Localized topological solitons in a ferromagnet

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We show that topological solitons which are localized in three dimensions may exist in a ferromagnet. A soliton corresponds to the Hopf invariant; soliton size is stabilized by the law of conservation of the number of spin deviations.

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Earlier studies of ferromagnets have dealt with either solitons that were nonlocalized in three dimensions (domain walls, "hedgehogs," see Ref. 1) or localized nontopological solitons.[2] Localized solitons correspond to a homogeneous distribution of magnetization at infinity **M** (**r**). Consequently, topological soliton analysis involves studying the properties of transformation of a 3-*D* space $\{\mathbf{r}\}$ with identically infinitely-distant points (which is equivalent to a 3-*D* sphere S^3) onto a 2-*D* sphere $\mathbf{m}^2 = 1$, where $\mathbf{m} = \mathbf{M}/M_0$, M_0 is the saturation magnetization. We know that the S^3 transformation corresponds to the Hopf invariant Z which takes on integer values. The Hopf tranformations may be easily constructed (see Ref. 3) having constructed transformation $\{\mathbf{r}\}$ onto a set of 3-*D* deflections $\hat{O} \in SO(3)$-degree of Z, such that that $O_{ik} \to \delta_{ik}$ for $|\mathbf{r}| \to \infty$ and having operated $\hat{O}$ on the vector $\mathbf{m}(\infty) = \hat{\mathbf{z}}$. Transformation with $Z = \pm 1$ corresponds to

$$\mathbf{m} = \hat{\mathbf{z}} \cos 2\chi + \hat{\mathbf{R}}(\hat{\mathbf{R}}\hat{\mathbf{z}})(1 - \cos 2\chi) + [\hat{\mathbf{R}}, \hat{\mathbf{z}}] \sin 2\chi , \tag{1}$$

where $\hat{\mathbf{R}}(\hat{\mathbf{r}})$ spans a unit sphere for the case of vector $\hat{\mathbf{r}}$ spanning unit spheres $\chi = \chi(r, \theta)$ $\chi(0, \theta) = \pi$, $\chi(\infty, \theta) = 0$, where r and θ are the spherical coordinates. The form of $\hat{\mathbf{R}}$, χ is determined by minimization of the ferromagnetic energy $W\{\mathbf{m}\}$ with allowance for Eq. (1). Even in the simple case of an isotropic ferromagnet

$$W_e\{\mathbf{m}\} = (\alpha M_0^2/2) \int (\nabla \mathbf{m})^2 d\mathbf{r}, \tag{2}$$

where $\alpha = (Isa^2/2\mu_0 M_0)$, I is exchange integral, s is atomic spin, a is lattice constant, equations for $\hat{\mathbf{R}}$, χ cannot be integrated. However, these equations are of the automodel-type, i.e., the form of the solution depends on an arbitrary constant R which characterizes the soliton dimensions. The solution asymptotes are as follows

$$\hat{\mathbf{R}}(\hat{\mathbf{r}}) \to \pm \hat{\mathbf{r}} \quad \text{at} \quad Z = \pm 1, \quad \chi(r, \theta) = \begin{cases} \pi - (r/R), & r \to 0, \\ \chi_0 (R/r)^2, & r \to \infty, \end{cases} \tag{3}$$

i.e., the soliton energy is proportional to R. However, magnetization dynamics equations contain an important motion integral—the number of spin deviations $S_z\{\mathbf{m}\}$

$$S_z = (M_0/2\mu_0) \int (1 - m_z) d\mathbf{r}. \tag{4}$$

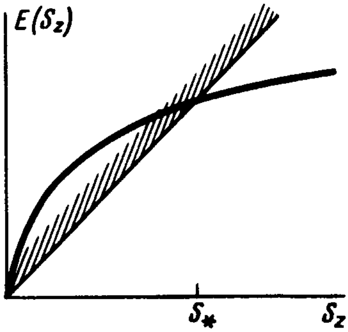

FIG. 1. Dependence of soliton energy on S_z; continuous spectrum region is shown cross-hatched; $E \geqslant \epsilon_0 S_z$.

A solution of Eq. (1) also calls for a nontrivial value of the moment of the magnetization field impulse **L**; however, **L** is expressed in terms of S_z, namely $\mathbf{L} = -(\hbar Z S_z)\hat{\mathbf{z}}$. By fixing S_z we thereby fix R and prevent a soliton collapse. It may be easily shown that

$$S_z \sim s(R/a)^3 \quad \text{or} \quad R \sim a(S_z/s)^{1/3}. \tag{5}$$

If we express the soliton energy by S_z, we get (see Fig. 1)

$$E \sim sI(s^2 S_z)^{1/3}. \tag{6}$$

Dissipation of soliton energy and the associated decrease in S_z, i.e., R, may only occur as a result of the slow process emission of magnons with momentum $\hbar k$ and energy $\epsilon(k)$. Since this process is possible at $\epsilon(k) \leqslant (dE/dS_z)$, i.e., $k \lesssim 1/R$, and its amplitude is proportional to $I(ak)^2$, the soliton lifetime $\tau = S_z/(dS_z/dt)$ at $S_z \gg s$ being large compared to $I/\hbar$

