Structure of a shock wave in a weakly ionized nonisothermal plasma

R. F. Avramenko, A. A. Rukhadze, and S. F. Teselkin
M. V. Lomonosov Moscow State University

(Submitted 29 July 1981)

Pis'ma Zh. Eksp. Teor. Fiz. **34**, No. 9, 485–488 (5 November 1981)

The structure of a plane ion-acoustic shock wave is analyzed for the case of a weakly ionized nonisothermal plasma which is sustained by a shock wave in the neutrals. Analytic expressions are derived for the profiles of the density and velocity of the charged particles.

PACS numbers: 52.35.Tc

1. Low-frequency ion-acoustic waves exist in a nonisothermal plasma with $T_e \gg T_i \gtrsim T_n$ of sufficiently low density, either highly ionized or weakly ionized. If the characteristic dimensions are much larger than the electron Debye length, such slow motions of the charged plasma particles can be described well in an arbitrary nonlinear approximation by the equations of single-fluid hydrodynamics with a constant temperature T_e (Refs. 1–4). These equations incorporate the friction between the ions and the neutral particles, which leads to a dissipation of some of the kinetic energy of the charged particles through momentum transfer to the neutral gas. There is the possibility, however, of a different situation, such that the motion of the neutral particles causes a motion of the charged particles by means of the friction force. This forced motion can be particularly important in a weakly ionized plasma, in which the motion of the neutrals is given, and it perturbs the charged particles; the inverse effect of the charged particles on the neutrals can be ignored [see expression (7) below].

Let us consider the particular case of the one-dimensional, forced motion of the charged particles of a weakly ionized plasma, which is caused by a relatively weak (nonionizing) plane shock wave that is propagating through the neutrals. Since the ion-acoustic velocity is much higher than the ion thermal velocity or the acoustic velocity of the neutrals, the shock wave may act as a source of ion-acoustic waves which lead the shock wave and which perturb the charged particles ahead of the shock wave. The result is a complex nonlinear motion; it is this motion with which we will be concerned below. Ahead of the shock front there is a large region in which the charged particles are perturbed; the shape of this region is reminiscent of a diffuse shock wave, and we accordingly refer to this region as an "ion-acoustic shock wave." In the absence of an external agent, ion-acoustic shock waves could not propagate through a nonisothermal electron-ion plasma.[5]

2. To describe the motion of the charged particles in the weakly ionized nonisothermal plasma, we begin from the equations of single-fluid hydrodynamics[3]:

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}(\rho V) = 0,$$
$$\frac{\partial}{\partial t}V + V\frac{\partial}{\partial x}V = -\frac{v_s^2}{\rho}\frac{\partial}{\partial k}\rho - \nu_{in}(V - V_n). \tag{1}$$

Here $\rho = MN_i$ is the mass density of the charged component of the plasma, V is its hydrodynamic velocity, $v_s^2 = z(T_e/M)$ is the ion-acoustic velocity ($z = |e_i/e|$), and ν_{in} is the rate of collisions of ions with neutrals, whose average hydrodynamic velocity is V_n.

As mentioned earlier, we assume that the motion of the shock wave through the neutrals is given:

$$V_n = u\theta(-\xi),$$
$$\rho_n = \rho_{no} + (\rho_{n1} - \rho_{no})\,\theta(-\xi). \tag{2}$$

Here $\xi = x - ct$, where c is the velocity of the shock front and u is the velocity of the neutrals behind the front, ρ_{n0} and ρ_{n1} are the densities of the neutral gas respectively ahead of and behind the shock front, and $\theta(\xi)$ is the unit step function.

3. In solving this problem, we assume that all quantities depend on ξ. From the first equation in (1) we then find

$$\rho = \rho_o(1 - V/c)^{-1}, \tag{3}$$

where ρ_0 is the unperturbed density of the charged component. Using (3), we find the following equation from the second equation in (1):

$$\frac{d}{d\xi}\left[\frac{y^2}{2} - v_s^2 \ln\left(\frac{y}{c}\right)\right] - \nu_{in}(y - y_n) = 0, \tag{4}$$

where $y = c - V$ and $y_n = c - V_n$.

In addition to y_n, there is another discontinuous quantity in Eq. (4): the rate of ion-neutral collisions, ν_{in}, since $\nu_{in} \sim \rho_n$. This equation should thus be solved separately in the regions $\xi > 0$ and $\xi < 0$, and the resulting partial solutions should be joined at the point $\xi = 0$ in accordance with the continuity of the function $y(\xi)$. Furthermore, in the absence of a discontinuity in the neutrals, i.e., with $\rho_{n1} = \rho_{n0}$, the solution of Eq. (4) is trivial: $V \equiv 0, y \equiv c$. Using all these considerations and also the inequality $v_s^2 \gg c^2$, we finally find

$$V = \begin{cases} \dfrac{c}{1 + \exp\left(\dfrac{\xi - \mu\xi_o}{\xi_o}\right)} & \text{for } \xi > 0, \\[2ex] u = c\left(1 - \dfrac{\rho_{no}}{\rho_{n1}}\right) & \text{for } \xi < 0; \end{cases} \tag{5}$$

$$\rho = \begin{cases} \rho_0 \left[1 + \exp\left(- \dfrac{\xi - \mu \xi_0}{\xi_0} \right) \right] & \text{for } \xi > 0, \\ \rho_0 \dfrac{\rho_{n1}}{\rho_{n0}} & \text{for } \xi < 0 . \end{cases} \quad (6)$$

