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SUPERCONDUCTIVITY NEAR A PARAMAGNETIC IMPURITY

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It is shown in [1] that introduction of a small amount ($\tilde{<}1\%$) of paramagnetic impurities in a superconductor exerts a strong influence on its properties. In particular, the energy gap of the spectrum of such an alloy no longer coincides with the magnitude of the ordering parameter Δ. The calculation in [1] was carried out in the Born approximation with respect to the impurity potential. It will be shown below that in the case of a superconductor allowance for the total amplitude of the scattering of the electrons by the magnetic impurity leads to a number of interesting results.

We consider first the case of one impurity (placed at the origin) with an interaction in the form

$$\hat{V}(r) = 2\pi/m[U(r) + J(r)\sigma\vec{S}], \tag{1}$$

where $\vec{S}$ - spin of the impurity, which we shall regard as a classical vector. Allowance for the quantum properties of $\vec{S}$ leads to the known Kondo anomaly [2], which arises also in a non-superconducting metal. The foregoing assumption is apparently justified if the spin of the impurity is sufficiently large. The spectrum of the superconductor at $T = 0$, in the presence of an external field $\hat{V}$, is determined by the following system of equations for the coefficients (u, v) of the Bogolyubov transformation [3]:

$$\begin{aligned} \epsilon u_\alpha(r) &= (p^2/2m - \mu)u_\alpha(r) + V_{\alpha\beta}(r)u_\beta(r) + i\Delta(r)\sigma^y_{\alpha\beta}v_\beta(r), \\ \epsilon v_\alpha(r) &= -(p^2/2m - \mu)v_\alpha(r) - V_{\alpha\beta}(r)v_\beta(r) - i\Delta(r)\sigma^y_{\alpha\beta}u_\beta(r) \end{aligned} \tag{2}$$

($\epsilon > 0$). The system (2) breaks up into two independent systems for the pairs (u , v) and (u , v), which are obtained from each other by reversing the sign of J. We choose $\vec{S}$ along the z axis and assume that the parameter Δ does not depend on the coordinates (we shall verify this later). Going over to the Fourier representation and expanding the functions $u(\vec{p})$, $v(\vec{p})$, and $V(\vec{p} - \vec{p}\,')$ in terms of the harmonics, we get

$$\begin{aligned} u_{\ell\uparrow}(\xi) &= p_0\,\frac{(U_\ell + J_\ell S)(\epsilon + \xi)\bar{u}_{\ell\uparrow} - \Delta(U_\ell - J_\ell S)v_{\ell\downarrow}}{\epsilon^2 - \Delta^2 - \xi^2}, \\ v_{\ell\downarrow} &= p_0\,\frac{-(U_\ell - J_\ell S)(\epsilon - \xi)v_{\ell\downarrow} + \Delta(U_\ell + J_\ell S)\bar{u}_{\ell\uparrow}}{\epsilon^2 - \Delta^2 - \xi^2}, \end{aligned} \tag{3}$$

where

$$\bar{f} = \int_{-\infty}^{+\infty} f(\xi)\,d\xi/\pi, \quad \xi = (p^2/2m) - \mu$$

and p_0 is the Fermi momentum. The expressions in the form $U_\ell \pm J_\ell S$ represent the ℓ-th harmonic of the total scattering amplitude of an electron having a spin directed upward or downward respectively by the impurity atom in the normal metal. It results from the bare interaction (1) by the usual allowance [4] for the momentum region far from the Fermi surface.

We shall show that the system (3) admits of a solution for energies ϵ lying inside the energy gap of the pure superconductor. Indeed, integrating with respect to ξ and equating to zero the determinant of the obtained system, we get

$$\epsilon_\ell = \Delta \frac{1 + (p_0 U_\ell)^2 - (p_0 J_\ell S)^2}{\sqrt{[1 + (p_0 U_\ell)^2 - (p_0 J_\ell S)^2]^2 + 4(p_0 J_\ell S)^2}} < \Delta \tag{4}$$

for $J < 0$. When $J > 0$, the same result is obtained for the pairs (u , v). Formula (4) becomes particularly simple if we introduce the scattering phases $\delta_\ell^\pm$:

$$\epsilon_\ell = \Delta \cos(\delta_\ell^+ - \delta_\ell^-), \qquad U_\ell \pm J_\ell S = \tan \delta_\ell^\pm. \tag{4'}$$

