taken in the form proposed by Hulthen [5], the amplitude of the \((KN + \pi\Sigma)\) reaction was taken from [6], and the amplitude of the \((\Sigma N + \Lambda p)\) reaction was assumed constant. It should be noted that in such an analysis of the experimental data we cannot claim to obtain estimates of the cross section of the \((\Sigma N + \Lambda p)\) reaction near threshold.

This is connected with the fact that the spectrum presented in Fig. 1 is not normalized, i.e., no data are given on the probability of this reaction. Therefore a comparison with experiment determines the amplitude \((1)\) accurate to within a constant factor. The property of the \((K^+d + \pi^-\Lambda p)\) reaction is expressed in terms of \((1)\) as follows

\[
w(\omega) = N|M(\omega)|^2 \rho(\omega),
\]

where \(N\) is a constant factor and \(\rho(\omega)\) is the phase volume. The unknown parameters of the problem were the real and imaginary parts of the constant \(C\). The normalization of the function \(w(\omega)\) was based on the value of this function at the threshold.

The obtained diagram is shown by the solid line in Fig. 1. We see that the height and half-width of this peak are in satisfactory agreement with experiment \((\chi^2 = 1.8\) for 21 degrees of freedom).

It can be concluded from the foregoing that to describe the 2130-Mev peak in the \((\Lambda p)\) mass spectrum there is no need to introduce a \((\Lambda p)\) resonance. An estimate of the \((\Sigma N + \Lambda p)\) reaction amplitude based on the study of the entire \((\Lambda p)\) mass spectrum, shows that

\[
\frac{d\sigma_{\Lambda p - \Sigma N}}{d\Omega} \sim (1.4 \text{F})^2 \frac{k\Sigma N}{k\Lambda p},
\]

i.e., the \(\Lambda p + \Sigma N\) amplitude at the threshold has the same order of magnitude as the \((\Lambda p)\) elastic scattering length \((a_{\Lambda p} = -1.6 \text{F}; k_{ij}\) is the relative momentum of the particles \(i\) and \(j\).

In conclusion, I wish to thank I.S. Shapiro for useful advice and a discussion of the results, and also V.M. Kolybasov for a discussion as a result of which the initial variant of the paper was substantially revised.


SCATTERING OF NEUTRONS BY NUCLEI IN THE REGION OF NON-OVERLAPPING RESONANCES OF THE NUCLEUS

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We have measured the differential cross sections of elastic scattering of neutrons with energies \((1.8 \pm 0.2)\) MeV by the nucleus \(^{208}\text{Pb}\) in the angle range
12 - 168° (the neutron source was the reaction $^{12}\text{C} - d^1$; the $^{208}\text{Pb}$ sample had 97.7% enrichment). The measurements were made with two neutron beams having the same energy boundaries, but differing greatly in the distribution of the number of neutrons in the beam with respect to the energy. The distribution of the neutron intensity in the beam as a function of their energy in the first beam was characteristic of the $^{12}\text{C} - d$ reaction under experimental conditions. The measured values of the differential cross section $\sigma(\theta)$ for elastic scattering of neutrons from this beam in the investigated sample are shown in the figure by the points, through which curve 1 is drawn. The second beam was obtained by passing the first beam through a $^{208}\text{Pb}$ filter with transparency $0.12^2$. The main difference between the first and second beams was that in the second the number of neutrons corresponding to resonances in the total cross section of the interaction of the neutrons with the $^{208}\text{Pb}$ nucleus$^3$ was smaller by one order of magnitude or more. Curve 2 is drawn through the points representing the results of the measurements of the differential cross section for elastic scattering of the neutrons from the filtered beam $\sigma^f(\theta)^4$. Both curves reveal a characteristic diffraction structure.

Curve 3 in the same figure is drawn through the points representing the difference of the cross sections $\sigma(\theta) - \sigma^f(\theta)$. The indicated errors take into account only the statistical accuracy of the individual measurements. When the relative normalization of curves 1 and 2 is changed within 4% (see footnote$^4$) the curve $\sigma(\theta) - \sigma^f(\theta)$ changes in form somewhat, but its diffraction structure in the positions of its minima remain practically unchanged. (For comparison, at the bottom of the figure we show the difference $\sigma(\theta) - \sigma^nf(\theta)$, where $\sigma^nf(\theta)$ is the differential cross section obtained in a control experiment with a neutral filter of paraffin.)

Data on the elastic scattering of neutrons is customarily interpreted with the aid of the optical model of the nucleus, and the conditions of our experiment correspond to that case of application of the model, when a large number

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1) The detectors were scintillation counters, and the pulse-height analysis of the neutron pulses made it possible to eliminate the contribution made to the detector counts by neutrons with energies exceeding 2 MeV (the neutrons from the reactions d-d and $^{13}\text{C} - d$ on the target).

2) The lead filter was placed near the source of the neutrons ahead of the collimator channel.

3) $^{208}\text{Pb}$ contains in the investigated neutron energy region not less than 20 non-overlapping resonances in the total interaction cross section [1].

