Primordial gravitons and possibility of their observation

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A mechanism for the formation of non-equilibrium isotropic gravitation-wave noise is indicated. Certain possibilities of its registration are considered and indicate that the corresponding observations are promising.

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At the present time, the efforts of the experimenters are aimed at observing rare gravitational-radiation pulses, accompanying cosmic catastrophes, as well as monochromatic signals from astronomic and laboratory sources. It must be persistently emphasized that there are weighty arguments in favor of the existence of gravitation radiation in the form of nonequilibrium isotropic cosmic noise containing an appreciable energy density and realized over a very broad spectrum. The mechanism of formation of such gravitation-wave noise (GWN) should be the super-adiabatic amplification of the gravitational waves and the spontaneous production of gravitons in the gravitational field of the metagalaxy during the earliest stages of its evolution. This mechanism follows even from the simplest premises of the standard cosmological model, namely that the smooth (background) gravitational field of the metagalaxy is nonstationary and isotropic.

The spectrum and intensity of GWN are determined by the character of the nonstationarity of the external gravitational field. Noise of the indicated origin
is completely absent only in the exceptional case when the scale factor of the background world obeys the law \( a(t) = a_0 \sqrt{t} \), up to the limits of applicability of modern gravitational theory, \( t = t_{P1} = \frac{3H_0}{c^3} \). If for some reason \( a(t) \) were to have a minimum at densities corresponding to \( t_{P1} \) or lower, then in the present epoch \( t_0 \) the GWN should consist of generated gravitons and contain also amplified gravitational waves of the contraction epoch\(^{29} \) in its low-frequency band \( \nu < \nu_c = a(t_{P1})/\nu_{P1} / a(t_0) \), where \( \nu_{P1} = t_{P1}^3 \). The cross sections of processes in which gravitons take part are so small\(^{28,31} \) that we can apparently expect the arbitrary initial spectrum to be "washed away" and converted into a Planck spectrum only in the region \( \nu > \nu_c \) (\( \nu_c \) is in the vicinity of \( 10^{11} \) Hz, \( 10^{11} \) Hz).

There are indirect limitations on the possible energy density \( \epsilon_\epsilon \) of the gravitational waves.\(^{31} \) They are based on additional assumptions and do not have the force of proof, but are nevertheless quite likely. The strongest limitation\(^{110} \) reduces to the fact that for waves with \( 0 < \lambda_{\epsilon} < \lambda_m \approx 3 \times 10^{17} \) cm \( \epsilon_\epsilon \) cannot greatly exceed the energy density \( \epsilon_\gamma \approx 4 \times 10^{-23} \) erg/cm\(^3\) of the 3 K electromagnetic radiation. Integral limitations on \( \epsilon_\epsilon \) say nothing concerning the spectrum. The mechanism of the GWN formation predicts a smooth frequency dependence of the spectral density of the flux. We represent the spectrum by the power law \( F_\nu = K_\nu^{-\alpha} \), where \( K \) is expressed in terms of \( \epsilon_\epsilon \) by integrating \( F_\nu / c \) with respect to \( \nu \) from \( \nu_m \) to \( \nu_c \). At \( \alpha > -2 \), the effective temperature \( T_\epsilon = c^2 F_\nu / 2 k_\nu^2 \) at low frequencies greatly exceeds the equilibrium temperature, thus facilitating the observation of the GWN.

The mean-squared amplitude \( \bar{A}_\epsilon^2 \) of randomly modulated oscillations of a gravitational antenna, in the absence of intrinsic (thermal) noise \( \bar{A}_b^2 \), is determined by the energy \( F_\nu_0 \sigma_t^* \), where \( \sigma_t^* \approx Q / \nu_0 \) is the relaxation, \( Q \) is the quality factor, and \( \sigma_t \) is the effective antenna cross section integrated over the angles and frequencies \( \Delta \nu \approx \nu_0 / Q \). Since it is impossible to be "shielded" from the gravitational radiation, or to "turn away" from it, an advantageous method of detecting the GWN against the background of the intrinsic noise of the antenna would be to use a correlation system (see also\(^{111,112} \)). Let two oscillators be oriented perpendicular to each other. Their intrinsic noises are not correlated (the mutual gravitational influence of the oscillators is negligible). We represent the GWN in the form of a frequent sequence of pulses with random amplitude and polarization. Each pulse arriving from directions in which the directionality pattern of the oscillators overlap exerts an opposite action on the oscillators, by virtue of the tensor character of the gravitational radiation. The product of the oscillator noises, averaged over an infinite time, would make it possible to find any \( \bar{A}_\epsilon^2 \). Averaging over an observation time containing \( n \) intervals \( \tau^* \) makes it possible to obtain a fraction \( 1 / \sqrt{n} \) of \( \bar{A}_\epsilon^2 \).

