

CONCERNING ONE-DIMENSIONAL SUPERCONDUCTIVITY

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The publication of Little's paper [1] has evoked great interest in one-dimensional conducting systems, such as the molecules of certain linear polymers. We have therefore investigated the model of a monovalent metal, taking into account both the interaction between the electrons, due to Coulomb forces and polarization [1], and the interaction between electrons and ions (phonons). It is important here that our system, being one-dimensional, is at the same time sufficiently "thick" with respect to the Coulomb forces. In this sense it is similar to the molecules of a polymer, in which the transverse dimension of the main chain always amounts to several Angstroms, while the Debye radius does not exceed  $1 \text{ \AA}$  [1]. The plasma frequency  $\omega_p$  of such a system does not differ in order of magnitude from  $\omega_p$  of a three-dimensional metal. Low-frequency oscillations of the electron density are thus forbidden, and this enables us to disregard completely electron interactions with small momentum transfer.

We chose an interaction with large momentum transfer in the form

$$V_{\alpha\beta, \gamma\delta}^{(0)}(p_1 p_2, p_3 p_4) = g(\delta_{\alpha\gamma} \delta_{\beta\delta} - \delta_{\alpha\delta} \delta_{\beta\gamma}) - \lambda^2 \delta_{\alpha\gamma} \delta_{\beta\delta} \quad (1)$$

The constant  $g$  describes the Coulomb interaction and the Little interaction [1], and is effective only at interatomic distances, so that the corresponding integrals are cut off at atomic energies  $\epsilon_0$ .  $\lambda$  is the constant of electron-phonon interaction which, being long-range, is cut off at the Debye frequency  $\omega_D$ . The constants  $\lambda$  and  $g$  were assumed to be small, and we retained in the calculations only terms of order  $g^{n+1} \ln^n \epsilon_0/T_c$  (or  $\lambda^{2n+2} \ln^n \omega_D/T_c$ ), where  $T_c$  is the transition temperature. A detailed analysis, the reasoning of which is briefly described below, has shown the following:

1) In the presence of effective attraction, the system always goes over into the superconducting state. The transition temperature (or the energy gap  $\Delta_{\text{eff}}$ , which coincides with it in our approximation) is equal to

$$\begin{aligned} T_c &= \epsilon_0 \exp(\pi v/g) && \text{if } (g/\pi v) \ln(\epsilon_0/\omega_D) < -1 \\ T_c &= \omega_D \exp[\pi v/(g - \lambda^2)] && \text{if } (g/\pi v) \ln(\epsilon_0/\omega_D) > -1 \end{aligned} \quad (2)$$

$$\bar{g} = g/[1 + (g/\pi v) \ln(\epsilon_0/\omega_D)]$$

where  $v$  is the velocity on the Fermi surface.

2) The superconducting transition is of necessity accompanied by doubling of the lattice period, i.e., by a real shift of the ions. Therefore the effective gap is given by  $\Delta_{\text{eff}}^2 = |\Delta|^2 + |\delta|^2$ , where  $\Delta$  is due to Cooper pairing and  $\delta$  to the doubling of the period. The superconducting current is in this case proportional to

$$I_{\text{super}} \sim |\Delta|^2 / \Delta_{\text{eff}}^2$$

and thus vanishes in the absence of pairing.

3) If the interaction is an effective repulsion ( $g > 0$ ,  $\bar{g} - \lambda^2 > 0$ ), the system remains a normal metal at all temperatures.

Doubling of the period in the superconducting transition

is a distinctive feature of one-dimensional systems. Whereas in the three-dimensional case it is essential, in order for superconductivity to occur, to have only interaction of two electrons (or two holes) lying in the Fermi surface, in the one-dimensional case it is just as important to have interaction between electron 1 and hole 3 (electron 2 and hole 4) (Fig. 1). Thus, a specific feature of one-dimensional superconductivity will be not pairs, but groups of four. In the diagram language this means that along with the Cooper diagram of Fig. 2a, the order of magnitude of which at small  $p_1 + p_2$  is  $g^2 \ln \epsilon_0 / (p_1 + p_2)$ , it is necessary to take into account also a diagram which has at  $p_1 - p_3 \approx 2p_0$  the same order  $[g^2 \ln \epsilon_0 / (p_1 - p_3 - 2p_0)]$ . It is easy to understand that in our approximation when  $g \ln \epsilon_0 / (p_1 + p_2) \sim v$ , all the diagrams of the type of Fig. 2c will also be important. A distinguishing feature of this class of diagrams is that any of them can be divided into two parts by cutting two internal lines.

