Instability in parallel pumping in almost isotropic ferromagnets

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(Submitted January 29, 1976)
Pis'ma Zh. Eksp. Teor. Fiz. **23**, No. 6, 320–323 (20 March 1976)

An equation is derived for the nonlinear dynamics of weakly-anisotropic ferrodielectrics in the case of parallel pumping. A new type of instability is found.

PACS numbers: 75.30.–m, 77.40.+i

Paramagnetic resonance in magnets, which has been investigated in detail both theoretically[1] and experimentally, is produced in a frequency region $\omega > \omega_0 = 2gH$. Here H is the magnetic field and g is the gyromagnetic ratio. The purpose of the present communication is to point out the possibility of an instability of an entirely different type, which is not connected with the decay of the electromagnetic wave into two spin waves. The new instability occurs at frequencies $\omega \ll \omega_0$.

We consider a ferrodielectric in an external homogeneous alternating magnetic field $\mathbf{H}(t)$ directed along the easy-magnetization axis. We assume that the characteristic frequency ω of the field satisfies the condition

$$\tau_{sp} << \omega^{-1} << \tau_{dd}, \qquad (1)$$

where τ_{sp} is the time of spin-phonon relaxation without a change of the moment, and τ_{dd} is the time of the moment relaxation due to the dipole-dipole interaction.

The existence of this frequency is guaranteed for ferrodielectrics whose Curie temperature T_C is larger than or of the order of the Debye temperature.[3] The strongest is the exchange interaction, which leads to establishment of thermal equilibrium in the spin-wave gas without changing the number of the waves. The spin-phonon interaction then equalizes the spin-system and the lattice temperatures, again without changing the moment. Finally the weak dipole-dipole interaction leads to a slow variation of the moment. This is the only process which we consider. It is convenient to introduce a fictitious magnetic field $H'(t)$, connected with its moment $M(t)$ by the relation

$$M(t) = M_0(H'), \tag{2}$$

where $M_0(H')$ is the moment as a function of the field in the equilibrium state. The fields H and H' are assumed to be weak in comparison with the saturation field. The equation of motion for H' takes the form

$$\chi_0(H')\frac{dH'}{dt} = Af\left(\frac{H'}{H}\right), \tag{3}$$

where A is a quantity that does not depend on H or H',

$$\chi_0 = \frac{\partial M_0}{\partial H'}, \quad f(x) = \operatorname{sgn} x(1-x) \iint\limits_{\substack{uv \geq 1/4 \\ u,\, v \geq 0}}^{\infty} \frac{du\,dv\,(4 + {}^3/_{uv})}{(u+|x|)(v+|x|)(u+v+|x|+\operatorname{sgn} x)}. \tag{4}$$

Eq. (3) was obtained from the kinetic equation using the known form[4] of the dipole-dipole collision integral. In the approximation linear in $H' - H$ the equation of motion was obtained by Kaganov and Tsukernik.[5] The derivation of Eq. (3) will be presented in a detailed paper. A fair approximation of $f(x)$ is the bilinear function

$$f(x) \approx \operatorname{sgn} x\,(1-x)\frac{a}{1+b|x|}, \tag{5}$$

$a = 37.4$ and $b = 5.68$ at $x > 0$ and $a = 156$, $b = 23.7$ at $x < 0$. The accuracy of the approximation (5) is about 10%. The values at $x = 0$ and $x = 1$, as well as the asymptotic value as $x \to \infty$, are exact. It was assumed here that **H** and **M** have the same direction. Eq. (3) retains its meaning also for oppositely directed **H** and **M** ($x < 0$), provided only that H' differs from zero. This means that in the case of a sudden change in the field direction, the moment first reaches its spontaneous value without changing direction ($H' = 0$) and then turns to the required direction without changing magnitude.

In this communication we consider the solution of Eq. (3) in only one particular case, when the field $H(t)$ can be represented in the form

$$H(t) = \bar{H} + h\cos\omega t, \tag{6}$$

with $\tau_{dd} \gg 1$. In this case the moment does not have time to change significantly over the period. Stationary states of the system $H' = \text{const}$ are possible, and their positions are determined, in accordance with (3), by the equation

$$\overline{f(H'/H)} = 0. \tag{7}$$

The bar denotes averaging over the period. Using the approximation (5), we obtain in the case $h \leq H$:

$$\frac{H'}{\bar{H}} = \frac{b \pm \sqrt{(b+1)^2 - (2b+1)\, h^2/\bar{H}^2}}{2b+1}, \quad (b = 5.68). \tag{8}$$

In the region $h \geq \bar{H}$, the dependence of $H'/\bar{H}$ on $h/\bar{H}$ is more complicated. An analysis shows that a plot of this dependence is close to the parabola (8) (Fig. 1). In the region lying between the solid curve and the coordinate axis, dH/dt

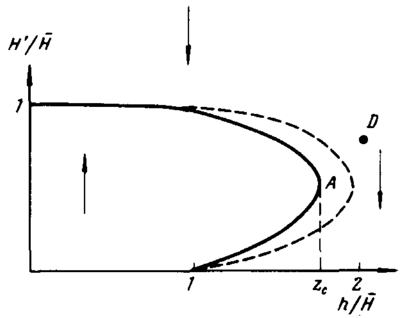

FIG. 1.

>0, and in the remaining part of the quadrant dH'/dt is negative. Therefore the upper part of the curve from 1 to A corresponds to stable stationary states. It is seen from Fig. 1 that the ratio $z = H'/H$ has a critical value such that at $z > z_c$ the magnet does not reach a stationary state. If the alternating field is turned on for a time too short for the moment to change, then one can land in principle on any point of the plane of Fig. 1. The subsequent motion is indicated by the arrows. In the instability region (the point D) the magnet first reaches the value $H' = 0$, and then the moment begins to rotate randomly. The motion of the moment during the "turbulence" stage is, of course, not described by Eq. (3). It is interesting that the state of the system also changes jumpwise at the instability threshold $z = z_c$, going from the point A to the axis $H' = 0$ with subsequent development of turbulence.

The stationary-state curve is also of definite interest, since the field H' can differ greatly from H. This can be verified by measuring the susceptibility $\chi = \partial M/\partial \bar{H}$ at a given value of h.

We present some estimates. For an yttrium garnet, the temperature region in which $\tau_{dd} > \tau_{sp}$ is determined by the inequality $T \geq 10\,°K$, and $\tau_{dd}^{-1} \approx 10^7$ Hz at $T \approx 100\,°K$ and $H \approx 100$ Oe. If $\tau_{sp} > \tau_{dd}$, then pumping leads to adiabatic oscillations of the temperature and to heating of spin waves. These temperature changes are negligibly small, and the arguments advanced above are valid if $T_C < T_D$. Consequently, the instability can be observed also in low-temperature magnets.

The dynamic instability considered above is of the same nature as the static instability of isotropic magnets[6]: the moment tends to follow the field. If the amplitude of the alternating field is so large that the moment and the field are oppositely directed for an appreciable fraction of the time, then a stationary state is impossible.

In conclusion, we thank Yu. P. Baglaev and N.I. Timofeev for help with the numerical calculations.

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