CHANGE OF RELATIVISTIC COLLAPSE INTO ANTICOLLAPSE AND KINEMATICS OF A CHARGED SPHERE

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Relativistic gravitational collapse of a spherical star ends, as is well known [1], with an asymptotic (as $t \to \infty$) stoppage of the contraction process, as seen by an external observer, at a gravitational radius $r \to r_g = 2Gm/c^2$. Deviations from exact spherical symmetry do not change this conclusion [2].

In its proper time, the star reaches $R_g$ after a finite time and continues to contract further. As a result of instability of the spherical contraction, the perturbations increase without limit. This raises the question of the subsequent fate of the matter in the star, since it is unknown whether the contraction can change to expansion or whether the material goes (formally) through an infinite density stage. Penrose has shown [3] that under the Schwarzchild sphere the occurrence of true singularities of space time is unavoidable, but did not explain whether the entire matter reaches $\rho = \infty$. It is shown in [2,5] that the star cannot expand again (even asymmetrically) in a manner so it would go out from under the Schwarzschild sphere ($r = r_g$) into a region visible to the same external observer [1)] who sees its collapse in his own (infinite) time.

The purpose of the present note is to show that the expansion of the star after contraction can occur just the same, and that the star goes out from under the Schwarzschild sphere, but into another external region (which is Euclidean at infinity) with the same properties as the first external region and situated in the absolute future relative to it [2)].

To show this, let us consider the collapse of a sphere of charged dust. This will enable us to study the change from contraction to expansion within the Schwarzschild sphere in a strictly spherical problem, without the matter going through infinite density (except for the particle in the center).

We assume that the matter in this sphere is originally rarefied, the sphere has a uniform charge distribution, and the charge does not become redistributed in the matter during the course of the collapse.

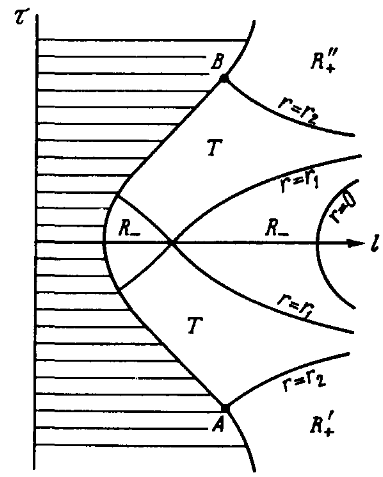
We investigate first the motion of a point on the surface of this sphere. The motion of this point can be regarded as the motion of a charged trial body in the external gravitational and electric fields of the charged sphere, determined by the well known Nordstrom metric

$$ds^2 = \left(1 - \frac{2Gm}{c^2 r} + \frac{G\epsilon^2}{c^4 r^2}\right)dt^2 - \left(1 - \frac{2Gm}{c^2 r} + \frac{G\epsilon^2}{c^4 r^2}\right)dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

where m is the mass of the sphere and $\epsilon$ its charge, with $m > \epsilon/G^{1/2}$. The component $g_{00}$ vanishes for two values of $r = r_1, r_2$, $(r_1 < r_2)$; $r_2$ corresponds to the gravitational radius of the Schwarzschild solution. The region $r > r_2$ is the usual space outside the Schwarzschild sphere ($r = r_2$), which goes over into Galilean space at infinity. We shall call it the $R_+$

region. The region $r_1 < r < r_2$ is the so-called T region [5]; the region $r < r_1$ will be called the internal $R_-$ region. The true singularity $r = 0$ of space-time is space-like and lies in the $R_-$ region.

The complete structure of the Nordstrom space-time is given in [7]. It is important that this space-time should consist of an infinite denumerable set of $R_+$ regions, which alternate with the T and $R_-$ regions (part of the total Nordstrom space is shown in the figure; for more details see [7]). The metric has an "oscillating" character. This means that as the trial body (in particular, a charged one) falls to smaller r it goes under the gravitational radius $r_2$, but does not reach $r = 0$, turning back in the $R_-$ region and moving towards larger r (see the figure). We call attention to the fact that after the surface of this sphere crosses $r = r_2$ at the point A of the figure it never appears again in the same region $R'_+$, but goes out from $r = r_2$ at the point B in an already different region $R''_+$ which lies in the absolute future relative to $R'_+$.




