Hadron multiplicity in jets in e^+e^- annihilation

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The Q^2 dependence of the average hadron multiplicity for jets in e^+e^- annihilation is calculated in quantum chromodynamics in the approximation of planar diagrams. Kinematic restrictions lead to a result different from that found previously by other investigators.

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The hadron multiplicity in e^+e^-annihilation cannot be calculated completely in quantum chromodynamics because of the unresolved problem of quark confinement. The average number of hadrons is related to the average multiplicity of colorless clusters, which consist of partons (a quark, an antiquark, and gluons) according to the hypothesis of preconfinement.[1]

According to this hypothesis, the colorless clusters form in e^+e^- annihilation as a result of the sequential decay of a highly virtual quark (or antiquark) with a mass $\sqrt{k^2} \sim \sqrt{Q^2}$, where Q is the 4-momentum of the photon. The average virtualities of the decay products, p^2, do not depend on Q^2: $p^2 \sim Q_0^2$. Bassetto *et al.*[2] have shown that the cluster masses are also limited and are on the order of Q_0. Each cluster decays into several hadrons.

The average multiplicity of the heavy partons with mass $\sim Q_0$ in e^+e^- annihilation is calculated by perturbation theory in quantum chromodynamics. Since a quark jet develops primarily through gluons, it is sufficient to calculate the parton multiplicity in the gluon jet, N_Q. The number of partons in the quark jet, N_q, is related to N_Q by[3]

$$N_q(Q^2, Q_0^2) \simeq \frac{C_F}{C_A} N_G(Q^2, Q_0^2),$$

where $C_F = (N^2 - 1)/2N$, and $C_A = N$ is the number of colors.

The asymptotic expression derived for N_G at large Q^2 by the method of summing leading (doubly logarithmic) terms is[2,4]

$$N_G(Q^2, Q_0^2) \sim \exp\left[2\sqrt{\frac{C_A}{\pi b} \ln \frac{Q^2}{\Lambda^2}}\right], \tag{1}$$

where $12\pi b = 11N - 2N_f$, N_f is the number of flavors, and Λ is the scale in the effective coupling constant, $a_s(Q^2) = 1/b \ln Q^2/\Lambda^2$. Only planar diagrams were considered in the derivation of (1).

We might note that the average multiplicities in other hard processes are determined in a universal way by the average number of partons in the quark jet in the e^+e^- annihilation, N_q (Ref. 5).

Mueller[6] recently derived an expression for N_G in the three-loop approximation. Incorporation of the cross diagrams led to an additional factor of $1/\sqrt{2}$ in the argument of the exponential function in the expression for N_G (Ref. 6):

$$N_G(Q^2, Q_0^2) \sim \exp\left[\sqrt{\frac{2C_A}{\pi b}\ln\frac{Q^2}{\Lambda^2}}\right]. \tag{2}$$

In the present letter we will show that expression (2) holds even in the approximation of planar diagrams if we consider (in contrast with Refs. 2 and 4) corrections of the type k^2/Q^2 to the decay functions P_G^{GG} (see the discussion below).

The number N_G is related to the parton density $n_G(k^2, Q_0^2)$ in a gluon with a fixed virtuality k^2:

$$N_G(Q^2, Q_0^2) = \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} n_G(k^2, Q_0^2).$$

In turn, n_G satisfies a Bethe-Salpeter equation in the approximation of planar diagrams in the light-cone gauge:

$$n_G(k^2, Q_0^2) = 1 + \frac{a(k^2)}{2\pi}\int_{Q_0^2}^{k^2} \frac{dl^2}{l^2} n_G(l^2, Q_0^2) \int_{max\left(0, \frac{l^2}{k^2} - \frac{k^2}{Q^2}\right)}^{1} dz\, P_G^{GG}(z, k^2/Q^2), \tag{3}$$

where z is that fraction of the longitudinal momentum of the initial parton ($k_{\|} = Q/2$) which is carried off by the gluon with virtuality l^2. In the gauge chosen here, the function P_G^{GG}, which gives the probability for the decay $G \to GG$, has the following form at small values of z:

$$P_G^{GG}(z, k^2/Q^2) \simeq 2C_A \frac{1}{z + k^2/Q^2}. \tag{4}$$

Analysis of the kinematics reveals a restriction on the range of the integration over z in Eq. (3). We wish to emphasize that in the leading logarithmic approximation the region $Q^2 \gg k^2 \gg l^2 \gg Q_0^2$ makes the leading contribution to n_G.

Using the explicit form of the function P_G^{GG} for small z given in (4), we find an equation which describes the Q^2 dependence of the average parton multiplicity in the gluon jet:

$$\left(Q^2 \frac{d}{dQ^2}\right)^2 N_G(Q^2, Q_0^2) = \frac{C_A}{2\pi} a_s(Q^2) N_G(Q^2, Q_0^2). \tag{5}$$

Solving this equation, we find the result in (2). The quantity k^2/Q^2 in Eq. (3) is usually ignored, so that we find expression (1).

An asymptotic expression for N_G at large values of Q^2 can also be calculated in noncovariant perturbation theory in the Altarelli-Parisi approach.[7] In this approach, N_G satisfies

$$N_G(Q^2, Q_0^2) = 1 + \int_{Q_0^2}^{Q^2} \frac{dp_\perp^2}{p_\perp^2} \frac{a(p_\perp^2)}{2\pi} N_G(p_\perp^2, Q_0^2) \int_0^1 dz P_G^{GG}(z, p_\perp^2/Q^2). \qquad (6)$$

Calculations show that at $z \sim 0$ the function P_G^{GG} has a singularity of the type

$$P_G^{GG}(z, p_\perp^2/Q^2) \simeq 2C_A \frac{1}{\sqrt{z^2 + p_\perp^2/Q^2}} .$$

Substituting this expression in Eq. (6), we again find that the Q^2 dependence of the average number of heavy partons in the gluon jet is of the form in (2).

We thus see that a summation of all the diagrams which contribute to the average multiplicity, which was carried out in sixth order in Ref. 6, is equivalent to a more accurate consideration of the kinematic restrictions in the approximation of planar diagrams.

There is a somewhat analogous situation in a calculation of the corrections to the quark fragmentation function in the region $z \to 1$. In Ref. 2b it was shown that the summation of these corrections reduces to a change in the argument k^2 in the effective coupling constant $a_s(k^2)$. This argument is determined by the maximum quantum-exchange virtuality which is permitted by the kinematics, $k^2(1-z)$.

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