Angular momentum and orbital waves in the anisotropic A phase of superfluid He^3

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The angular-momentum density of He^3 in the A phase is calculated. Equations for the orbital waves are derived on the basis of the commutation relations for the components of the angular momentum. The effective mass of the orbital excitations turns out to be of the order of $m(T_c/\epsilon_F)^2\ln(\epsilon_F/T_c)$.

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As is well known (see the review[1]), the anisotropic A phase of superfluid He^3 has three types of collective oscillations, connected with different violations of symmetry. These are: a) density oscillations connected with violation of gauge invariance, b) spin waves connected with violation of the invariance to rotations in spin space, c) orbital waves connected with violation of invariance to rotations in ordinary space. The first two types of oscillations have been well investigated theoretically (see, e.g., [2,3]), since the equations for the phase of the condensate and for the angle of rotation of the spin can be obtained from the conservation laws for the number of particles and for the spin. On the other hand, no dispersion of the orbital waves was obtained, since there is no corresponding conservation law leading to an equation of motion for the unit vector $\mathbf{l}$ that indicates the direction of the orbital momentum of the pair. An equation for $\mathbf{l}$ was specified in [4] with the aid of a set of phenomenological parameters.

The first attempt to obtain an equation for $\mathbf{l}$ in the hydrodynamic regime was undertaken in[5], where a commutation relation was obtained for the components of the angular momentum density vector $\mathbf{L}$:

$$[L_i(\mathbf{r}), L_j(\mathbf{r}')] = ie_{ijk} L_k(\mathbf{r})\,\delta(\mathbf{r}-\mathbf{r}'),\ \hbar = 1. \tag{1}$$

It was postulated in that reference that the vector $\mathbf{L}$ is directed along $\mathbf{l}$, i.e., $\mathbf{L} = L\mathbf{l}$, and consequently, at small deviations $\delta\mathbf{l}$ from the equilibrium position, one can assume $L^{1/2}\delta l_x$ and $L^{1/2}\delta l_y$ to be canonically conjugate variables (the z axis is directed along $\mathbf{l}$). Therefore these quantities satisfy the Hamilton equation

$$L\delta\dot{l}_x = \delta F/\delta l_y,\ L\delta\dot{l}_y = -(\delta F/\delta l_x). \tag{2}$$

The free energy F can be easily calculated in the weak-coupling approximation, so that only knowledge of the value of L is lacking to define the situation completely. In[5], as in [6], it was assumed that the angular momentum L is connected with the tangential component of the current-density correlator:

$$L \overset{?}{=} \int d^3R\, R \langle \mathbf{j}(\mathbf{R})\rho(0)\rangle_\phi \sim (T_c/\epsilon_F)\hbar\rho_s \tag{3}$$

an assumption for which there are not sufficient grounds (see the criticism of this assumption in[1]).

In the present paper we calculate L by the method of the matrix kinetic equation (see[7] and also[7])

$$\omega\delta n_{\mathbf{k}} = \delta n_{\mathbf{k}}\epsilon^0_{\mathbf{k}+\mathbf{q}/2} - \epsilon^0_{\mathbf{k}-\mathbf{q}/2}\delta n_{\mathbf{k}} + n^0_{\mathbf{k}-\mathbf{q}/2}\delta\epsilon_{\mathbf{k}} - \delta\epsilon_{\mathbf{k}} n^0_{\mathbf{k}+\mathbf{q}/2} + I(n), \tag{4}$$

where $\delta n_{\mathbf{k}}$, $n^0_{\mathbf{k}}$, $\epsilon^0_{\mathbf{k}}$, and $\delta\epsilon_{\mathbf{k}}$ are 4×4 matrices and are defined as follows (we use the notation of[3]):