$$\tau \sim (I/\hbar s)(s/S_z)^{5/3}. \tag{7}$$

We should note that in addition to the static solutions of the form of Eq. (1), the equations permit time-dependent (but stationary from a quantum-mechanical viewpoint) solutions that contain the topological charge, in which magnetization precesses around the z-axis at a fixed frequency ω,[2], i.e.,

$$\hat{R}_z = \hat{R}_z(\hat{\mathbf{r}}), \quad \hat{R}_x + i\hat{R}_y = R^{(+)}(\hat{\mathbf{r}})e^{-i\omega t} \quad \text{or} \quad m_x + im_y = m^{(+)}(\mathbf{r})e^{-i\omega t}. \tag{8}$$

The solutions correspond to a mimimum of the function $[W\{\mathbf{m}\} + \hbar\omega S_z]$, see Ref. 2, for which $\chi \sim (1/r)\exp[-r\sqrt{\omega/2a\mu_0 M_0}]$ for $r \to \infty$. The question concerning which value of ω coresponds to a soliton energy minimum for a given value of topological charge Z and S_z remains open.[1)]

We shall take into account the energy of the magnetic anisotropy

$$W_a\{\mathbf{m}\} = (\beta M_0^2/2)\int(1 - m_z^2)d\mathbf{r}. \tag{9}$$

This gives rise to the appearance in the problem of a characteristic length

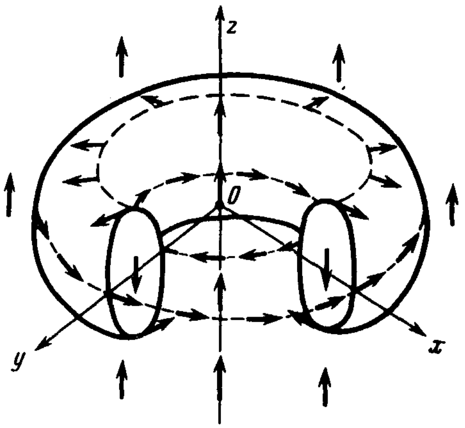

FIG. 2. Soliton shape for $R \gg x_0$. Arrows indicate direction of magnetization at certain points outside and inside a torus and at the center of domain boundary.

$x_0 = (\alpha/\beta)^{1/2} \gg a$ which has the sense of the 180° thickness of the domain boundary. The effect of anisotropy is significant only for $R \gtrsim x_0$; in particular, the asymptotics of Eq. (3) change: $\chi(r) \sim (1/r)\ \exp(-r/x_0)$ for $r \to \infty$. At $R \gg x_0$, an energetically favored magnetization configuration is one for which there exists in a soliton a region with volume $R^3 \gg x_0^3$ where $m_z \simeq -1$. This region contributes to S_z without contribution to the energy (see Eqs. (3), (9) and (4)).

A soliton of this type and allowing for Eq. (1) may be constructed, assuming that the above mentioned region is shaped by dimensions R, $R \gg x_0$ and it is separated from the remaining portion of the magnet by the domain boundary (see Fig. 2), i.e., at $R \gg x_0$ (or at $S_z \gg S_* = s(x_0/a)^3$) the problem again becomes scale-invariant. Having evaluated E and S_z, we get

$$E \sim \sigma R^2 + sl(R/a), \quad S_z \sim s(R/a)^3 = S_* (R/x_0)^3, \tag{10}$$

where σ is the boundary energy density, $\sigma = 2(\alpha\beta)^{1/2} M_0^2$, the second term in E is associated with the inhomogeneity $\mathbf{m}$ at the boundary center and it is imposed by the topology (see Eq. (1)), however, it is small at $R \gg x_0$. Using Eq. (10) we get

$$E \sim \epsilon_0 (S_z^2 S_*)^{1/3} \quad \text{at} \quad S_z \gg S_*, \tag{11}$$

where $\epsilon_0 = 2\mu_0\beta M_0$ is the magnon activation. Since $(dE/dS_z) \sim \epsilon_0 (S_*/S_z)^{1/3} \ll \epsilon_0$, magnon emission is forbidden energetically. Soliton energy dissipation at $S_z \gtrsim S_*$ may only occur due to processes which fail to conserve S_z; for example, processes dependent on a weak dipole-dipole spin interaction. One of the authors (B. Ivanov) thanks A. S. Kovalev for fruitful discussions.

[1)]The problem of continuity of the derivative of the solution for a fixed ω also remains an open question. Derivative discontinuities at the surface $|\mathbf{r}| \sim R$ occur in the analysis of certain spherically-symmetric transformations; however, they present no significant difficulties in the analysis.[3] In our case the symmetry is lower (axial) and there are reasons to assume that there exists a solution with a derivative discontinuity $\mathbf{m}$ only on the line or even only on a point (for $r \to 0$).

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[2]B.A. Ivanov and A.M. Kosevich, ibid. **72**, 2000 (1977) [**45**, 1050 (1977)].
[3]G.E. Volovik and V.P. Mineev, ibid. **73**, 767 (1977) [**46**, 401 (1977)].