Quantum fluctuations and a nonsingular universe

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Over a finite time, quantum fluctuations of the curvature disrupt the nonsingular cosmological solution corresponding to a universe with a polarized vacuum. If this solution held as an intermediate stage in the evolution of the universe, then the spectrum of produced fluctuations could have led to the formation of galaxies and galactic clusters.

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A nonsingular cosmological model with a polarized vacuum has been attracting particular interest recently.[1] It has been pointed out elsewhere that quantum fluctuations may prove important in cosmology at energy densities comparable to the Planck value.[2] Since these are in fact the energy densities characteristic of the nonsingular polarized-vacuum model,[1] we believe it is worthwhile to study the role of quantum fluctuations in order to determine whether there is a singularity in this model.

For an isotropic metric, the single-loop corrections describing the polarization of the vacuum of physical fields in a strong gravitational field lead to the following Einstein equations[3]:

$$R^i_k - \frac{1}{2}\delta^i_k = \frac{1}{H^2}\left(R^i_l R^l_k - \frac{2}{3} R R^i_k - \frac{1}{2}\delta^i_k R^l_m R^m{}_l + \frac{1}{4}\delta^i_k R^2\right)$$

$$- \frac{1}{6M^2}\left(2R^{;i}{}_{;k} - 2\delta^i_k R^{;l}{}_{;l} - 2RR^i_k + \frac{1}{2}\delta^i_k R^2\right), \qquad (1)$$

where the coefficients M^2 and H^2 result from the sum over the effects of all the fields. For stability of the Minkowski space, M^2 must be positive. For $H^2 > 0$, Eqs. (1) have a particular solution of the "de Sitter" type,[1]

$$ds^2 = g_{ik}dx^i dx^k = a^2(\eta)\left(d\eta^2 - \sum_{\alpha=1}^{3}(dx^\alpha)^2\right), \qquad (2)$$

$$a(\eta) = -\frac{1}{H\eta}, \quad -\infty < \eta < 0, \qquad R = -12H^2 = \text{const}.$$

The curvature invariants, in particular R, have no singularities in the limit $\eta \to -\infty$, showing that there is no actual singularity in the universe described by metric (2).

Let us consider, against the background of this metric, some small perturbations that satisfy Eqs. (1). We restrict the discussion to scalar perturbations. In a synchronous frame of reference ($\delta g_{i0} = 0$) these perturbations have the following tensor structure:

$$h^{\alpha}_{\beta} = -\frac{1}{a^2}\delta g_{\alpha\beta} = \left(\nabla^{\alpha}\nabla_{\beta} - \frac{1}{3}\delta^{\alpha}_{\beta}\Delta\right)\lambda - \frac{1}{3}\delta^{\alpha}_{\beta}\Delta\mu, \tag{3}$$

where $\nabla^{\alpha} = \nabla_{\alpha} = \partial/\partial x^{\alpha}$, and Δ is the Laplacian.

The convolution of Eqs. (1), linearized with respect to h^{α}_{β}, yields a second-order equation for perturbations of the curvature scalar δR:

$$\delta R'' + 2\frac{a'}{a}\delta R' - \Delta\delta R - M^2 a^2 \delta R = 0, \tag{4}$$

where the prime denotes differentiation with respect to η, and $a(\eta) = -1/H\eta$.

Also using Eqs. (1), we can show that all the quantities in which we are interested (in particular, the h^{α}_{β}), can be expressed in terms of δR. This quantity plays a special role because of its invariance with respect to so-called fictitious perturbations[4] which stem from transformation of the coordinate system which does not disrupt the synchronism. Fictitious perturbations of scalar functions result from a change in the origin for the time scale[5]; since R is independent of the time ($R = \text{const}$) in the de Sitter model, with which we are concerned here, we have $\delta R = 0$ for these fictitious modes.

Adopting a perturbation of the curvature scalar as a physical variable, we find the corresponding action in the form[6]

$$\delta S_b = \frac{1}{2}\int d^4x\left[\phi'^2 - \nabla_{\alpha}\phi\nabla^{\alpha}\phi + \left(\frac{a''}{a} + M^2a^2\right)\phi^2\right], \tag{5}$$

where $\phi = 1/\sqrt{18(4H^2 - M^2)}\ a\delta R/Ml$, and $l = (8\pi G/3)^{1/2} = 4.37 \times 10^{-33}$ cm is the Planck length.

In quantizing we should note that the physical system under consideration (a polarized vacuum in a gravitational field) has a finite energy density, which must undergo fluctuations according to the uncertainty principle. These fluctuations are zero-point oscillations of the field of collective excitations of the ordinary physical fields ("scalarons" with mass M).

