Superconducting system with weak coupling to the current in the ground state

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We consider a superconducting ring with a Josephson junction containing magnetic impurities. It is shown that if the tunneling through the magnetic impurities is large enough, then the ground state of such a system is a state with a current and a magnetic flux.

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We consider a superconducting ring with a Josephson junction containing magnetic impurities. We show that if the tunneling through the magnetic impurities is large enough, then the ground state for such a system is a state with nonzero electric current and magnetic flux.

1. Consider a Josephson junction with magnetic impurities inside the dielectric layer. The tunnel Hamiltonian of this junction can be written in the form

$$H_T = \sum_{\substack{\mathbf{k},\mathbf{k}',\mathbf{n}\\ s,s'}} (t_{\mathbf{k}\mathbf{k}'}\delta_{ss'} + v_{\mathbf{k}\mathbf{k}'n}\vec{\sigma}_{ss'}\mathbf{S}_{\mathbf{n}})a^{+}_{\mathbf{k}s}b_{\mathbf{k}'s'} + \text{h.c.}, \tag{1}$$

where $a_{\mathbf{k}s}(b_{\mathbf{k}s})$ is the annihilation operator for a conduction electron with wave vector $\mathbf{k}$ and spin s in the layer $A(B)$, $\mathbf{S}_n$ is the operator of the localized spin at site n in the layer, and $\boldsymbol{\sigma}$ are Pauli matrices. [1–3]

In the case of a tunnel Hamiltonian (1), we obtain for the stationary Josephson current and for the energy of the junction

$$J = (J_0 - J_s)\sin\phi, \qquad E = -\frac{\hbar}{2l}(J_0 - J_s)\cos\varphi,$$
$$J_0 - J_s = 2\pi^2 e\,[\,t^2 - \sum_{\mathbf{n}} v_{\mathbf{n}}^2 S(S+1)\,]\,N^2(0)\Delta\,\mathrm{th}\frac{\Delta}{2T}, \tag{2}$$

where $J_0 \propto t^2$ corresponds to the first term in the round brackets of (1) (with the Hermitian-conjugate term), and J corresponds to the second term, which takes into account the possibility of electron tunneling with spin flip (the localized spins are assumed to be disordered), t^2 and v_n^2 are the mean values of $|t_{\mathbf{kk}'}|^2$ and $|v_{\mathbf{kk}'\mathbf{n}}|^2$ on the Fermi surface, $N(0)$ is the density of states on the Fermi surface, $2\Delta(T)$ is the energy gap, and ϕ is the phase discontinuity on the Josephson junction. [1,4]

According to[2,3] we have

$$v_{\mathbf{kk}'\mathbf{n}} \sim [\epsilon_d(\epsilon_d + U)]^{-1}, \tag{3}$$

where $\epsilon_d < 0$ is the energy of one electron in a localized magnetic state, reckoned from the Fermi energy, while $2\epsilon_d + U > 0$ is the energy of two electrons in this state with allowance for their Coulomb repulsion U. By proper choice of the energy ϵ_d, the absolute value of $v_{\mathbf{kk}'\mathbf{n}}$ can be made large enough. The parameter $t_{\mathbf{kk}'}$ in (1) is a sum of two terms, one of which is $-\frac{1}{2}\sum_{\mathbf{n}} v_{\mathbf{kk}'\mathbf{n}}$, and the second describe all the remaining forms of tunneling. [3] In principle, these terms of $t_{\mathbf{kk}'}$ can cancel each other and cases are possible when the tunneling described by the second term of (1) predominates. It is precisely such contact with $J_s > J_0$ which will be considered below. Both the energy of the junction and the current, as functions of the phase discontinuity ϕ in the junction, have opposite signs in comparison with the corresponding expressions for an ordinary junction without magnetic impurities. The ground state of a junction with $J_s > J_0$ corresponds to a phase discontinuity $\phi \cong \pi$, and not zero as in the usual case, and we name this junction a π junction.

2. We consider a superconducting system consisting of a bulky superconductor short circuited by a π junction (see Fig. 1). Such a system corresponds to the ring of Silver and Zimmerman[5] in which the usual Josephson junction was replaced by a π junction. For simplicity we assume a pointlike junction, i.e., with dimensions smaller than the Josephson length. For the total current J in the π junction and for the current density j in the superconductor we have

$$J = -J_c\sin\phi, \qquad \mathbf{j} = b\left(\hbar\nabla\alpha - \frac{2e}{c}\mathbf{A}\right), \tag{4}$$

where $J_c = |J_0 - J_s|$, $\nabla\alpha$ is the gradient of the phase in the superconductor, while the coefficient b depends on the concentration of the superconducting electrons. The usual procedure (see[6]) for integrating the phase over a closed con-