We write down the condition for the observation in the form \( F_\nu_0 \sigma_t^* > n^{-1/2} k T \). For an electromagnetic antenna–resonator with characteristic dimensions \( l_2 \approx \lambda_\epsilon \) and with a constant–field intensity \( H \) we have \( \sigma_\epsilon \approx (G/c^3) H^2 \lambda_\epsilon^2 \). A rigid antenna with \( l_2 \approx \lambda_\epsilon / 2 \approx (1/2) (v_\epsilon / c) \lambda_\epsilon \) (\( v_\epsilon \) is the speed of sound) and mass \( M \) yields \( \sigma_m \approx (G/c^3) (M c^2) (v_\epsilon / c)^2 \approx (G/c^3) M/2 v_0^2 \). The most suitable for observation is a spectrum with \( \alpha \approx 1 \), which corresponds to reasonable models of the initial state, although it is not mandatory. Substituting \( F_\nu_0 \approx c \epsilon_\epsilon \nu_0^{-1} \) and, accordingly \( \sigma_m \) and \( \sigma_\epsilon \), we rewrite the observation condition in the form

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where $\mu$ is the combination of the unaccounted for numerical coefficients and can reach a value $\mu \sim 10^2$. We use a stringent estimate $c_\varepsilon \approx 10^{-12}$ erg/cm$^3$. For the first antennas of the Weber type (and for $\sqrt{\pi} = 3 \times 10^5$) the left-hand side of (1) approximately equal to $10^{-3}$. An electromagnetic variant with $\nu = 10^7$ Hz, $H = 3 \times 10^5$ G, $Q = 10^{13}$, $T = 5 \times 10^{-3}$ sK, and $n = 1$ yields a value on the order of $3 \times 10^{-2}$ in the left-hand side of (1'). The required level will probably be reached by the next-generation rigid-body antennas with gigantic $Q$ factors and maintained at temperatures lower than that of liquid helium.  

R.B. Braginskii$^\dagger$ pointed out to me that the use of satellites free of drift is promising. Indeed, two satellites located a distance $l$ apart can acquire within a time $\Delta t \sim l/v$ a relative velocity $v \sim l \sqrt{G/c^3} c_\varepsilon$. At $l = 3 \times 10^{13}$ cm we have $v \approx 2 \times 10^{-7}$ cm/sec, which comes close to the attained sensitivity level. Of course, the GWN should manifest itself also in excessive "chatter" of the earth, planets, and other astronomical systems. The registration of GWN, or else an experimental determination of the outline of its spectrum, can yield fundamental information on exceedingly early stages of the evolution of the metagalaxy.

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1) In$^{[3]}$ I proved the possibility of graviton production, and not at all the opposite, as might be the impression gained by reading.$^{[6]}$ For this process it is necessary to have $a(t) \neq a_0 t$ regardless of whether this is caused by a "true" or "effective" energy-momentum tensor.
2) The low-frequency part of the spectrum of the gravitons produced on going from contraction to expansion in accordance with a power law is calculated in.$^{[7]}$ It is also noted there that the near-equilibrium spectrum, on which$^{[8]}$ insists, is obtained only in a particular case.

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$^7$A. A. Starobinski$^\dagger$, Phys. Lett. (in print).
Dynamic similarity at critical points of arbitrary order

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The dynamic exponents at the critical points of higher order are determined and their dependence on the conservation law is investigated.

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Systems with critical points, which are simultaneously critical for several phases, are presently intensively investigated. It is the number $\sigma$ of these phases which determine the order of a critical point. At $\sigma > 2$, the second-order phase transition curves go over in such points into first-order phase transition curves. Such a behavior of the curves is possible for the mixture $^3$He–$^4$He, for metamagnets, and for compressible magnets.$^{[1-5]}$ A significant change in the static critical properties has been observed as a function of the number $\sigma$ of phases near the critical point. A change takes place also in the dimensionality $d_\sigma$ of space starting with which a stable nontrivial solution appears for the renormalization group equations for the coupling constants of the fluctuating fields.$^{[6,7]}$ Expansion in powers of the deviation $e_\sigma = d_\sigma - d$, of the dimensionality $d$ of the system from $d_\sigma = 2\sigma/(\sigma - 1)$ makes it possible to calculate the resultant deviations of the critical exponents from the predictions of the mean-field theory.$^{[7-9]}$ In this paper this approach is generalized for the investigation of the dynamics at a critical point of arbitrary order. Let the Hamiltonian of the system be a Ginzburg–Landau functional

$$\mathcal{H}(\bar{\psi}) = \int d^d x \left[ \frac{1}{2} \left| \nabla \bar{\psi}(x) \right|^2 + \sum_{k=1}^{\sigma} \frac{u_{2k}}{(2k)!} (\bar{\psi} \psi)^k - h \bar{\psi} \right]$$

with a single $n$-component order parameter $\bar{\psi}(x)$, where $h(x,t)$ is the field conjugate to it. The dynamic equation can be obtained from the condition that the rate of the change of the order parameter be proportional to the conjugate thermodynamic force. The proportionality coefficient $\Gamma_0$, which sets the time scale, plays the role of the nonrenormalized kinetic coefficient and depends on the law governing the conservation of the order parameter. The action of the