The Kapitza–Dirac effect for atoms in a strong resonant field

A. P. Kazantsev and G. I. Surdutovich

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
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It is shown that the scattering of atoms in a strong resonant field of a standing wave proceeds more effectively than scattering of electrons. A resonant field of power 0.1–10^2 W is needed to deflect Na atoms through an angle 0.01–0.1 rad.

The Kapitza-Dirac effect[1] constitutes elastic scattering of electrons in the field of a strong standing electromagnetic wave

$$E(x)e^{-i\omega t} + \text{c.c.}, \qquad E(x) = E_0 \cos kx. \tag{1}$$

The electron is then acted upon by an effective force[2]

$$F_x^e = -\frac{dU^e}{dx}, \qquad U^e = \frac{(eE(x))^2}{m\omega^2}. \tag{2}$$

Using the optical analogy, it can be stated that the electron wave is scattered by a diffraction grating having a period $\lambda/2 = \pi/k$. The diffraction angles θ_n are determined by the Bragg condition, which takes at small θ the form

$$\theta_n = 2\hbar kn/P, \tag{3}$$

where P is the momentum of the incident particles.

The theory of the Kapitza-Dirac effect was considered in[3,4]. In[5,6] there was observed scattering of electrons in high-power optical fields in the first diffraction maximum. For an electron energy 10 eV, as used in[5], we have $\theta_1^e \sim 10^{-3}$.

In this paper we discuss the features of atom scattering in a strong resonant field of a standing wave. The angles between the diffraction maxima are somewhat smaller for the atoms than for the electrons. Thus, for sodium atoms at thermal velocity the deflection angle per quantum in the resonant transition is $\theta_1^a \sim 10^{-4}$. Such angles are measured in experiments on the deflection of an atomic beam by the light pressure.[7,8] However, the number of quanta scattered by the atoms can be large. Therefore in a strong resonant field the maximum deflection angles is much larger for atoms than for electrons.

The force $F_x^a = 2p[dE(x)/dx]$ acting on the atom (p is the induced dipole moment) depends essentially on the frequency detuning $\Delta = \omega - \omega_0$, where ω_0 is the transition frequency. At large detunings $\hbar\Delta \gtrsim dE_0$ we have an average gradient force[12]

$$F_x^a = \frac{dU^a}{dx}, \qquad U^a = \frac{(dE(x))^2}{\hbar\Delta}, \tag{4}$$

where Δ is the matrix element of the dipole moment of the transition. As resonance is approached, the average gradient force tends to zero. The exact potential of the average gradient force was calculated in[9]. At small detunings, an important role is assumed by fluctuations of the gradient force. Under the conditions of exact resonance we have $U^a = \pm dE(x)$. The "±" sign is connected with the fact that in this case the number of atom trajectories doubles.[10] Since both the average and the fluctuating gradient forces lead to scattering of the atoms, to estimate the effective potential of the atom at $\hbar\Delta \lesssim dE_0$ we can use the expression

$$U^a = dE(x). \tag{5}$$

In the optical region, for not too strong fields, the atomic potential (5) greatly exceeds the electronic potential: $U^e/U^a \sim E/E_{at} \ll 1$, where $E_{at} \sim 10^9$ V/cm is the characteristic atomic field intensity. When the number of scattered quanta becomes large, the transverse velocity v_x of the atoms (see the figure) can be obtained from classical considerations by using the energy conservation law

$$Mv_x^2/2 + U^a(x) = U^a(x_0). \tag{6}$$

It is obvious that the maximum transverse energy on the order U^a can be acquired by an atom entering with zero transverse velocity only in a light beam of sufficiently large thickness $l > l_c$, where the critical thickness l_c is determined from the condition

$$l_c/v_y = \lambda/4v_x. \tag{7}$$

Thus, the maximum angle of deflection of the atoms in a light beam of thickness $l > l_c$ amounts to

$$\theta_{max}^a = \frac{v_x}{v_y} = \left(\frac{2U^a}{Mv_y^2}\right)^{1/2}. \tag{8}$$

A similar estimate is obtained for electrons. It is clear that the atom deflection angle can be much larger than for electrons. The number of quanta n_c, the scattering of which deflects the atom through an angle θ_{max}^a, is given by the relation namely the square root of the ratio of the potential energy to the recoil energy. Numerical estimates with the aid of relations (7)—(9) for sodium atoms are listed in the table (wavelength $\lambda = 0.6\ \mu$).

$$n_c = \left(\frac{MU^a}{2(\hbar k)^2}\right)^{1/2}, \tag{9}$$

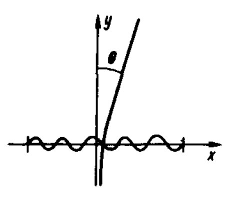

FIG. 1.

θ	n_c	$U^a/\hbar$	I
0.1	10^3	$4.2\cdot10^{11}$ гц	10^2 W
0.01	10^2	$4.2\cdot10^{9}$ гц	0.1 W

Here I is the power of the light beam. In the calculation it was assumed that the light beam is focused in only one direction (is compressed along the x axis) to a dimension on the order of l_c. The height of the beam along the z axis is constant at ~0.1 cm. The average thermal velocity of the atoms is $v_y \sim 5\times10^4$ cm/sec.

We note the following advantage of the scattering of atoms by a standing wave as compared with a traveling wave. In the traveling wave the atom is acted upon (in the saturation regime) by a force $F_{sp}=\hbar k\gamma/2$,[11] which is limited by the decay rate γ of the upper level. In a strong standing-wave field the atom is acted upon by the gradient force (4) due to the induced transitions. The quantity $F/F_{sp}\sim U_a/\hbar\gamma$ is the ratio of the rate of the induced transition to that of the spontaneous one. For the examples considered here we have $F/F_{sp}\sim10^5-10^3$. A deflection through an angle $\theta\sim0.1$ as a result of F_{sp} will occur in a beam of width of about 10 cm, whereas in the strong field of a standing wave the same angle is produced in a beam several wavelengths thick.

From the energy point of view, the deflection of atoms by a standing wave is also more convenient, for the photons are not scattered in this case (the time of passage of the atom through the light beam is less than the spontaneous emission time). On the other hand in the case of the force F_{sp}, one photon of the light beam is lost for each recoil pulse $\hbar k$ acquired by the atom.

Thus, the atoms can be effectively scattered by a resonant standing-wave field of low power, if the light beam is focused to a dimension of several dozen wavelengths.

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