Kelvin-Helmholtz instability and the Jovian Great Red Spot

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(Submitted 28 July 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **36**, No. 6, 190–193 (20 September 1982)

It has been shown experimentally, for the first time, that the Kelvin-Helmholtz instability is manifested by a predominant generation of anticyclonic Rossby solitons in a model of a homogeneous atmosphere of a rotating planet. These solitons drift opposite the global rotation. Their properties and the conditions for their existence are reminiscent of the vortex at the Jovian Great Red Spot.

PACS numbers: 47.20. + m, 96.30.Kf

The Kelvin-Helmholtz instability of shear flows in a shallow liquid is of considerable interest, particularly for the physics of planetary atmospheres and plasma physics. This instability has recently been observed in some remarkable experiments.[1–3] A distinguishing feature of the experiments which we are reporting here is that the apparatus may be regarded as a model of a homogeneous planetary atmosphere. The following conditions are met simultaneously: 1) The liquid (water) is rotating as a whole around a vertical symmetry axis at an angular velocity Ω_0; 2) the depth of the liquid, H_0, is approximately constant and small in comparison with the Rossby radius $r_R = (g^*H_0)^{1/2}/f_0$, where g is the acceleration due to gravity, $g^* = g/\cos\alpha$, $f_0 = 2\Omega_0 \cos\alpha$, α is the polar angle, and f_0 is the Coriolis parameter, which has a gradient over latitude (the so-called β effect). Under these conditions the manifestations of the instability and the conditions for its appearance are extremely reminiscent of the Great Red Spot of Jupiter.[4–8]

The experimental apparatus is basically a paraboloid which is rotating around a vertical axis (Fig. 1). We used a similar vessel in Ref. 9, but in the present experiments there are two depressions in the central part of the bottom of the vessel in which two rings, each 3 cm wide, can move, rotating around the common symmetry axis. The gaps between the rings and the walls of the depressions are 1 mm wide. The distance between the rings is $\Delta y =$ 3 cm, and the center of the line connecting the rings lies 10 cm from the rotation axis. Each ring is rotating at the same angular velocity Ω_{shear} with respect to the paraboloid. In the case of an anticyclonic shear, the upper ring lags behind the rotation of the paraboloid, while the lower ring leads it; in the case of a cyclonic shear, the upper ring rotates more rapidly than the paraboloid, and the lower more slowly. The period of the main rotation, $T_0 = 2\pi/\Omega_0$, is 0.58 s. At the corresponding rotation velocity, in the absence of shear, the liquid covers the surface of the paraboloid in a level layer of constant depth H_0; this depth was varied over the range 5–10 mm.

The experiments show that if the velocity shear lies below a certain threshold, laminar flows are established in the system with two abrupt velocity changes over a distance $\sim H_0$ in the gaps between the rings and the adjacent parts of the bottom of the vessel. If this threshold is exceeded, the motion of the liquid, which changes radically

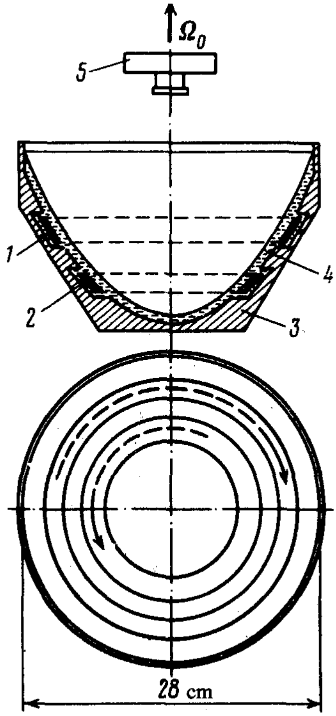

FIG. 1. The experimental apparatus. 3—Vessel with a parabolic bottom profile, which rotates around a vertical axis; 4—liquid (water), whose surface assumes a parabolic shape; 1,2—rings which create oppositely directed flows with a velocity shear (the arrows, which illustrate the flow in a top view, correspond to an anticyclonic shear); 5—camera, which is rotating along with the parabolic bottom of the vessel.

