Thermo-emf and the critical temperature of the superconducting transition of thallium and rhenium accompanying a topological transition

N. V. Zavaritskii, V. I. Makarov, and A. A. Yurgens

Institute of Physics Problems, Academy of Sciences of the USSR Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR

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Thermo-emf in rhenium and thallium produced as a result of a pressure-induced topological transition exhibits large anomalies (on the order of a measurable quantity) which uniquely correspond to a nonlinear variation of the critical superconducting transition temperature $T_c$. The intrinsic lifetime of quasiparticles (charge carriers) has a considerable effect on the magnitude of the thermo-emf anomaly.

Topological transitions in an electron system were predicted theoretically by Lifshitz and observed experimentally in several metals under hydrostatic compression and under an elastic uniaxial deformation. These transitions are also accompanied by thermo-emf anomalies $\alpha$ manifested as extrema in the dependence of $\alpha$ on the parameter which determines the Fermi energy. These particular features of the thermo-emf were studied in 2D systems by varying the surface-charge density, under an elastic uniaxial deformation of metals, and in the $\text{Li}_{1-x}\text{Mg}_x$ alloys.

In the present letter we report a study of pure thallium and rhenium and also thallium with a mercury impurity, in which the transitions where previously detected from the $T_c$ anomalies upon a hydrostatic compression.

The thallium samples with $RRR \sim 6000$ and with a small mercury impurity ($\lesssim 1.0\%$) were prepared the same way as in Ref. 2, by pressing the material through a 4-mm-diam hole and then annealing it for 50 h at 100 $^\circ$C. The $RRR$ of rhenium single crystals $2\times2$ mm in cross section, with the (0001) and $\bar{1}$(0001) axes directed along the sample was in the range from 500 to 22 000.

To measure the thermo-emf under a pressure of up to 13 kbar, we used a high-pressure chamber with an 8-mm channel diameter. The temperature gradient along the samples ($\sim 0.02$ K) and the average temperature $T$ (up to 7 K) of the samples were measured with a 3LZh-superconductor thermocouple, whose characteristics were constant up to 12 kbar. The potential difference was determined through a compensation with a SKIMP as the null indicator (the sensitivity is $10^{-14}$–$10^{-15}$ V). The temperature dependence of the sample's thermo-emf and its critical temperature $T_c$ were measured at the given pressure. The pressure was determined from the manganin and superconducting pressure gauges to within 200 bar. The agreement between $\alpha(T)$ measured in a vacuum and $\alpha(T)$ measured in a high-pressure chamber at $P \sim 0$ kbar was a test of the validity of the measurement results.

The measurement results of the thermo-emf of thallium and rhenium are shown as a plot for $a = \alpha/T$ versus $T^2$ in Fig. 1 which is generally used in the analysis of the
thermo-emf. The peak on the $a(T^2)$ curves at $T^2 \sim 20(T \sim 4.5 \text{ K})$ is attributed, as in Ref. 14, to the transfer processes in the phonon system. The most intriguing result of those we obtained is the nonmonotonic dependence of the thermo-emf of the Tl + 1% Hg sample on the pressure. As the pressure is increased from 0 to 6 kbar, we observe a significant increase (up to a factor of three) in the thermo-emf. As the pressure is further increased to 12 kbar, the thermo-emf decreases almost to the original value. This behavior correlates well$^1$ with the particular features of $(\partial T_c/\partial P)(P)$ (Fig. 2). The thermo-emf anomaly we have observed is asymmetric with respect to the maximum, a characteristic feature of the topological transitions.$^{1,2}$ The pressure at which the transition occurs, which is determined from these results, is $P_c \sim 5$ kbar for this sample. There is also a correlation between the $T_c(P)$ and $a(P)$ curves for rhenium (Fig. 2). It turned out in this case, however, that $P_c \sim 10-12$ kbar for the samples studied by us. This value is greater than that reported previously$^4$: $P_c \sim 8$ kbar, so that we can detect a change in the values only in the region $P \leq P_c$.

The features on the $a(P)$ curves for rhenium and Tl + 1% Hg are smeared with respect to pressure, presumably because of the considerable scattering of electrons by the impurities.

Evidence for this conclusion comes from the fact that the $a(P)$ curve for the Tl + 0.2% Hg sample decreases more sharply as $P \to P_c$ (estimates based on this sample show that $P_c \sim -1$ kbar). At $P > P_c$ the thermo-emf anomaly varies as $\Delta a \sim (P - P_c)^{-1/2}$ (the insert in Fig. 2) in a first approximation, a typical feature in topological transitions.
The mean free path \( l \) of an electron, which is determined by the scattering by the impurities at the topological transition point,\(^2\) is\(^7\)

\[
l(p, \epsilon) = l_0(p, \epsilon) + \delta \nu(\epsilon) l_1(p, \epsilon),
\]

where \( \epsilon \) is the electron energy, and \( p \) is the electron momentum. The functions \( l_0 \) and \( l_1 \) depend smoothly on \( p \) and \( \epsilon \). \( \delta \nu \) is an irregular part of the electron-state density which is associated with a change in the topology of the surface \( \epsilon(p) = \epsilon \).