$$\delta n_{\mathbf{k}}(\mathbf{q}) = \begin{pmatrix} \delta n^e_{\mathbf{k}\alpha\beta}(\mathbf{q}), & \delta n^+_{\mathbf{k}\alpha\beta}(\mathbf{q}) \\ \delta n^-_{\mathbf{k}\alpha\beta}(\mathbf{q}), & \delta n^t_{\mathbf{k}\alpha\beta}(\mathbf{q}) \end{pmatrix}$$
$$= \begin{pmatrix} \langle c^+_{\mathbf{k}-\mathbf{q}/2\,\alpha} c_{\mathbf{k}+\mathbf{q}/2\,\beta}\rangle, & \langle c^+_{\mathbf{k}-\mathbf{q}/2\,\alpha} c^+_{-\mathbf{k}-\mathbf{q}/2\,\beta}\rangle \\ \langle c_{-\mathbf{k}+\mathbf{q}/2\,\alpha} c_{\mathbf{k}+\mathbf{q}/2\,\beta}\rangle, & \langle c_{-\mathbf{k}+\mathbf{q}/2\,\alpha} c^+_{-\mathbf{k}-\mathbf{q}/2\,\beta}\rangle \end{pmatrix}, \tag{5}$$
$$\epsilon^0_{\mathbf{k}} = \begin{pmatrix} \xi_{\mathbf{k}}\delta_{\alpha\beta}, & \Delta^+_{\mathbf{k}\alpha\beta} \\ \Delta_{\mathbf{k}\alpha\beta}, & -\xi_{\mathbf{k}}\delta_{\alpha\beta} \end{pmatrix}, \quad \delta\epsilon_{\mathbf{k}}(\mathbf{q}) = \begin{pmatrix} 0, & \delta\Delta^+_{\mathbf{k}\alpha\beta}(\mathbf{q}) \\ \delta\Delta_{\mathbf{k}\alpha\beta}(\mathbf{q}), & 0 \end{pmatrix},$$

$$n^0_{\mathbf{k}} = (1/2) - (1/2)(\epsilon^0_{\mathbf{k}}/E_{\mathbf{k}})\,\mathrm{th}(E_{\mathbf{k}}/2T),$$

where

$$E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + (\Delta^+_{\mathbf{k}}\Delta_{\mathbf{k}})},\quad \xi_{\mathbf{k}} = (k^2/2m) - (k_F^2/2m).$$

Here $\Delta_{\mathbf{k}\alpha\beta}$ is the order-parameter matrix. For the Anderson-Morel axial state[8], which is assumed to be realized in the A phase, we have

$$\Delta_{\mathbf{k}\alpha\beta} = i(\vec{\tau}\mathbf{V}\tau_y)_{\alpha\beta}(\hat{\mathbf{k}}\vec{\Delta}), \tag{6}$$

where $\vec{\tau}$ are Pauli matrices, $\mathbf{V}$ is a unit vector, $\vec{\Delta} = \vec{\Delta}' + i\vec{\Delta}''$, the vectors $\vec{\Delta}'$, $\vec{\Delta}''$, and $\mathbf{l}$ are perpendicular to one another, $|\vec{\Delta}'| = |\vec{\Delta}''| = \Delta(T)$, and $\hat{\mathbf{k}}$ is a unit vector in the direction of $\mathbf{k}$.

To calculate L we introduce the inhomogeneous stationary deviation $\delta\mathbf{l}_q$ from the equilibrium position. This deviation corresponds to a change in the order parameter

$$\delta\Delta_{\mathbf{k}\alpha\beta}(\mathbf{q}) = -i(\vec{\tau}\mathbf{V}\tau_y)_{\alpha\beta}(\hat{\mathbf{k}}\mathbf{l})(\vec{\Delta}\delta\mathbf{l}_{\mathbf{q}}). \tag{7}$$

