The Aaronov–Bohm effect in disordered conductors

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It is shown that the Aaronov-Bohm effect, which is manifested in the oscillations of the kinetic coefficients as a function of the magnetic flux that penetrates the sample, must exist in disordered normal conductors. The period of these oscillations is $\Phi_0 = bc/2e$, i.e., it is half as large as in the ordinary Aaronov-Bohm effect.

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It is well known that the wave functions and the energy spectrum of electrons depend on the vector-potential of the magnetic field **A** (Aaronov-Bohm effect). This can be observed, for example, in the magnetic-flux quantization in a superconducting ring and in the temperature oscillations of the superconducting transition as a function of the mag-

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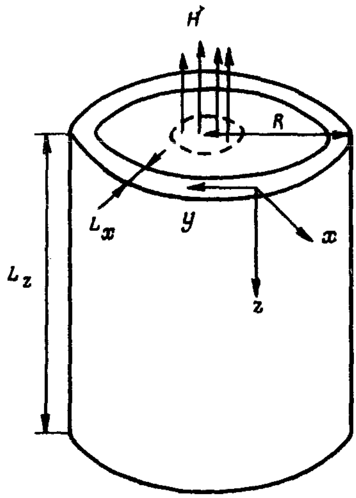

FIG. 1.

netic flux.[1] Such effects can also occur in normal metals: Dingle[2] showed that the thermodynamic values oscillate as a function of the flux that penetrates a metal cylinder. Analogous oscillations of kinetic values, which presumably were observed in Refs. 4 and 5, have been analyzed in a number of papers.[3]

All such effects vanish, however, as a result of electron scattering when the mean free path of electrons is equal to the sample size. We shall show in this paper that the Aaronov-Bohm-type interference effects occur in dirty metals; the absolute value of these effects may even increase with decreasing mean free path in the impurities and the oscillation period is $\Phi_0 = hc/2e$, i.e., it is half as large as in the ordinary Aaronov-Bohm effect, $\Phi_0' = hc/e$. For simplicity, we shall examine a hollow, thin-walled, metallic cylinder (Fig. 1) that contains a solenoid outside of which the magnetic field is equal to zero; however, the sample has a nonvanishing vector-potential **A** which has only a constant tangential component in the bulk of the sample.

It is known that the main quantum correction for the conductivity[6,7] is obtained from the "fan-shaped" diagrams (Fig. 2). The use of these diagrams in a magnetic field gives a negative magnetic resistance.[8] The correction for the conductivity in the presence of a vector-potential has the form[8]

$$\frac{\Delta\sigma}{\sigma_0} = -\frac{2}{\pi\nu} C_\omega(\mathbf{r},\mathbf{r}), \tag{1}$$

where $C_\omega(\mathbf{Q})$ is the sum of the fan diagrams for a small total momentum **Q** and small transmitted frequency ω, amd ν is the density of states of the electrons at the Fermi level.

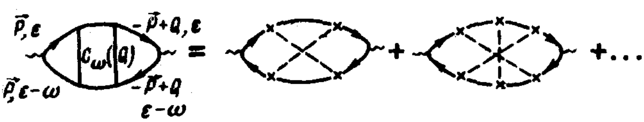

FIG. 2

The equation for $C_\omega(\mathbf{r},\mathbf{r}')$ has the form[8]

$$\hbar\left\{D\left(-i\nabla-\frac{2e}{c}\mathbf{A}\right)^2+i\omega+\frac{1}{\tau_\epsilon}\right\}C_\omega(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'), \tag{2}$$

where τ_ϵ is the energy relaxation time of electrons, D is their diffusion coefficient, and ω is the frequency of the external field. Equation (2) is an equation for the Green's function of a particle with a mass $1/2D \ll m$ (m is the electron mass) and with a charge $2e$. Equation (2) must be solved with periodic boundary conditions along the Y axis (around the circumference of the cylinder). The solution of Eq. (2) has the form