The determination of the sum of all diagrams of this type, the so-called "parquet," reduces to a solution of the system of nonlinear integral equations derived by Dyatlov, Sudakov, and Ter-Martirosyan [2]. The concrete form of the system depends on the spin structure of the interaction. In our case, when the spin dependence of the total vertex part at  $p_1 p_4 \sim -p_2 p_3 \sim p_0$  is determined by the formula

$$\Gamma_{\alpha\beta, \gamma\delta}(p_1 p_2, p_3 p_4) = \gamma_1 (\delta_{\alpha\gamma} \delta_{\beta\delta} - \delta_{\alpha\delta} \delta_{\beta\gamma}) + \gamma_2 \delta_{\alpha\delta} \delta_{\beta\gamma},$$



Fig. 1

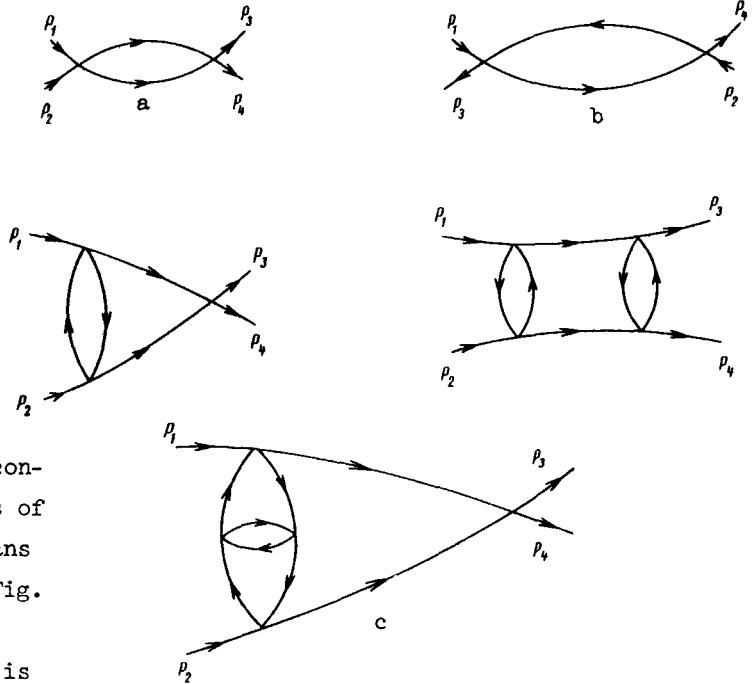


Fig. 2

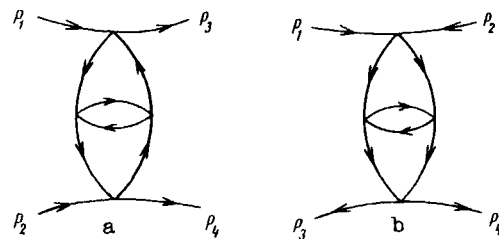


Fig. 3

the system is written in the form

$$\begin{aligned}\gamma_{2,3}(\eta, \xi) &= s_{2,3}(\eta) + s_{2,3}(\eta) \int_{\eta}^{\xi} \gamma_{2,3}(\zeta, \xi) d\zeta + \int_0^{\eta} s_{2,3}(\zeta) \gamma_{2,3}(\zeta, \xi) d\zeta, (\eta < \xi); \\ \gamma_{\pm}(\xi, \eta) &= \sigma_{\pm}(\xi) \mp \sigma_{\pm} \int_{\xi}^{\eta} \gamma_{\pm}(\zeta, \eta) d\zeta \mp \int_0^{\xi} \sigma_{\pm}(\zeta) \gamma_{\pm}(\zeta, \eta) d\zeta, (\eta > \xi); \\ \gamma_1(\xi, \xi) &= s_1(\xi) + \sigma_1(\xi) - \gamma_1^{(0)}, \quad \gamma_2(\xi, \xi) = s_2(\xi) + \sigma_2(\xi) + \sigma_2(\xi) - \gamma_2^{(0)} \quad (\eta = \xi)\end{aligned}$$

Here

$$\begin{aligned}\gamma_3 &= -2\gamma_1 + \gamma_2, & \gamma_{\pm} &= \gamma_1 \pm \gamma_2; \\ \xi &= (1/2\pi v) \ln \epsilon_0/v|p_1 + p_2|, & \eta &= (1/2\pi v) \ln \epsilon_0/v|p_1 - p_3 - 2p_0|\end{aligned}$$

The quantities  $s$  and  $\sigma$  are the sums of all the diagrams which can not be cut respectively along two parallel lines with total momentum  $p_1 + p_2$  (for example, Fig. 3a) and along two antiparallel lines with momentum difference  $p_1 - p_3$  (Fig. 3b). In the definition of the quantities  $\xi$  and  $\eta$ , the momenta must be taken to mean the largest of the three following quantities: the momentum proper, the frequency, and (in the case of diagram technique at  $T \neq 0$ ) the temperature.