Following Ref. 15, we see that the thermo-emf anomaly, which is attributable to the second term in (1), is described by the equation

\[
\delta(\alpha \sigma) = -\frac{2e}{T} I_0(\mu) \int_{-\infty}^{+\infty} (\epsilon - \mu) \delta \nu(\epsilon) \frac{\partial f_0}{\partial \epsilon} \, d\epsilon,
\]

where \( f_0 \) is the Fermi distribution function, \( e \) is the electron charge, \( \mu \) is the chemical potential, and \( \sigma \) is the electrical conductivity of the metal.

The function \( I_0(\mu) \) for a crystal of cubic symmetry is

\[
I_0(\mu) = \frac{1}{3(2\pi \hbar)^3} \int \frac{dS_p}{|v_p|} \left( \epsilon(p) = \epsilon v_p = \partial \epsilon / \partial p \right)
\]

where \( dS_p \) is the area element of a constant-energy surface, and \( \epsilon(p) = \epsilon v_p = \partial \epsilon / \partial p \) is the electron velocity.
If $\delta \alpha / \alpha_0 \gg \delta \sigma / \sigma_0$ (which is usually the case), the quantity $\delta \alpha$ is proportional to the anomalous part of the electronic component of the thermal expansion coefficient $\delta \beta$; i.e.,

$$\delta \alpha = c \delta \beta. \quad (3)$$

The numerical factor $c$ can easily be found by comparing Eq. (2) with Eq. (3) of Ref. 16.

We see that expressions (2) and (3) are valid even when the finite lifetime of quasiparticles, $\tau_p$, is taken into account. This lifetime is governed by the interaction of electrons with impurities for an arbitrary relationship between the quantities

$$\mu \sim \varepsilon_c, \; T \text{ and } \Gamma, \text{ and } \mu \gg T, \; \Gamma \quad (\Gamma = \hbar / \tau_p).$$

For a topological transition corresponding to the formation of a new part of the Fermi surface, we have in this case

$$\delta V(\varepsilon) \sim \left[ \sqrt{\left( \varepsilon - \varepsilon_c \right)^2 + \Gamma (P_c)^2 + (\varepsilon - \varepsilon_c)^2} \right]^{1/2}, \quad (4)$$

where $\varepsilon_c$ is the critical energy.

A blurring of the features in Fig. 2 shows that a comparison of theory with experiment should be based on the use of Eq. (2) with the help of (4) [see Eq. (3) in Sec. 2 of Ref. 16].

After such a comparison, we can estimate the parameters characterizing a topological transition. In $T1 + 1\%$ Hg we have

$$\Gamma^2 (P_c) \sim 1 \text{ K}^2; \; P_c \approx 4.6 \text{ kbar.}$$

From the sign of $\delta \alpha > 0$ we conclude that the metals we have studied form new regions of the Fermi surface under hydrostatic compression.

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1The results of the calculation of the dependence $T_c(P)$ for T1 and Re are consistent with those obtained previously. 2

2A change in the phonon drag due to a topological transition was ignored in this study.
Oscillations of the boundary between $A$ and $B$ phases of a superfluid $^3$He

A. V. Markelov

Institute of Physics Problems, Academy of Sciences of the USSR

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The hydrodynamics of the boundary between the $A$ and $B$ phases of superfluid $^3$He is analyzed. The existence of two weakly damped surface modes is predicted.

The boundary between coexisting $A$ and $B$ $^3$He phases is apparently a unique example of an interface between two superfluid liquids. The peculiar behavior of this system stems from the presence of a condensate. The $^3$He atoms in the condensate can in a thermodynamically equilibrium state tunnel through the boundary which is a potential barrier of height on the order of the energy gap $\Delta$. The dissipative current flowing across the boundary, which is associated with the supercondensate particles, is, on the other hand, exponentially small [within $\sim \exp(-\Delta/T)$] because of an exponential decrease of the density with the temperature in the $B$ phase and because of the elastic reflection of quasiparticles of energy lower than the energy gap $\Delta$ upon collision with the boundary in the $A$ phase. Consequently, in contrast with the ordinary liquids, we are dealing with a two-parameter family of Goldstone transformations that conserve the energy of the system. This process first involves a uniform displacement of the interface in the case of a liquid at rest and, secondly, it involves a flow of liquid in the case of a stationary boundary. Accordingly, the oscillation spectrum of the boundary between $A$ and $B$ phases should have two weakly damped modes.

Let us consider the wave spectrum at $T = 0$. Since the energy of the $A$ phase decreases $\sim \chi H^2$ in a magnetic field and the energy of the $B$ phase remains virtually the same, this situation can actually occur in $\sim 10^4$-G fields. We assume that the boundary is described by the equation $z = \xi(x,t)$ (at equilibrium we have $z = 0$), the $A$ phase occupies the half-space $z > \xi$, the $B$ phase occupies the half-space $z < \xi$, the