Tverd. Tela (Lengrad) **29**, 2442 (1987) [Sov. Phys. Solid State **29**, 1405 (1987)].
[7]Z. Tesanovic and B. I. Halperin, Phys. Rev. B **36**, 4888 (1987).

Translated by D. Parsons