The $Q^2$ dependence of the average multiplicity in deep inelastic processes

A. V. Kiselev and V. A. Petrov

Institute of High Energy Physics

(Submitted 10 November 1985)
Pis'ma Zh. Eksp. Teor. Fiz. 43, No. 1, 5–7 (10 January 1986)

An equation obtained by the authors previously is used to calculate a detailed $Q^2$ dependence of the average hadron multiplicity in $\mu p$ collisions. The results are in agreement with the latest experimental data of the EMC collaboration.

The multiple production processes in deep inelastic scattering of leptons by hadrons are aimed at achieving a better understanding of the transformation of energy into matter as a function of the size of the interaction region. This size is determined by the $Q^2$ transfer of the 4-momentum from the initial lepton to the hadron. The diameter of the interaction region decreases with increasing $Q^2$. Simple considerations based on a graphic model in which a large energy $W$ is concentrated in a small volume of space show that the average number of secondary particles in this case will increase with increasing $Q^2$. Such an increase may conceivably be accompanied by a change in the nature of the functional dependence on $W$.

The dependence of the average multiplicity of charged hadrons $\bar{n}(W^2,Q^2)$ on variable $Q^2$ was studied in $\nu p$ and $\bar{\nu} p$ collisions in the energy interval $3 < W < 14$ GeV (the WA21 experiment on VEVS). The experimental data reported by Schmitz$^2$ are consistent, within the error margin, with the slight increase of the average multiplicity in $Q^2$ at $Q^2 > 10$ GeV$^2$. However, because of the large errors of this experiment, an unambiguous conclusion about $Q^2$ dependence of $\bar{n}$ cannot be made.

The latest exact measurements of the average multiplicity of charged hadrons in $\mu^+p$ interaction, carried out by the EMC collaboration$^3$ (the NA9 experiment), have shown that $\bar{n}$ does in fact increase with $Q^2$ at constant $W$ in the range $4 < W < 20$ GeV (Fig. 1). The derivative of the average multiplicity $\partial \bar{n}/\partial \text{ln} Q^2$ in the energy range indicated above tends to decrease$^3$ with increasing $W$ at fixed $Q^2$ (Fig. 2). At the same time, at reasonably high energies ($W > 10$ GeV) this tendency becomes indistinct and $\partial \bar{n}/\partial \text{ln} Q^2$ begins to oscillate around the mean value of 0.23. These data are clearly at variance with the popular LUND model,$^3$ for example, in which $\bar{n}$ decreases with increasing $Q^2$. The dashed curves in Figs. 1 and 2 are the predictions of the LUND model.

Kiselev and Petrov$^4$ proposed the following equation for $n$ which was derived by them$^5$ on the basis of perturbative QCD:

$$\bar{n}(W^2, Q^2) = \bar{n}_{e^+e^-} \bar{W}_{\text{eff}} + \bar{n}_{\text{diq}},$$

where $\bar{n}_{e^+e^-}$ is the average hadron multiplicity in $e^+e^-$ annihilation and $\bar{n}_{\text{diq}}$ is the
The diquark component $\bar{n}_{\text{dij}},$ which is bounded above by a $Q^2$-independent quantity, is a phenomenological parameter.

The invariant mass $W_{\text{eff}}$ required to produce hadrons in the parton subprocess is given by

$$W_{\text{eff}}^2 = \frac{\xi(Q^2)}{\xi(Q^2) + n} W^2,$$  \hspace{1cm} (2)

where

$$\xi(Q^2) = \frac{4}{3} \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s k^2}{2\pi}$$

is the evolutionary QCD parameter, and $n$ is the vanishing factor for the $u$-quark distribution in the proton.

Kiselev and Petrov\textsuperscript{4} used Eq. (1) to describe the behavior of $\bar{n}$ with respect to variable $W$ in deep inelastic $\nu p$ and $\bar{\nu} p$ processes.
Equation (1) implies that the hadron multiplicity in $\mu p$ interaction increases with $Q^2$ at fixed $W$, since $W_{\text{eff}}$ in (2) is a monotonically increasing function of the variable $Q^2$. The solid curves in Fig. 1 are the calculations based on Eq. (1) in the region in which it is applicable ($W > 10$ GeV) with
\[ \bar{n}_{\text{diss}} = 2.25. \] (3)

For $\bar{n}_{e^+e^-}$ the following equation is quite suitable, beginning at energies $W = 7-8$ GeV:
\[ \bar{n}_{e^+e^-}(W^2) = 2.85 + 0.09 \exp\sqrt{2.88 \ln W^2 / \text{GeV}^2} \] (4)

We should point out that the $W^2$ dependence of $\bar{n}_{e^+e^-}$ and the argument of the exponential function in (4) were found within the framework of perturbative QCD (see Ref. 6, for example). From (1)-(4) we find
\[ \frac{\partial \bar{n}}{\partial \ln Q^2} = 1.84 \frac{\bar{n} - 5.1}{\ln(\bar{n} - 5.1)} + 2.4 \frac{1}{\xi(\xi + n) \ln Q^2 / \Lambda^2}. \] (5)

It follows from expression (5) that the derivative of the average multiplicity $\partial \bar{n} / \partial \ln Q^2$ is 1) always positive and 2) increases with increasing $W$ at fixed $Q^2$.

The calculations based on Eq. (5), which are represented by a solid curve in Fig. 2, show that in the energy region where Eqs. (1) and (2) apply, they are consistent with the experimental data within the measurement errors. This cannot be said of the LUND model and the dual unitarization scheme, in which $\bar{n}$ decreases with $Q^2$. The theoretical value of $\partial \bar{n} / \partial \ln Q^2$ increases from 0.21 at $W = 10$ GeV to 0.25 at $W = 20$ GeV. It follows from (5) that $\partial \bar{n} / \partial \ln Q^2$ will increase further with energy. In particular, at $W^2 = 1000 \text{ GeV}^2$ a value of 0.28 is predicted.

As we can see, the proposed equations for the average multiplicity give clear qualitative and quantitative conclusions on the $Q^2$ dependence of $\bar{n}(W^2, Q^2)$. Further experimental studies of finite hadron states in deep inelastic processes at higher $W$ (the E 665 experiment to be performed on the FERMILAB tevatron, for example) will presumably give a definite answer as to whether the theoretical understanding of the deep inelastic processes developed in Refs. 1, 4, and 5 is correct.


Translated by S. J. Amoretty