in nature, is determined by the sign of the shear. If the shear is cyclonic, cyclonic vortices of small dimensions (less than r_R) and extremely small amplitude arise in the system. This amplitude does not increase even if Ω_{shear} is increased to Ω_0. If the shear is anticyclonic, raising the flow velocity even slightly above the threshold gives rise to large-amplitude vortices which have the following properties: 1) Their dimensions are much larger than the Rossby radius r_R. 2) They are stable, elongated, steady-state anticyclones. 3) They drift opposite the global rotation of the liquid at a velocity essentially equal to the Rossby velocity $V_R = H_0\Omega_0 \sin\alpha$. 4) The drift velocity of the vortices increases with increasing H_0 and Ω_0 (the latter is due primarily to the gradient in the depth of the liquid[9]). Near the threshold for the (large-scale) instability there are four vortices; if the shear is large, there are three. These facts are illustrated in Fig. 2, where part *a* refers to cyclonic shear, part *b* to anticyclonic shear of the same absolute value, and part *c* to a larger anticyclonic shear. Comparing these facts with the properties of Rossby solitons,[9] we conclude that the observed anticyclonic vortices are in fact Rossby solitons, whose global shape is a consequence of the flows. There is yet another important fact: The threshold flow velocity (the magnitude of the shear) at which this large-scale instability appears is several times the Rossby velocity V_R.

The condition for the occurrence of the Kelvin-Helmholtz instability (and the threshold for this instability) in the case of the β effect is determined by the Rayleigh-Ho criterion,[10]

$$\beta - \frac{\partial^2 u}{\partial y^2} = 0, \qquad (1)$$

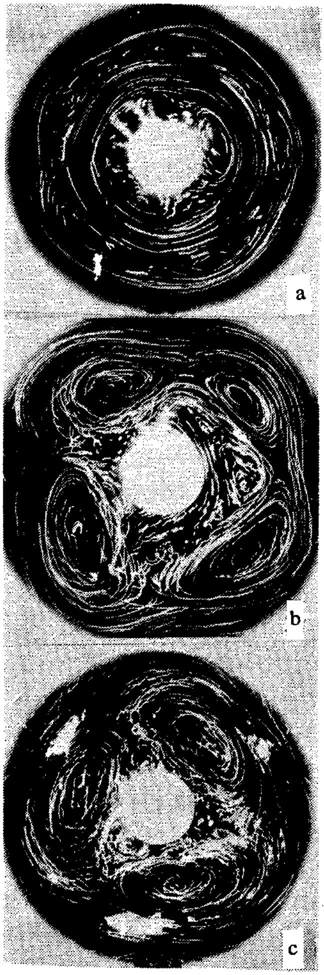

FIG. 2. Liquid flow patterns. a—Cyclonic shear; b,c—anticyclonic shear. These are photographs of the paths traced out by white tracer particles which float on the surface of the water against the background of the black bottom. The photographs were taken with a rotating camera at an exposure time of 0.25 s. The white spot at the center is part of the apparatus used to rotate the vessel. The rotation period of the paraboloid is $T_0 = 0.58$ s, and the period of the shear is (a, b) $T_{\text{shear}} = 1.7$ s or (c) 0.8 s. The Rossby velocity is $V_R = 9$ cm/s; the velocity of the outer flow is (a, b) $u = 43$ cm/s or (c) 91 cm/s.

where, according to Ref. 9,

$$\beta = -\frac{1}{r_R^2}\,\frac{\partial}{\partial y}\,(f_0 r_R^2),$$

u is the flow velocity ($u > 0$ if the flow is directed along the global rotation of the system), and y is the latitudinal coordinate (along the meridian). We then find the stability condition to be

$$V_R + u + r_R^2\,\frac{\partial^2 u}{\partial y^2} = 0. \tag{2}$$

(In the estimates below we assume $|\partial^2 u/\partial y^2| \approx |u|/\delta^2$, where δ is the scale dimension of the steady-state flow gradient.) Whether criterion (2) is satisfied depends in a fundamental way on the sign of the flow velocity. *If $u > 0$,* an instability can occur only if $\delta < r_R$; i.e., the vortices which result from the instability must be small in comparison with the Rossby radius. *If* $u < 0$, an instability is possible at any value of δ. For a large-scale instability ($\delta \gtrsim r_R$) the threshold (minimum) flow velocity is $|u| \gtrsim V_R$. This is the case of most interest, since the instability of the flow (the flow is directed opposite the rotation of the system) in this case can give rise to Rossby solitons. It is not difficult to see that both of these instability cases occur in the present experiments (Fig. 2); the case $u > 0$ corresponds to cyclonic shear, and the case $u < 0$ to anticyclonic shear. The basic effect is attributable to the outer ring, whose linear velocity is higher than that of the inner ring (for a given Ω_{shear}). In particular, large anticyclones—with dimensions larger than the Rossby radius (Rossby solitons)— are in fact generated in the case of an anticyclonic shear at velocities of the (lagging) flow above the Rossby velocity. (In analyzing the possibility that large-scale vortices—with dimensions greater than r_R— may be excited, one must take into account the fact that the anticyclonic Rossby vortices are stable, while the cyclonic vortices decay rapidly.[9]).