The system can be solved exactly by the method proposed by Ansel'm [3]. At temperatures above the transition point we obtain

$$\begin{aligned}\gamma_1(\xi, \xi) &= g/(1 + 2g\xi), \quad \gamma_2(\xi, \xi) = (1/2)[\gamma_1(\xi, \xi) - g], \quad \xi < \xi_0 \\ \gamma_1(\xi, \xi) &= (\bar{g} - \lambda^2)/[1 + 2(\bar{g} - \lambda^2)(\xi - \xi_0)], \quad \xi > \xi_0 = (1/2\pi v) \ln \epsilon_0/\omega_D\end{aligned} \quad (4)$$

From this follow at  $p_1 = -p_2$  and  $p_3 = p_0$  the formulas given above for  $T_c$ . The Green's function of the phonons at  $k = 2p_0$  also has a singularity at the point  $T = T_c$

$$D \sim (1 + 2g\xi)^{-3/4} \quad (5)$$

At absolute zero, the superconductor is described by three Green's functions:

$$G = -i\langle Ta_p a_p^+ \rangle, \quad F^+ = \langle Ta_p^+ a_p^+ \rangle, \quad \bar{G} = -i\langle Ta_p a_{p-2p_0}^+ \rangle \quad (6)$$

for which we can write a closed system of equations analogous to the three-dimensional case (see, for example, [4]). Its solution is

$$\begin{aligned}G &= (\omega + \epsilon_{0p})/(\omega^2 + E_p^2), \quad F^+ = [-i\Delta^*(p)]/(\omega^2 + E_p^2), \quad \bar{G} = [i\delta(p)]/(\omega^2 + E_p^2); \\ E_p^2 &= \epsilon_{0p}^2 + |\Delta(p)|^2 + |\delta(p)|^2\end{aligned} \quad (7)$$

The functions  $\Delta$  and  $\delta$  satisfy the integral equations ( $\xi_{\text{eff}} = (1/2\pi v)\ln(\epsilon_0/\Delta_{\text{eff}})$ )

$$\begin{aligned}\Delta[\eta = (1/2\pi v)\ln(\epsilon_0/v|p - p_0|)] &= s(\eta) \int_{\eta}^{\xi_{\text{eff}}} \Delta(\zeta) d\zeta + \int_0^{\eta} s_3(\zeta) \Delta(\zeta) d\zeta, \\ \delta[\eta = (1/2\pi v)\ln(\epsilon_0/v|p - p_0|)] &= \sigma_+(\eta) \int_{\eta}^{\xi_{\text{eff}}} \delta(\zeta) d\zeta - \int_0^{\eta} \sigma_+(\zeta) \delta(\zeta) d\zeta\end{aligned} \quad (8)$$

which have nonvanishing solutions only when  $1 + 2g\xi_{\text{eff}} = 0$ .

The system of equations for  $G$ ,  $F^+$ , and  $\bar{G}$  enables us to find the current flowing in the superconductor as a result of application of an alternating electric field  $E$ . Calculations

with account of the specific nature of the Coulomb forces (see [4], Sec. 37) yield

$$I(\omega) = i(2e^2v/\pi\omega)(|\Delta|^2/\Delta_{\text{eff}}^2) E(\omega) \quad (9)$$

We note, finally, that Ferrell's arguments [5] against the existence of one-dimensional superconductors do not seem convincing to us. It can be shown (this will be done in a detailed article) that the presence of ion oscillations (there are no electron oscillations in our system) will merely cause the one-dimensional superconductor to be always similar to an alloy.

- [1] W. A. Little, Phys. Rev. 134, A1416 (1964).
- [2] Dyatlov, Sudakov, and Ter-Martirosyan, JETP 32, 767 (1957), Soviet Phys. JETP 2, 631 (1957).
- [3] A. A. Ansel'm, JETP 38, 1887 (1960), Soviet Phys. JETP 11, 1356 (1960).
- [4] Abrikosov, Gor'kov, and Dzyaloshinskii, *Metody kvantovoi teorii polya v staticheskoi fizike* (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz, 1962.
- [5] R. A. Ferrell, Phys. Rev. Lett. 13, 330 (1964).

#### ERRATUM

Figure 1 of the article by Yu. Kagan and V. Sobakin, "Anisotropy of Odd Photomagnetic effect in Semiconductors of Cubic Symmetry," in JETP Letters 2, 46 (1965), was erroneously identified in the caption as pertaining to the even photomagnetic effect, in place of the odd effect.