Contraction and expansion of a charged sphere: τ - time coordinate, *l* - radial spatial coordinate.

The region inside the sphere is not described by Nordstrom's solution and is shown shaded in the figure. Motion of any particle of the sphere occurs only under the influence of the masses and charges lying inside the sphere drawn through the particle. Qualitatively the motion is the same as that of the surface, and the particles do not reach $r = 0$. It follows from Einstein's equations that no infinite density is reached when the charge is uniformly distributed [3] (except for the particle at $r \equiv 0$), and the contraction gives way to expansion not simultaneously, but beginning on the edge of the sphere and moving toward the center. The maximum contraction of each layer is $\rho \approx c^6 m^4/\epsilon^6$. Outside the charged sphere, in the vacuum, there is a true singularity at $r = 0$ (see the figure).

If the oscillations of the sphere continue for an infinite number of times, it becomes necessary to consider an infinite number of $R_+$ regions.

As already noted, the introduction of the charge is an artificial strategem, indicating the character of the solution. It can be assumed that in the general case of neutral matter the increase in the perturbations upon contraction (or processes at $\rho > 10^{93}$ g/cm$^3$) will cause the contraction to change to expansion, but in an external space which is different (in a certain sense).


[1] L. D. Landau and E. M. Lifshitz, Teoriya polya (Field Theory), Fizmatgiz, 1962.

[2] A. G. Doroshkevich, Ya. B. Zel'dovich, and I. D. Novikov, JETP 49, 170 (1965), Soviet Phys. JETP 22, 122 (1966).

[3] R. Penrose, Phys. Rev. Lett. 14, 57 (1965).

[4] J. A. Wheeler, in Gravitatsiya, neitrino i Vselennaya (Gravitation, Neutrinos, and the Universe) (Russ. Transl.) IIL, 1962.

[5] I. D. Novikov, Soobshcheniya (Reports), Main Astronomical Observatory, No. 132, 3 (1964).
[6] I. D. Novikov, Astron. zh. 41, 1075 (1964), Soviet Astronomy AJ 8, 857 (1965).
[7] J. C. Graves and D. R. Brill, Phys. Rev. 120, 1507 (1960).

1) It is assumed that the observer is always far from the star, where the field is weak. In the Friedmann cosmological model this is not the case; the change of contraction into expansion was considered for this case in [6].

2) We emphasize that these external spaces differ in principle from the spaces that are made continuous through Wheeler's topological "knobs" [4]. The latter can be joined by a single space-like section.

3) In the case of a charge concentrated in the center, the equation can be integrated rigorously [8]. Crossings of the dust particles are then unavoidable.

## QUANTUM OSCILLATIONS OF SURFACE RESISTANCE OF ZINC AT 1 Mc

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It is presently known that low-frequency resistance oscillations in zinc, corresponding to a needle-like Fermi surface, do not constitute the ordinary Shubnikov - de Haas effect. The giant amplitude of these oscillations is attributed to magnetic breakdown of the Fermi surface of zinc [1,2].

To investigate in greater detail these unusual oscillations, we measured the surface resistance R(H) of zinc single crystals at 1 Mc, using a type IMI-2 nuclear magnetic-field pickup. The sample was placed in the oscillator coil. The detector output was fed to a low-frequency narrow-band amplifier with phase detector. The absorption signal was recorded with a two-coordinate automatic plotter. This method of investigating quantum oscillations was first used in [3].

The measurements were made in fields up to 22 kOe at 1.4°K. The modulation frequency of the external magnetic field was 20 cps. The zinc samples were the same as used in [2], with $\rho_{300°K}/\rho_{4.2°K} \approx 18{,}000$. The samples were cylinders ~2 mm in diameter and ~30 mm long or plates measuring 1 x 2 x 20 mm. Different polarizations of the high-frequency current relative to the direction of the constant magnetic field $\bar{H}$ were used for the same crystallographic direction. The field could be either parallel or perpendicular to the skin-layer surface. The geometry of the samples made it possible to carry out dc measurements simultaneously with the high-frequency measurements.

The measurements disclosed oscillations of $\partial R(H)/\partial H$, connected with different parts of the Fermi surface of zinc (with period up to $1 \times 10^{-7}$ Oe$^{-1}$). Principal attention was paid, however, to oscillations due to the needle-like Fermi surface, and the following results were obtained for this case.