We see therefore that for a nonmagnetic impurity ($\delta_\ell^+ = \delta_\ell^-$) the excitation spectrum begins with Δ, as expected. For pure exchange scattering ($\delta_\ell^+ = -\delta_\ell^-$) we have $\epsilon_\ell = \Delta \cos 2\delta_\ell$. Thus, in the presence of a paramagnetic impurity in the system, there are a number of discrete levels inside the forbidden band, and their sequence is determined by the difference $\delta_\ell^+ - \delta_\ell^-$ for different harmonics. The origin of the levels has a clear-cut physical meaning, namely, the spin of the impurity tends to polarize the spins of the electrons of the Cooper pair in one direction, so that its binding energy decreases. The wave functions of these stages can be readily calculated. We confine ourselves here to the simplest case of isotropic scattering ($\ell = 0$). For distances that are large compared with atomic distances ($rp_0 \gg 0$) we have

$$u_{0\uparrow}, v_{0\downarrow} \quad \frac{\sin(p_0 r - \delta_0^\pm)}{p_0 r} e^{-r/\xi |\sin(\delta_0^+ - \delta_0^-)|}, \tag{5}$$

where $\xi = \hbar v_F/\Delta$ is the coherence length of the superconductor at $T = 0$. The wave functions (5) describe states localized near the impurity at distances $r \sim \xi/|\sin(\delta_0^+ - \delta_0^-)| = \xi(1 - \epsilon_0^2/\Delta^2)^{-1/2}$. In the normal metal ($\xi = \infty$), formulas (5) describe a free electron in a centrally-symmetrical field.

The change of the parameter $\Delta_1(r)$ under the influence of the impurity can be determined if one knows the system of functions (u, v) for the continuous spectrum. For isotropic scattering at $T = 0$, the result is

$$\frac{\Delta_1(r)}{\Delta} = -\frac{\sin^2(\delta_0^+ - \delta_0^-)}{(p_0 r)^2} \phi(r/\xi) \qquad \text{if } r \lessdot \xi, \tag{6}$$

where $\varphi(x)$ - dimensionless functions on the order of unity. The factor $(p_0 r)^2$ in the denominator ensures the aforementioned smallness of the relative change of Δ. The largest change takes place at distances $r \sim v_F/\tilde{\omega}$ and amounts to $\sim 10^{-4}$.

It is similarly possible to consider the case of two impurities located a distance R apart. It is immediately clear that the arising splitting and shift of the levels of the isolated impurities will be small for $Rp_0 \gg 1$ as a result of the presence of a rapidly oscillating factor $\sin(p_0 R \pm \delta)_{p_0 R}$ in (5). For impurities with antiparallel spins, the energy level remains degenerate and is determined by

$$\bar{\epsilon}_{\uparrow\downarrow} = \epsilon_0 \left[1 + \frac{1}{4} \frac{\mathrm{tg}^2 2\delta}{(p_0 R)^2} e^{-(2R/\xi)\sin 2\delta} \right] \tag{7}$$

for pure-exchange isotropic scattering ($\delta^+ - \delta^- = \delta$). The bar denotes averaging over distances $R \gg p_0^{-1}$. For the case of parallel spins, the corresponding expression is

$$\bar{\epsilon}_{\uparrow\uparrow} = \epsilon_0 \left[1 + \frac{1}{4} \frac{\sin^2 2\delta}{(p_0 R)^2} \left(1 + \frac{4R\sin 2\delta}{\xi} \right) e^{-(2R/\xi)\sin 2\delta} \right]. \tag{8}$$

In the latter case, splitting of the levels takes place and has an oscillatory character

$$\epsilon = \epsilon_0 \left[1 \pm \mathrm{tg} 2\delta \frac{\sin p_0 R}{p_0 R} e^{-(R/\xi)\sin 2\delta} \right]; \tag{9}$$

the minus (plus) sign pertains to the symmetrical (antisymmetrical) solution relative to the center of the impurity pair. The averaging of the expressions (7) and (8) over the entire sample for finite impurity concentrations leads to the conclusion that the isolated levels cease to exist when the electron mean free path becomes comparable with the coherence length ξ. If the scattering is not too weak, this corresponds to concentrations at which the superconductivity is completely destroyed in the system.

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JOSEPHSON EFFECT IN SUPERCONDUCTING POINT CONTACTS

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Radiation from a superconducting point contact was observed in experiments [1] when DC was made to flow through the contact. Exposure of the contact to an external electromagnetic