Interestingly, the vortices observed in the case of an anticyclonic shear (Figs. 2b and 2c) are extremely reminiscent of the Great Red Spot of Jupiter in terms of their shape, physical properties, drift direction, and conditions for existence. The Great Red Spot is an anticyclonic vortex with dimensions greater than the Rossby radius.[4] It is drifting opposite the rotation of the planet and is surrounded by latitudinal zonal winds. The Spot is an oval with long dimension in the drift direction, like the vortex in Fig. 2. The Spot is apparently a Rossby soliton.[5–8] In view of the results of the present experiments, the following fact appears to be of fundamental importance: The Great Red Spot is "tied" to that band of latitudes in the Jovian atmosphere where the shear of the zonal flows is anticyclonic, while it "ignores" the corresponding latitudinal belt slightly to the north, where the shear—no smaller in absolute value—is cyclonic. These facts are clearly similar to the cyclonic-anticyclonic asymmetry of the nonlinear Kelvin-Helmholtz instability demonstrated in Fig. 2.

Regarding the threshold for the instability, we note the following. The maximum velocity of the zonal wind in the region of the Great Red Spot is[4] $|u|_{\max} = 50\text{–}60$ m/s, while the Rossby velocity is[8] $V_R = 160$ m/s, so that the condition $|u|_{\max} \gtrsim V_R$ does not hold. One way out of this difficulty is (for example) to argue that the Spot is, from the wave standpoint, more a three-dimensional formation than a two-dimensional one, and in this situation the wave motion along the vertical direction causes the drift velocity of the Spot to decrease and to become much lower than the value of V_R introduced above.[8] With regard to why there is only one Spot (more than ten vortices of this scale could be fitted in along the circumference of the planet), the following might be suggested: If the Spot arose from a prethreshold state of the system as a result of some "local" process, then in a system with hysteresis this azimuthally inhomogeneous state might be maintained by the existing shear of the zonal flows, even if this shear was insufficient to excite a chain of vortices (under the conditions of Fig. 2, this chain consists of three or four vortices). There thus might be only a single Spot over the entire zonal belt.

We wish to thank V. V. Kadomtsev for stimulating discussions; A. M. Obukhov, F. V. Dolzhanskiĭ, and Yu. L. Chernous'ko for consultations and assistance; and S. V. Antipov, V. K. Rodionov, and A. N. Khvatov for assistance.

[1]E. B. Gledzer, F. V. Dolzhanskiĭ, and A. M. Obukhov, Sistemy gidrodinamicheskogo tipa i ikh primenenie

(Systems of the Hydrodynamic Type and Their Applications), Nauka, Moscow, 1981.
[2]O. V. Dolzhanskiĭ, Izv. Akad. Nauk SSSR, Ser. Fiz. Atm. Okeana **17**, 563 (1981).
[3]Yu. L. Chernous'ko, Izv. Akad. Nauk SSSR, Ser. Fiz. Atm. Okeana **16**, 423 (1980).
[4]B. A. Smith and J. Hunt, in: Jupiter (ed. T. Gerels) (Russ. transl. Mir, Moscow, 1979, Vol. 2); E. J. Reese and B. A. Smith, Icarus **9**, 474 (1968).
[5]V. I. Petviashvili, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 632 (1980) [JETP Lett. **32**, 619 (1980)].
[6]T. Maxworthy and L. G. Redekopp, Icarus **29**, 261 (1976); Science **210**, 1350 (1980).
[7]R. Z. Sagdeev, V. D. Shapiro, and V. I. Shevchenko, Pis'ma Astron. Zh. **7**, 505 (1981) [Sov. Astron. Lett. **7**, 279 (1981)].
[8]M. V. Nezlin, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 83 (1981) [JETP Lett. **34**, 77 (1981)].
[9]S. V. Antipov, M. V. Nezlin, E. N. Snezhkin, and A. S. Trubnikov, Zh. Eksp. Teor. Fiz. **82**, 145 (1982) [Sov. Phys. JETP **55**, 85 (1982)]; Pis'ma Zh. Eksp. Teor. Fiz. **33**, 368 (1981) [JETP Lett. **33**, 351 (1981)].
[10]L. A. Dikiĭ, Gidrodinamicheskaya ustoĭchivost' i dinamika atmosfery (Hydrodynamic Stability and Dynamics of the Atmosphere), Gidrometeoizdat, Leningrad, 1976, p. 63; P. H. Leblond and L. A. Mysak, Waves in the Ocean, Elsevier, New York, 1977 (Russ. transl. Mir., Moscow, 1981, Vol. 2).

Translated by Dave Parsons
Edited by S. J. Amoretty