Transmission coefficient and the localization length of an electron in \( N \) bound disordered chains

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A model of \( N \)-coupled chains with defects for which it is possible to prove rigorously the presence of localization with arbitrarily strong coupling between chains is examined. \( N \) different localization lengths arise under these conditions.

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The exact solution of the problem of localization of single-electron states is known only for an elementary metallic chain, when the localization length \( l^* \) is of the order of the free path \( l \). On the other hand, there are strong arguments in favor of localization of all single-electron states in a metallic wire and in the two-dimensional case when \( l^* \gg l \) with weak disordering \( p_{\text{F}} \gg 1 \). For this reason, it is interesting to examine this problem for \( N \) coupled chains.

As is well known, the localization length enters explicitly into the expression for the decrease in the electron transmission coefficient with increasing dimensions \( L \) of the disordered region: \( T \sim \exp\left( -L/l^* \right) \). Starting from this, in order to determine \( l^* \) we shall investigate the behavior of the transmission coefficient \( T(L) \). This approach to the problem turns out to be simpler than investigating the density-density correlation function. It is assumed that the disordered segment \((0, L)\) consists of \( N \) chains, which can be represented as lying on a cylindrical surface parallel to its axis. The electron, together with the predominant motion along the chain, can hop in a perpendicular direction. Thus, in the system of functions \( \{ \psi_n(x) \} \), where \( \psi_n(x) \) corresponds to the \( n \)-th chain, the Hamiltonian has the form

\[
H_{nn'} = \left[ -\frac{1}{2m} \frac{d^2}{dx^2} + U_n(x) \right] \delta_{nn'} + t (\delta_{n-1}, n' + \delta_{n+1}, n').
\] (1)

Here \( U_n(x) \) is a random impurity potential for the \( n \)-th chain and \( t \ll E_\text{F} \) is the overlap integral describing electron hops to neighboring chains. In order to restrict the analysis to \( N \) chains, we shall supplement (1) with the following boundary condition:
\( \psi_N(x) = e^{i\alpha} \psi_0(x). \)  

The phase factor \( e^{i\alpha} \), which is responsible for the phase increase \( \alpha \) with each revolution around the cylinder axis, destroys the symmetry with respect to time inversion (\( \psi \rightarrow \psi^* \)). This can be attributed to some magnetic flux inside the cylinder.

It is convenient to go over to a new system of wave functions:

\[
\tilde{\psi}_n(x) = \frac{1}{\sqrt{N}} \sum_{n'=0}^{N-1} \exp(2\pi i n'/N) \psi_{n'}(x),
\]

\[
\tilde{H}_{nn'} = \left[ -\frac{1}{2m} \frac{d^2}{dx^2} + 2t \cos \left( \frac{(2\pi n + a)}{N} \right) \right] \delta_{nn'} + \frac{1}{N} \sum_{n''=0}^{N-1} U_{n_0}(x) \exp(2\pi i n_0(n - n' - 1)/N).
\]

In the ordered regions, where \( U_n(x) = 0 \), it is possible to write immediately the solution of the Schrödinger equation in the form

\[
\tilde{\psi}_n(x) = A_n \exp(ik_n x) + B_n \exp(-ik_n x),
\]

\[
k_n = \sqrt{2mE - 2t \gamma^{-1} \cos \left( \frac{(2\pi n + a)}{N} \right)}.
\]

The constants \( A_n^L, B_n^L \) and \( A_n^R, B_n^R \), which correspond to solution (5) on the left and right of the disordered section \((0, L)\), are related by a linear transformation

\[
\begin{bmatrix}
A_n^R \\
B_n^R
\end{bmatrix} = \hat{m} \begin{bmatrix}
A_n^L \\
B_n^L
\end{bmatrix},
\]

where \( \hat{m} \) is a 2\(N\)-dimensional complex matrix. The necessary information on scattering is contained in the matrix \( \hat{M} = \hat{m}^* \hat{m} \), which was already introduced in Ref. 6 for a single chain. The condition for flux conservation imposes certain constraints on \( \hat{m} \) and \( \hat{M} \). We shall immediately write \( \hat{M} \) in a form that satisfies these requirements:

\[
\hat{M} = \begin{bmatrix}
\hat{u}^* & \hat{u}^* \\
\hat{v}^* & \hat{v}^*
\end{bmatrix} \begin{bmatrix}
\hat{\Gamma} & \hat{\gamma} \\
\hat{\gamma} & \hat{\Gamma}
\end{bmatrix}.
\]

Here \( \hat{u}, \hat{v} \) are unitary matrices, while \( (\hat{\Gamma})_{nn'} = \Gamma_n \delta_{nn'} \) is a diagonal real matrix which determines the \( N \) transmission coefficients. In fact, if there is an incident flux \( j_A^L = (A^+ A)^L \) and a reflected flux \( j_B^L = (B^+ B)^L \) on the left of the disordered region, while on the right there is only a transmitted flux \( j_A^R = (A^+ A)^R \), then it is possible to express \( j_A^R \) in terms of the amplitude \( A_n^R \) of the incident flux:

\[
j_A^R = (A^+)^L \hat{u}^* \left( 2[\text{ch} \hat{\Gamma} + 1]^{-1} \right) \hat{\gamma} (A^+)^L.
\]