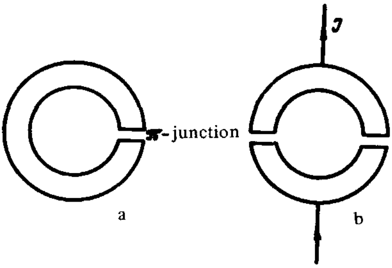

FIG. 1. a) Superconducting ring with π junction, b) superconducting quantum interferometer with two Josephson junctions.

tour that passes through the superconductor in the region with zero current density in the absence of an external field yields the following equations for the flux Φ and for the system energy E:

$$\Phi = \frac{\phi}{2\pi}\Phi_0, \qquad \sin\phi - k\phi = 0, \qquad k = \frac{c\Phi_0}{2\pi L J_c}, \qquad E(\phi) = \frac{\hbar}{2e} J_c(\cos\phi + \tfrac{1}{2}k\phi^2), \tag{5}$$

where Φ_0 is the magnetic-flux quantum and L is the inductance of the system. At $k>1$ the ground state of the system corresponds to a solution with $J=0$, $\Phi=0$, and $E_0=E(0)=\hbar J_c/2e$. This is just the solution (but with opposite sign of E_0) realized for a ring with an ordinary junction.

At $k<1$, the ground state corresponds to a solution with nonzero current and nonzero magnetic field. In the ground state of the system, the current J increases as k decreases from unity to $2/\pi$, reaches a maximum ($J=J_c$, $\Phi=\Phi_0/4$) at $k=2\pi$, and then decreases with further decrease of k. The energy E_0 decreases monotonically from $E(0)$ to $-E(0)$ when k decreases from unity to zero,

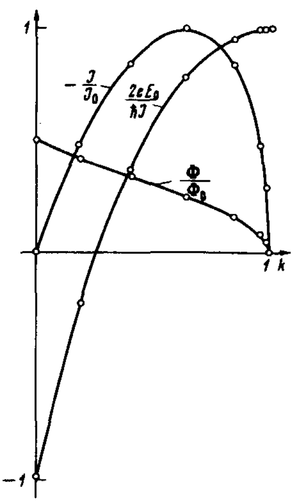

FIG. 2. Dependence of the energy, current, and magnetic flux through the ring in the ground state of the system on the parameter k.

while the magnetic flux increases monotonically with decreasing k, approaching $\Phi_0/2$ as $k \to 0$ (see Fig. 2).

At $k \ll 1$ we obtain

$$\Phi = \tfrac{1}{2}\Phi_0 (1 - k), \quad J = - J_c \pi k = - \frac{c\Phi_0}{2L}, \quad E_0 = E(0)\left(- 1 + \frac{\pi^2}{2} k\right). \tag{6}$$

In this case there is located, at an energy lower than that of the currentless states, a ground state with $\Phi \cong \Phi_0/2$ and approximately $1/\pi\sqrt{k}$ excited states of the system, with a magnetic flux close to half-integer multiples of the flux quantum Φ_0. From among these states with $E < E(0)$, the maximum energy and flux is reached in the state with $\Phi \cong \sqrt{2LJ_c\Phi_0/\pi}$. If the junction has dimensions much larger than the Josephson length, then the ground state corresponds to flux $\Phi = \frac{1}{2}\Phi_0$. The current flows over the surface of the ring, decreasing at the London depths in the interior of the superconductor and at the Josephson depth in the interior of the junction.

Thus, if the π junction can be realized experimentally, then an undamped current and an associated magnetic flux should appear (spontaneously) in a ring with such a junction below the superconducting-transition point. The spontaneous flux is close to $\Phi_0/2$ for a point junction at $k \ll 1$ and is equal to $\Phi_0/2$ for a broad junction. This flux can be observed experimentally.

It is convenient to investigate the influence of magnetic impurities on the properties of Josephson junctions by using a "quantum interferometer"[7] (Fig. 1(b); see also[6]).

If Φ is the flux through the closed contour of such an interferometer, and J_{c1} and J_{c2} are the critical currents of the Josephson junctions of the circuit, then the expression for the critical interferometer current J_m is

$$J_m = \left[(J_{c1} - J_{c2})^2 + 4 J_{c1} J_{c2} \cos^2 \frac{\pi\Phi}{\Phi_0}\right]^{1/2} .$$

If magnetic impurities are introduced in one of the junctions of the interferometer, the amplitude of the oscillations of the critical current J_m will first decrease, the oscillations will vanish at $J_c = |J_0 - J_s| = 0$, and then, with further increase of the impurity concentration, the maxima and the minima on the $J_m(\Phi)$ curve exchange places, thus indicating that a π junction has been produced.

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