The canonical quantization is carried out by a procedure similar to that of Ref. 7. As a result, we can calculate various correlation functions, e.g.,

$$\langle 0|\delta\hat{R}(\mathbf{x})\delta\hat{R}(\mathbf{x}+\mathbf{r})|0\rangle = \frac{1}{2\pi^2}\int P^2(k)\frac{\sin kr}{kr}\frac{dk}{k}, \tag{6}$$

where k and r are the physical wave vector and the physical length, expressed in units of reciprocal centimeters and centimeters, respectively. The spectrum $P(k)$ is expressed in terms of Bessel functions of index $3/2\nu = 3/2\sqrt{1 + 4/9\, M^2/H^2}$ and argument k/H, which are the exact solutions of Eq. (4). The spectrum $P(k)$ is shown in Fig. 1. A distinguishing feature of this spectrum is a maximum at $k \sim H(\eta/\eta_0)$ (η_0 is the initial time at which the vacuum of the scalaron field is given); the perturbation

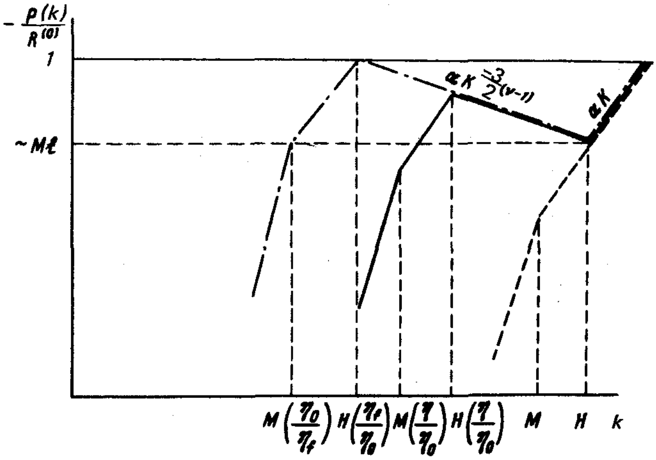

FIG. 1. Spectrum of relative curvature fluctuations, $P(k)/R^{(0)}$, plotted as a function of the wave number k. Dashed lines —spectrum of vacuum fluctuations which is specified at some initial time η_0; solid line—the spectrum into which the vacuum spectrum transforms at some later time η; dot-dashed line—the spectrum at an even later time η_f.

amplitude at this maximum increases over time. Over a finite time, which in the most interesting case $(M^2 \ll H^2)$ is

$$\delta t_f = \frac{3}{2H}\,\frac{H^2}{M^2}\ln\left(\frac{1}{2(\ell M)^2}\right) \tag{7}$$

the amplitude of the curvature perturbations at the maximum reaches values characteristic of the original background model, and the universe enters a stage of Friedmann expansion with ordinary hydrodynamic matter as a result of multiple production of scalarons with finite wave numbers [primarily with $k = H(\eta/\eta_0)$]. Thus we see that the quantum fluctuations, which are necessarily present in the system, cause the universe to spend a finite time in the de Sitter state and thus cast doubt on the possibility of a nonsingular beginning of the universe. Regardless of the nature of the singularity (classical or quantum), we believe that this fact significantly detracts from the esthetic value of the original model.

A finite duration of the de Sitter stage does not by itself rule out the possibility that this stage may exist as an intermediate stage in the evolution of the universe. An interesting question arises here: Might not perturbations of the metric, which would be sufficient for the formation of galaxies and galactic clusters, arise in this stage? To answer this question, we need to calculate the correlation functions for the fluctuations of the metric after the universe goes from the de Sitter stage to the hydrodynamic stage. By analogy with (6) we find

$$< 0 |\, \hat{h}(\mathbf{x})\hat{h}(\mathbf{x}+\mathbf{r}) \,|\, 0 > = \frac{1}{2\pi^2}\int Q^2(k)\,\frac{\sin kr}{kr}\,\frac{dk}{k}\,, \tag{8}$$

where $h = h^{\alpha}_{\alpha}$ and where, for the most interesting region, $H > k > H\exp(-3H^2/M^2)$

$(M^2 \ll H^2)$,

$$Q(k) \approx 3\hbar M\left(1 + \frac{1}{2}\ln\frac{H}{k}\right). \quad (9)$$

The fluctuation spectrum is thus nearly flat. The quantity $Q(k)$ is a measure of the amplitude of perturbations with scale dimensions $1/k$ at the time the universe begins the ordinary Friedmann expansion. With $M\hbar \sim 10^{-3}$-10^{-5} and $M/H \lesssim 0.1$—these values are consistent with modern theories of elementary particles—the amplitude of the perturbations of the metric on the scale of galactic clusters turns out to be equal to 10^{-3}-10^{-5}, and these perturbations can lead to the observed large-scale structure of the universe. The form of spectrum (9) is completely consistent with modern theories for the formation of galaxies.[5]

To summarize: Using a de Sitter model as an example, we have shown that quantum fluctuations (zero-point vibrations) cause the universe to spend a finite time in a state with a polarized vacuum. This result casts doubt on the possibility of a nonsingular origin for the universe. However, models in which the de Sitter stage exists only as an intermediate stage in the evolution are attractive because fluctuations of the metric sufficient for the formation of galaxies can occur. Thus we have one possible approach for solving the problem of the appearance of the original perturbation spectrum.

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