The diagonal matrix in braces in (9) contains \( N \) transmission coefficients \( T_n = 2/(\text{ch} \Gamma_n + 1) \).
In order to obtain a Fokker-Planck equation for the distribution function \( W(L; \hat{\Gamma}, \hat{\mu}, \hat{\nu}) \), it is necessary to follow the change in \( W(L \ldots) \) with a small increase in the length \( L \) of the disordered section. We shall assume that the transmission amplitude through the separate impurity \( d = \cos \gamma \) is close to unity: \( \gamma \approx 1, \delta = 0 \). Thus the scattering matrix \( \hat{m} \) for an isolated impurity at the point \( x_0 \) of chain \( n_0 \) has the form:

\[
\hat{m}(x_0 n_0) = 1 + \begin{bmatrix}
0 & \hat{\gamma}(x_0 n_0) \\
\hat{\gamma}^*(x_0 n_0) & 0
\end{bmatrix},
\]

\[
[ \hat{\gamma}(x_0 n_0) ]_{nn'} = -i \gamma N^{-1} \exp[-i(k_n + k_{n'}) x_0 + 2\pi n_0 (n - n') / N].
\] (10)

Here we shall consider only the simplest case when, in deriving the Fokker-Planck equation, the averaging over the rapid dependences is performed with the help of the relation

\[
\gamma_{nn'} \hat{\gamma}_{mm'}^* = \gamma^2 N^{-2} \delta_{nn'} \delta_{mm'}.
\] (11)

Using (6) and (10), we see that (11) is satisfied when \( N \) is an odd number, that there is no symmetry with respect to time inversion (i.e., \( \alpha / \pi \) is not an integer) and that the following inequality is satisfied:

\[
tl/v_F N^3 \gg 1.
\]

Relation (11) is the key to simplifying the problem. In this case the distribution of \( \hat{\Gamma} \) separates from the distributions of the matrices \( \hat{\mu}, \hat{\nu} \). We note that this simplification does not occur for systems consisting of \( N \) chains lying on a surface and, in particular, for two chains.

The equation for the distribution function \( W(L, \hat{\Gamma}) \), where \( \hat{\Gamma} = c \Gamma \hat{\Gamma} \), (i.e., \( T_n = 2/(F_n + 1) \)) has the form

\[
NL \frac{\partial W}{\partial L} = \sum_{n = 0}^{N - 1} \frac{\partial}{\partial F_n} \left( F_n^2 - 1 \right) \left[ \frac{\partial}{\partial F_n} - 2 \sum_{n' \neq n} \frac{1}{F_n - F_{n'}} \right] W(L, \hat{\Gamma}).
\] (12)

An analysis of Eq. (12) shows that it contains \( N \) different exponential dependences \( F_n \sim \exp(L / \lambda_n^*) \) and in addition the exponents differ generally by a quantity of the order of \( L / NL \). This means that at distances \( L \gg NL \) all \( F_n \) have different orders of magnitude. Let us renormalize \( F_n \) in increasing order. Thus for \( L \gg NL \) we have

\[
F_0 \ll F_1 \ll \ldots \ll F_{N - 1}.
\] (13)

The strong inequality (13) allows decoupling Eq. (12) into \( N \) equations for the distribution functions \( W_n(L, F_n) \):

\[
NL \frac{\partial W_n}{\partial L} = \left[ \frac{\partial}{\partial F_n} \left( F_n^2 - 1 \right) \frac{\partial}{\partial F_n} - 2n \frac{\partial}{\partial F_n} F_n \right] W_n(L, F_n),
\] (14)
where the number 1 next to $F^2_n$ should be retained only for $n = 0$, when (14) coincides with the equation for a single chain\(^5\) to within the substitution $l \to Nl$.

An equation for $N$ different localization lengths follows from (14) ($l^*$ is the localization length in an isolated chain):

$$I_n^* (N) = Nl^* / (1 + 2n); \quad n = 0, 1, \ldots, N - 1.$$  \hspace{1cm} (15)

They have the curious property that the average of their inverses is independent of $N$ and is equal to $1/l^*$. The largest localization length $I^*_N(N) = Nl^*$ will make the greatest contribution to long-wavelength kinetic phenomena.

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**Structure functions for the electrodisintegration of the deuteron at a momentum transfer of 1.8 GeV\(^2\)**


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Structure functions are found for the electrodisintegration of the deuteron near the threshold in the relativistic region with $q^2_{\text{max}} = 1.8 \text{ GeV}^2$ and $\nu = 0.5–0.65 \text{ GeV}$.

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One of the fundamental problems of hadron physics is to determine the nucleon-nucleon ($NN$) potential. Information on the $NN$ interaction slightly off the mass shell is particularly important. Such information can be found from the electrodisintegration of the deuteron at large momentum transfer. Under these conditions there should be a cumulative effect, interpreted as scattering by a state which is an “overlap” of two