

CONCERNING ONE MODEL IN THE THEORY OF THE GUNN EFFECT

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The analytic theory of the Gunn effect [1-4] is based on phenomenological equations in which the carrier mobility μ and diffusion coefficient D are assumed to be functions of the electric field intensity E . They describe correctly the main qualitative laws governing the Gunn effect, but in most integrals involved in the theory a predominant contribution is made by regions in which the field is highly inhomogeneous, say the enrichment region forming the rear wall of the strong-field domain. Under these conditions the use of the functions $\mu(E)$ and $D(E)$ is incorrect for the following reasons:

First, when the diffusion current is comparable with the field current, the power acquired by the carriers from the field is determined by the total carrier flux, and not only by the field flux.

Second, generally speaking, the thickness of the enrichment region is of the order of the cooling length of the carriers, $l_\epsilon \approx kT_e/eE_p$, where T_e is the electron temperature and E_p is the field corresponding to the maximum drift velocity. Indeed, in the enrichment layer the diffusion and field currents should be of the same order, thus yielding directly the estimate $l \sim D/\mu E_p \sim l_\epsilon$ for the thickness of this layer. Inasmuch as the concentration and the field experience large changes within the length l_ϵ , the phenomenological theory is no longer valid and the problem must, strictly speaking, be solved with the aid of the kinetic equation (which entails tremendous difficulties).*

With the exception of special cases, therefore, the phenomenological theory can claim to provide only a model description of the effect. Nevertheless, it is advantageous to develop new models in order to obtain a more complete picture and to be able to compare different models with experiment and to estimate their relative advantages. We consider below a model in which the first of the aforementioned defects of the theory is eliminated.

We put $x_p = \epsilon E_p / 4\pi en_0$ and introduce new variables $\mathcal{E} = E/E_p$, $\xi = x/x_p$, $\mathcal{D} = D/x_p$, and the current density $f = i/en_0$. We can then write for the space charge waves moving with velocity c the well known equation for \mathcal{E} , which takes diffusion and drift into account (see [2]),

$$-\mathcal{D} \mathcal{E}_\xi \frac{d\mathcal{E}_\xi}{d\xi} + (\mu \mathcal{E} - c) (\mathcal{E}_\xi + 1) = f - c, \quad \mathcal{E}_\xi = \frac{d\mathcal{E}}{d\xi}. \quad (1)$$

The power acquired by the carrier in the field \mathcal{E} is

$$w = \mathcal{E} \bar{v} = \mathcal{E} \frac{f + c\mathcal{E}_\xi}{1 + \mathcal{E}_\xi}, \quad (2)$$

where $f + c\mathcal{E}_\xi$ is the current carried by the electric charges, $(1 + \mathcal{E}_\xi)$ is their density in n_0 units, and \bar{v} is the average carrier velocity.

We shall assume that μ and \mathcal{D} are determined at each point by the value of w at this point; we denote them as functions of w by $\tilde{\mu}(w)$ and $\tilde{\mathcal{D}}(w)$. These functions are connected with $\mu(\mathcal{E})$ and $\mathcal{D}(\mathcal{E})$, defined for a homogeneous field \mathcal{E} when $w = \mathcal{E}^2 \mu(\mathcal{E})$, in the following manner:

$$\tilde{\mu}(w) = \tilde{\mu}(\mathcal{E}^2 \mu(\mathcal{E})) = \mu(\mathcal{E}), \quad \tilde{\mathcal{D}}(w) = \tilde{\mathcal{D}}(\mathcal{E}^2 \mu(\mathcal{E})) = \mathcal{D}(\mathcal{E}). \quad (3)$$

If we transform somewhat Eq. (1) and apply to it the Bendikson condition for the existence of closed integral curves, then we can show, with allowance for (2), that this criterion is satisfied when $f = c$. Then (2) goes over into $w = \mathcal{E}f$, and (1) takes the form

$$\frac{\mathcal{E} \mathcal{E}}{\mathcal{E} \mathcal{E} + 1} \frac{d\mathcal{E}}{dw} = \frac{w \tilde{\mu}(w) - f^2}{f^2 \tilde{\mathcal{D}}(w)}; \quad (4)$$

the integral between the extremal values of the field in the domain is therefore

$$\int_{w_{\min}}^{w_{\max}} \frac{w \tilde{\mu}(w) - f^2}{\tilde{\mathcal{D}}(w)} dw = 0. \quad (5)$$

In order to go over to the usual functions μ and \mathcal{D} , we introduce an auxiliary field E defined with the aid of $w = E^2 \mu(E)$ and assume that w is a monotonic function of E . Then (5) takes the form

$$\int_{E_{\min}}^{E_{\max}} dE \frac{v^2(E) - f^2}{\mathcal{D}(E)} \frac{d(vE)}{dE} = 0, \quad (6)$$

$$v(E) = E \mu(E)$$

Since $\mathcal{E} = E v(E)/f$, formula (6) makes it possible to determine, in analogy with [1,2], the extremal field in the domain and to find the limiting current that separates the regions of existence of the strong and weak fields; in this case the results can differ greatly from those of [1,2]. The results agree only when f is close to v_p or v_v - to the experimental values of the drift velocity. It is of interest to note, however, that in these cases, in spite of the fact that the domain boundaries become broad compared with l_e , the equations in (1), written in terms of the functions $\tilde{\mu}$ and $\tilde{\mathcal{D}}$, are not the same as those in terms of μ and \mathcal{D} , and differ in the limit by a factor 2 in front of $(\mu \mathcal{E} - c)$.

However, the thermal-diffusion current was not taken into account in (1). Yet it was shown in [7] that it can greatly alter the velocity of domains whose width is not large. If we introduce the thermal diffusion current by simply replacing** $\tilde{\mathcal{D}}(dn/d\xi)$ by $(d/d\xi)(\tilde{\mathcal{D}} n)$, then in our case the changes can be even more appreciable, because the coefficient of the highest-order derivative $\mathcal{E}_{\xi\xi}$ will change, owing to the dependence of $\tilde{\mathcal{D}}(w)$ on \mathcal{E}_{ξ} , from $\tilde{\mathcal{D}}$ to $\tilde{\mathcal{D}} + (c\mathcal{E} - w)\tilde{\mathcal{D}}_w$, and the latter quantity, generally speaking, is not sign-invariant.

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* This difficulty was circumvented in the detailed paper [5] by introducing the electron temperature and investigating its dynamics. However, the Maxwellian distribution does not have time to become established at the actual temperatures and concentrations (see [6]), and therefore the approach of [5] is likewise a model approach.

** This follows from the kinetic equation in the presence of a relaxation time.

E R R A T U M

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On page 179, line 13 from the bottom, correct "... v_v - to the experimental values"
to read "... v_v - to the extremal values."