4) The statistical accuracy of the cross section measurements was ~2%; the values of the cross sections on curves 1 and 2 were determined relative to each other with accuracy ~4%; the accuracy of determination of the absolute value of the cross section is ~7%.
of non-overlapping resonances of the investigated nucleus lie within the energy spread of the neutron beam employed in the measurements. It is customarily assumed that the difficulty arising in the description of the experimental data in this case are resolved on the basis of physical notions concerning the existence of two processes of elastic scattering of neutrons by nuclei: potential (shape-elastic) scattering characterized by the cross section \( \sigma_{se}(\theta) \) and scattering via a compound nucleus (compound-elastic), characterized by a cross section \( \sigma_{ce}(\theta) \). Inasmuch as the amplitudes of both processes are assumed to be non-interfering (the processes are separated in time), the experimentally observable differential cross section \( \sigma(\theta) \) is represented in the form of a sum of the cross sections of the two processes, \( \sigma(\theta) = \sigma_{se}(\theta) + \sigma_{ce}(\theta) \). It is stated that the angular distribution of the cross section \( \sigma_{ce}(\theta) \) is isotropic, or else is symmetric about the angle 90°, and its magnitude can be calculated if one knows the levels of the intermediate nucleus. It is precisely in this manner that one interprets the experimental data on the elastic scattering of the neutrons with energy of several MeV and below [2 - 8].

In our experiment, the filtered beam of neutrons contained much fewer, compared with unfiltered beam, neutrons capable of being scattered resonantly (i.e., via the compound nucleus) by the \(^{208}\text{Pb} \) nucleus, and therefore the cross section difference \( \sigma(\theta) - \sigma^2(\theta) \) should, in accord with the foregoing, describe the angular distribution of the scattering via the compound nucleus. However, one cannot speak of any symmetry of curve 3 about the 90° angle. This result apparently indicates that the idea of the existence of two interfering elastic neutron-scattering processes is not applicable to interpretation of these experiments, at any rate in the region of the isolated resonances of the nucleus.

We add that the observed difference in the shapes of the curves 1 - 3 is very similar to that resulting from the scattering of the same beam of neutrons by three nuclei with different dimensions. The picture of scattering with the filtered beam corresponds to scattering by a nucleus of small dimensions, the picture of scattering of the unfiltered beam corresponds to a nucleus of medium dimensions, and the picture of scattering by neutrons removed by the filter corresponds to scattering by a large nucleus. This is precisely the phenomenological behavior of the shift of the positions of the minima and maxima of the angular distributions shown in the figure. One cannot exclude the possibility that this is not an accident, and that the position of the minima correlates with the total interaction cross section. Then the excessively small depth of the minima in the experimental angular distributions may be the consequence of a superposition of a number of pictures of angular distributions, corresponding to different neutron energies in the investigated beam.


\(^{5}\) The cross section at the minima greatly exceeds the value predicted by the theory.
POSSIBLE CONNECTION BETWEEN THE AMPLITUDES OF THE PROCESSES $e^+e^- \rightarrow 3\pi$, $\gamma\gamma \rightarrow 3\pi$, and $\pi^0 \rightarrow 2\gamma$

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We consider in the single-photon approximation an invariant matrix element (ME) of the process $e^+e^- \rightarrow 3\pi$ (see Fig. 1) near the threshold

$$\frac{e^{i(k')_\nu u(k)}}{q^2} J_\nu(p_1, p_2, p_3),$$

where

$$J_\nu = \langle p_1 a, p_2 b, p_3 c | i_\nu(0) | 0 \rangle = ih^{abc} \epsilon_{\nu \alpha \beta} p_1^\alpha p_2^\beta p_3^\gamma.$$  

(2)

Here $a$, $b$, and $c$ are the isotopic indices of the pions, $J_\nu$ is the electromagnetic current of the hadrons ($J_\nu = (1/2)e\bar{\psi}(1 + i\tau_3)\gamma_\nu \psi + \ldots$). We assume that $h$ is constant, a reasonable assumption near threshold.

We consider further the ME of the decay $\pi^0 \rightarrow 2\gamma$ (see Fig. 2):

$$M_{\nu \mu} = f_{\nu \mu \alpha \beta} q_1 a q_2 b,$$

where $f$ is connected with the lifetime of the $\pi^0$ meson: $\tau = 64\pi/f^2\mu^3$, where $\mu$ is the pion mass. Under very likely assumptions, we shall show further that

$$h = f/eF_\pi^2,$$

where $F_\pi = 0.93\mu/\sqrt{2}$ is the constant of the $\pi^+$-meson.

The derivation of relation (4) follows, strange as it seems, from a consideration of the properties of the more complicated amplitude $R_{\nu \mu}$, which describes the process $\gamma\gamma \rightarrow 3\pi$ (see Fig. 3a):

$$R_{\nu \mu}^{abc} = i\int dx e^{-iq_1 x} \langle p_1 a, p_2 b, p_3 c | T(i_\nu(x) i_\mu(0)) | 0 \rangle = R_{\nu \mu}^{pole} + R_{\nu \mu}^{cont}.$$  

(5)

After explicitly separating the pole terms ($R_{\nu \mu}^{pole}$), the remaining contact part of the amplitude ($R_{\nu \mu}^{cont}$) should be expanded in powers of the momenta $p_1$ and $q_1$. Confining ourselves to the lowest terms of the expansion and taking into account the requirements of $G$ parity and Bose symmetry, we find that the contact part is equal to:

$$R_{\nu \mu}^{cont} = f_{\nu \mu \alpha \beta} (-A(q_1 - q_2) a P_1^\beta + B q_1 a q_2 b) \delta_{\alpha \delta} \delta_{bc}$$

$$+ \text{perm: (} p_1, a - p_2, b - p_3, c \text{),}$$

(6)

where $A$ and $B$ are certain constants.

The pole part of the amplitude (5) is equal to the sum of the diagrams of Fig. 3b and Fig. 3c. The pole term in Fig. 3b includes the amplitude (3) and the $\pi\pi$ scattering amplitude, which