We substitute this expression in (4) and solve it relative to $\delta n^e_{\mathbf{k}\alpha\beta}(\mathbf{q})$ at $\omega = 0$. Using the fact that we can discard the collision integral at $\omega = 0$, we obtain for $\delta n^e_{\mathbf{k}}$, after lengthy but straightforward calculations, the expression

$$\delta n^e_{\mathbf{k}\alpha\beta}(\mathbf{q}) = -\frac{\dfrac{\mathrm{th}\,\dfrac{E_{\mathbf{k}+\mathbf{q}/2}}{2T}}{2E_{\mathbf{k}+\mathbf{q}/2}} - \dfrac{\mathrm{th}\,\dfrac{E_{\mathbf{k}-\mathbf{q}/2}}{2T}}{2E_{\mathbf{k}-\mathbf{q}/2}}}{E^2_{\mathbf{k}+\mathbf{q}/2} - E^2_{\mathbf{k}-\mathbf{q}/2}}$$
$$\times[(\xi_{\mathbf{k}} - (1/2)\mathbf{qk} + (q^2/8))\delta\Delta^+_{\mathbf{k}\alpha\gamma}\Delta_{\mathbf{k}+\mathbf{q}/2\,\gamma\beta}$$
$$+ (\xi_{\mathbf{k}} + (1/2)\mathbf{qk} + (q^2/8))\Delta^+_{\mathbf{k}-\mathbf{q}/2\,\alpha\gamma}\delta\Delta_{\mathbf{k}\gamma\beta}]. \tag{8}$$

With the aid of this expression we can find the change $\delta\mathbf{L}$ of the angular momentum density. Since $\delta\mathbf{L} = \mathbf{r}\times\delta\mathbf{j}$, its Fourier component $\delta\mathbf{L}_q$ is expressed in terms of $\delta n^e_{\mathbf{k}}$ as follows:

$$\delta\mathbf{L}_{\mathbf{q}} = -i\sum_{\mathbf{k}}\mathbf{k}\times(\partial/\partial\mathbf{q})\,\delta n^e_{\mathbf{k}\alpha\alpha}(\mathbf{q}). \tag{9}$$

Substituting (8) in (9) and letting $\mathbf{q}$ go to zero, we obtain with logarithmic accuracy

$$\delta\mathbf{L} = L\delta\mathbf{l},\ L = -(\Delta^2(T)/2)\sum_{\mathbf{k}}(\hat{\mathbf{k}}\mathbf{l})^2(\partial/\partial\xi)[\mathrm{th}(E/2T)/E]$$
$$= (\hbar\rho/8)(\Delta^2(T)/\epsilon_F^2)\ln[\epsilon_F/\max[T,\Delta(T)]]. \tag{10}$$

That is to say, as expected, the angular momentum $\mathbf{L}$ is directed along $\mathbf{l}$, but differs significantly in magnitude from expression (3), since it is of order $\rho_s(T_c/\epsilon_F)^2\hbar\ln(\epsilon_F/T_c)$.

The fact that Eq. (2) contains the value of L given by (10) rather than by (3) can be verified also in a different manner. The point is that at $T = 0$ Eq. (2) can be easily derived from (4), in which we can neglect the collision integral, and from the equation for the order parameter $\delta\Delta_{\mathbf{k}\alpha\beta}(\mathbf{q}) = \Sigma_{\mathbf{k}'}V_{\mathbf{k}\mathbf{k}'}\delta n^-_{\mathbf{k}'\alpha\beta}(\mathbf{q})$. We then obtain for L the expression $(\hbar\rho/8)(\Delta^2(0)/\epsilon_F^2)\ln(\epsilon_F/\Delta(0))$, which agrees with (10).

The orbital-excitation effective mass, obtained just as in[5] from Eqs. (2) but with formula (10) taken into account rather than formula (3), is of the order of $m_{eff} \sim m(T_c/\epsilon_F)^2\ln(\epsilon_F/T_c)$. A more detailed calculation of the dispersion of the orbital waves with allowance for the Fermi-liquid corrections and for the motion of the normal component will be published later.

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