Here $\mu = \ln(\rho_{n1}/\rho_{n0} - 1)$, and $\xi_0 = v_s^2/cv_{in}^{(0)}$, where $v_{in}^{(0)} \sim \rho_{n0}$ is the rate of ion-neutral collisions in the unperturbed plasma.

4. We will begin our discussion of these solutions by determining their range of applicability. In Eq. (4) we ignored the collisions of ions with other ions (the ion viscosity), as we are justified in doing at a sufficiently low degree of ionization, such that $\alpha = \rho_0/\rho_{n0} < \sigma_0 T_i^2 D^2 \cdot 2 \cdot 10^4$, where σ_0 is the cross section for the scattering of ions by neutrals, and D is the Mach number. For air, this cross section is $\sigma_0 \approx 10^{-15}$ cm^2, and at $T_i \approx 3 \times 10^2$ K we have $\alpha < 10^{-5}$. Furthermore, we assumed $v_s^2 \gg c^2$, so that the shock wave in the neutrals must not be so strong that the Mach number exceeds a few units. This requirement fits in with the assumption that the shock wave does not cause ionization.

We should emphasize that we have ignored the structure of the shock wave in the neutral component of the plasma, which is known to have dimensions on the order of the mean free path of the neutrals. The spatial structure of solutions (5) and (6) is determined by the quantity $\xi_0 = v_s^2/c^2 l_i$, where $l_i \approx l_n$ is the mean free path of the ions and the neutrals. The inequality $v_s^2 \gg c^2$ allows us to ignore the structure of the shock wave in the neutrals.

Figures 1a and 1b show the spatial structure of solutions (5) and (6) for the case $\mu > 1$, in terms of the velocity V and the density ρ, respectively. Ahead of the shock front which is propagating through the neutral gas there is a large region with a linear dimension $(1 + \mu)\xi_0$ in which the charged particles are perturbed. At $\mu > 1$, the structure of this perturbation of the charged particles is reminiscent of that of the shock wave, and on this basis this perturbation may be called an "ion-acoustic shock wave."

Finally, we note that the perturbed region ahead of the shock front forms in a time $\sim 1/v_{in}^{(0)}$, and we can use this time to estimate the power dissipated by the shock wave on the formation of this region, It is easy to show that the ratio of this power to the power of the shock wave is given in order to magnitude by

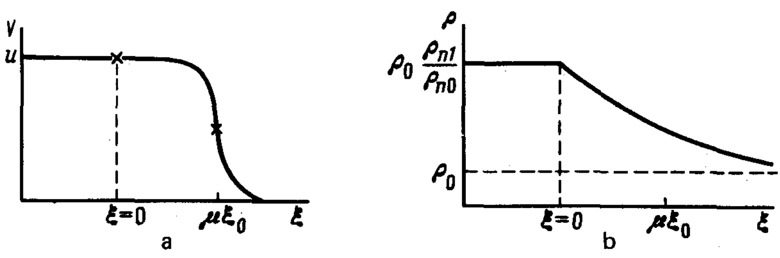

FIG. 1.

$$\eta \approx a \frac{T_e}{T_n} \frac{\rho_{n1}}{\rho_{n0}} \ln\left(\frac{\rho_{n1}}{\rho_{n0}} - 1\right). \qquad (7)$$

Obviously, we must require $\eta \ll 1$ for the validity of all these results, which are based on the assumption that the inverse effect of the motion of the charged particles on the neutral particles is negligible.

1. Yu. L. Klimontovich and V. P. Silin, Zh. Eksp. Teor. Fiz. **40**, 1213 (1961) [Sov. Phys.–JETP **13**, 852 (1961)].
2. E. E. Lovetskiĭ and A. A. Rukhadze, Zh. Eksp. Teor. Fiz. **41**, 1845 (1961) [Sov. Phys.–JETP **14**, 1312 (1962)].
3. E. Abu-Asali, B. A. Al'terkop, and A. A. Rukhadze, Plasma Phys. **17**, 189 (1975).
4. A. F. Aleksandrov, L. S. Bogdankevich, and A. A. Rukhadze, Osnovy élektrodinamiki plazmy (Introduction to Plasma Electrodynamics), Moscow, 1978.
5. L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred, Moscow, 1954, Chap. IX (Fluid Mechanics, Addison-Wesley, Reading, Mass., 1959).

Translated by Dave Parsons
Edited by S. J. Amoretty