$$C_\omega(\mathbf{r},\mathbf{r}')=\frac{1}{\hbar L_y}\int\frac{dQ_\perp}{2\pi^2}\sum_{l=-\infty}^{\infty}\frac{e^{iQ(\mathbf{r}-\mathbf{r}')}}{i\omega+\frac{1}{\tau_\epsilon}+DQ_\perp^2+D\left(Q_y^l-\frac{2e}{c}A\right)^2}, \tag{3}$$

where $\mathbf{Q}_\perp=(Q_x,Q_z)$, $Q^l_y=2\pi l/L_y$, $L_y=2\pi R$ is the circumference of the cylinder. Substituting Eq. (3) in Eq. (1), we obtain

$$\Delta\sigma=-\frac{2e^2}{\pi\hbar}\frac{1}{L_y}\int\frac{dQ_\perp}{2\pi^2}\sum_{l=-\infty}^{\infty}\left[Q_\perp^2+\left(\frac{2\pi}{L_y}\right)^2\left(l-\frac{\Phi}{\Phi_0}\right)^2+\tilde{L}_\epsilon^{-2}\right]^{-1}, \tag{4}$$

where $L_\epsilon=\sqrt{D\tau_\epsilon}$ and $\tilde{L}^2_\epsilon=(L^2_\epsilon/1+i\omega\tau_\epsilon)$. If the thickness L_x of the cylinder is smaller than L_ϵ, then the integral over Q_x must be replaced by a sum and only the term with $Q_x=0$ must be retained in the denominator (quasi-two-dimensional case). If, however, the length L_z of the cylinder is small in comparison with L_ϵ, then only the term with $\mathbf{Q}_\perp=0$ should be left in the denominator in the summation over Q_x and Q_z (quasi-one-dimensional case[9]).

We obtain

$$\Delta g=-\frac{e^2}{\pi^2 h}\tilde{L}_\epsilon\frac{\mathrm{sh}\,L_y/\tilde{L}_\epsilon}{\mathrm{ch}\,L_y/\tilde{L}_\epsilon-\cos 2\pi\,\Phi/\Phi_0} \tag{5}$$

for the conductivity per unit length in the quasi-one-dimensional case. We can see in Eq. (5) that the conductivity oscillates with the period $\Phi_0=hc/2e$ with the variation of the flux. The modulation amplitude decreases exponentially with increasing circumference of the cylinder.

In the two-dimensional case (i.e., for a long cylinder) the conductivity of a square-shaped film that was folded into a cylinder for $\omega=0$ and $g=\sigma/L_x$ has the form (when $L \gg L_\epsilon$)

$$\Delta g=\frac{e^2}{\pi^2\hbar}\left\{\ln\frac{\tau}{\tau_\epsilon}-\frac{1}{2}\int_{\infty}^{\infty}\frac{dt}{\sqrt{(L_y/2\pi L_\epsilon)^2+t^2}}\times\cos\left(t+2\pi\frac{\Phi}{\Phi_0}\right)\right\}. \tag{6}$$

Just as in a short cylinder, the conductivity oscillates with the period Φ_0. The oscillation amplitude, which is equal to $K_0(L_y/2\pi L)$ [$K_0(x)$ is the McDonald function], decays exponentially when $L_y \gg L_\epsilon$. It should be noted, however, that expressions (4) and (5) are correct for any relationship between L_y and L_ϵ. As $T \to 0$ $L_\epsilon \to \infty$; hence, Δ_g, as can easily be seen in Eq. (5), no longer depends on the disorder and the oscillation amplitude approaches infinity.

L_ϵ may be very large in the experiment. If τ_ϵ is determined by the collisions between the electrons and the Fermi energy is of the order of 10^5 K, then $\tau_\epsilon \sim 10^{-4}$ sec at $T \sim 0.1$ K. Therefore, $L_\epsilon \sim 1$ cm when the mean free path is $l \sim 10^{-3}$ cm. Thus, the macroscopic samples can be used to observe the effect. The radius of the samples used in the reported experiments is equal to $10^{-4} - 10^{-5}$ cm.[1,4,5]

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