Critical-current oscillations as a function of the exchange field and thickness of the ferromagnetic metal (F) in an S-F-S Josephson junction

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The Josephson current in an S-F-S junction is calculated for a short weak link. The current amplitude depends in an oscillatory manner on the exchange field of the pure ferromagnetic metal. When certain conditions are satisfied in a superconducting ring with an S-F-S junction, the energy minimum of the system corresponds to the state with spontaneous current and magnetic flux.

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In standard S-N-S Josephson junctions the oscillatory dependence of the current on the magnetic field is a consequence of the interference of the phases of the electrons of a Cooper pair, which are determined by the vector potential. We shall show that this interference appears in a superconductor–pure ferromagnetic metal–superconductor junction, but in this case the phases of the electrons in the Cooper pair are determined by the exchange field of the ferromagnetic metal. The S-F-S junctions can therefore be used to investigate exchange fields in pure ferromagnetic metals.

We shall examine an S-F-S junction with the geometry shown in Fig. 1, which corresponds to the ODSEE model.[1] We assume that the thickness L of the ferromagnet and its transverse dimension are small compared with the superconducting correlation length ξ_0 and with the electron mean free path length l, i.e., $L \ll d \ll \xi_0 \ll l$. We also assume that the effective electron-phonon interaction parameter of the BCS model is equal to zero in the ferromagnet. To describe the system, we use the Eulenberger equations with singlet pairing of electrons

$$\left(\omega + ih + \frac{1}{2} v_x \frac{\partial}{\partial x}\right) f(x) = \Delta(x) g(x), \quad f^+ f + g^2 = 1, \tag{1}$$
$$\left(\omega + ih - \frac{1}{2} v_x \frac{\partial}{\partial x}\right) f^+(x) = \Delta^*(x) g(x), \quad \omega = 2\pi T\left(n + \frac{1}{2}\right),$$

where the x axis is chosen as shown in Fig. 1. Inside the superconductors 1 and 3 the Weiss exchange field $h = 0$ and the order parameter $\Delta(x)$ in regions 1 and 3 is equal to $|\Delta|\exp(\pm i\varphi/2)$, where φ is the phase difference at the junction. Inside the ferromagnetic metal we find $\Delta(x) = 0$. We shall examine a one-domain sample, i.e., the field h is constant in region 2. We also assume that $h \gg \Delta$, and $h \gg v_F/l$ where v_F is the Fermi velocity of the electrons, which we assume, for simplicity, is the same in the superconductors and ferromagnetic metal. The condition $h \gg v_F/l$ makes it possible to ignore electron scattering by impurities. In the derivation of Eqs. (1) in region 2 we take account of the fact that in the Hamiltonian for the elec-

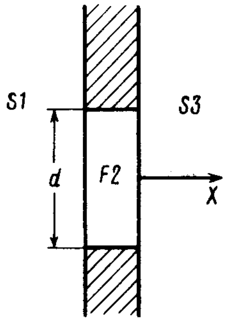

FIG. 1. Regions 1 and 3 are superconductors, 2 is a normal ferromagnetic metal, and L and d are the thickness and transverse dimension of the ferromagnet respectively. The shaded area is an insulator.

trons there is the exchange interaction $\mathbf{h}\psi^{+}(r)\,\vec{\sigma}\,\psi(r)$, where $\vec{\sigma}$ are the Pauli matrices, and ψ^{+} and ψ are electron operators. In this region the g function describes the electron motion with projection of the spin (-½) on the axis parallel to h. The influence of the magnetic field on the electron motion results in the replacement of φ by the gradient-invariant phase difference.

The solution of Eqs. (1) in region 1 and 3 has the form of an exponential function with the exponents 0, $\pm\Omega/|V_x|$, and $\Omega = \sqrt{\omega^2 + \Delta^2}$. In region 2 we find

$$f_2(x) = C_0 e^{-2(\omega + ih)x/v_x}, \quad f_2^{+}(x) = C_0^{+} e^{2(\omega + ih)x/v_x}, \tag{2}$$

where C_0 and C_0^{+} are constants. Taking into account the boundedness of the functions f and f^{+} at infinity and their continuity at the boundaries of regions 1, 2 and 2, 3, we determine the function $g^2(\omega, \nu_x)$. By using it we can easily calculate the current $I_s(\varphi)$ that flows through the contact:

$$I_s(\varphi\ a) = \frac{\pi\,\Delta\,a^2}{2eR_N}\int_a^{\infty}\frac{dy}{y^3}\left[\sin\frac{\varphi - y}{2}\ \mathrm{th}\,\frac{\Delta\cos((\varphi - y)/2)}{2T} + \sin\frac{\varphi + y}{2}\ \mathrm{th}\,\frac{\Delta\cos((\varphi + y)/2)}{2T}\right] \tag{3}$$

where $\alpha = 2|h|L/\nu_F$, and R_N is the resistance of the weak link in the normal state. The familiar expression for the Josephson current in a short, weak S-N-S link follows from Eq. (3) for $\alpha = 0$.[1,2]

The current $I_S(\varphi,\alpha)$ oscillates as a function of φ and α. The $I_S(\varphi,\alpha)$ dependence is simplified near T_c, where

$$I_s(\varphi, a) = \frac{\pi\Delta^2}{4eR_N T}\,F(a)\sin\varphi, \quad F(a) = a^2\int_a^{\infty}\frac{dy}{y^3}\cos y\,.$$

As a result of varying α, the function $F(\alpha)$ oscillates and passes through zero, and $F(\alpha) = -\sin\alpha/\alpha$ for $\alpha \gg 1$. The oscillations of the maximum critical current I_c as a function of α are preserved in the region $T \ll T_c$; however, since the relative variations of the quantity I_c are smaller, it does not vanish. In the limiting case of a dirty ferromagnetic metal $h \ll \nu_F/l$ the oscillations vanish over the entire temperature region.

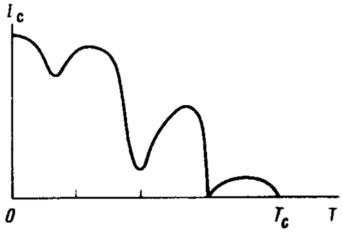

FIG. 2. Dependence of the maximum Josephson current on the temperature in an *S-F-S* junction for the parameters, $T_c \approx 4$ K, $\Theta \approx 10$ K, $h_0 \approx 300$ K, and $v_F \approx 2 \times 10^7$ cm/sec (shown schematically).

The oscillations of the maximum current as a function of α can be detected experimentally from the temperature dependence of I_c, since the exchange field in the ferromagnet changes with the temperature. We shall examine a ferromagnetic with a RKKI interaction and a Curie point Θ. For this case, $h_0 \approx \sqrt{\Theta \epsilon_F}$, where ϵ_F is the Fermi energy in the ferromagnetic metal, and h_0 is the exchange field at $T=0$. Setting $\epsilon_F \approx 1$ eV, $T_c \approx 4$ K, $\Theta < 10$ K, and $L \approx 10^{-3}$ cm, we find large, nonmonotonic variations of the current I_c by changing the temperature by about 0.5 K. Figure 2 shows schematically the $I_c(T)$ dependence for the parameters $T_c \approx 4$ K, $\Theta \approx 10$ K, $h_0 \approx 300$ K, and $v_F \approx 2 \times 10^7$ cm/sec.

We note that a closed superconducting ring with an *S-F-S* junction inserted into it has a spontaneous current and magnetic flux in the ground state if $F(\alpha) < 0$ and the ring inductance is sufficiently large.[3]

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