Vortices with free ends in superfluid He^3-*A*

G. E. Volovik and V. P. Mineev

Institute of Solid State Physics, USSR Academy of Sciences and L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
(Submitted April 29, 1976)
Pis'ma Zh. Eksp. Teor. Fiz. **23**, No. 11, 647–649 (5 June 1976)

It is shown that vortex lines with free ends exist in the superfluid *A* phase of He^3. The circulation of the superfluid velocity about an infinitesimally small contour surrounding the vortex line is equal to $2\pi\hbar/m$. The velocity field near the end of the vortex coincides with the vector-potential field of the Dirac monopole.

PACS numbers: 67.50.Fi

It is known that in superfluid He^4 the vortices are either closed or terminate at the walls or on the free surface of the helium. The impossibility of existence of vortices that terminate in the volume of the liquid follows from the fact that the phase of the order parameter is a single-valued function of the coordinates (accurate to a factor $2\pi n$, where n is an integer). In fact, if we assume the existence of a vortex that terminates in the volume of the liquid, we find that the circulation of the velocity on an infinitesimally small contour surrounding the vortex filament, which is equal to $2\pi n\hbar/m$, decreases continuously to zero when the contour is removed from the vortex, and this contradicts the uniqueness of the phase.

As first shown by Ambegaokar, de Gennes, and Rainer[1] (see also[2]), in the superfluid *A* phase of He^3 the phase of the order parameter, specified by e triad of orthogonal vectors $\vec{\Delta}'$, $\vec{\Delta}''$, $\mathbf{l}$ is not a unique function of the co- ·dinates. The reason is that the phase difference of the order parameter be- ·een two neighboring points is equal in He^3-A to the angle of rotation of the ·ctors $\vec{\Delta}'$ and $\vec{\Delta}''$ about the direction of $\mathbf{l}$. The ambiguity of the phase is the ›nsequence of noncommuativity of the group of three-dimensional rotations. ıis leads to the possibility, in principle, of the existence of vortices that rminate in the volume of the He^3-A.

We consider the Ginzburg-Landau equation for the order parameter $=\vec{\Delta}' + i\vec{\Delta}''$

$$\frac{\delta F}{\delta \vec{\Psi}^*} = -\alpha\vec{\Psi} + \beta|\vec{\Psi}|^2\vec{\Psi} - \gamma(\Delta\vec{\Psi} + 2\vec{\nabla}(\vec{\nabla}\vec{\Psi})) + \lambda\vec{\Psi}^* = 0, \qquad (1)$$

ıere (see[3])

$$\alpha = \frac{1}{6}N_F\left(1 - \frac{T}{T_c}\right), \quad \beta = \frac{\alpha}{2\Delta^2(T)}, \quad \gamma = \frac{1}{16}\frac{\rho^s}{\Delta^2(T)}, \quad N_F = \frac{m^* p_F}{\pi^2},$$

d λ is a Lagrange multiplier that ensures orthogonality of $\vec{\Delta}'$ and $\vec{\Delta}''$ as well equality of their moduli.

We choose the origin at the end point of the vortex. It can be verified that the lution at $r \ll \xi$ [ξ is a coherence length of order $(\gamma/\alpha)^{1/2}$] is

$$\vec{\Psi} = \mathrm{const}\, r^2(1 + \cos\theta)e^{i\phi}(\mathbf{e}_\theta + i\mathbf{e}_\phi) \times F\ \frac{3+\sqrt{13}}{3}, \frac{3-\sqrt{13}}{2}, 3; \frac{1+\cos\theta}{2}. \qquad (2)$$

·re $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ are unit vectors of the spherical coordinate system. From (2) can obtain an expression for the superfluid velocity

$$\mathbf{v}^s = \frac{\hbar}{4mi|\vec{\Psi}|^2}(\Psi_i^*\vec{\nabla}\Psi_i - \Psi_i\vec{\nabla}\Psi_i^*) = \frac{\hbar\mathbf{e}_\phi}{2mr}\frac{1-\cos\theta}{\sin\theta}. \qquad (3)$$

is seen from this equation that the circulation $\mathbf{v}^s$ on an infinitesimally small ıtour surrounding the polar axis is equal to zero at $z>0$ and equal to $2\pi\hbar/m$ $z<0$, i.e., the vortex line lies along the semiaxis $z<0$. We note that the locity field $\mathbf{v}^s$ coincides with the field of the vector potential $\mathbf{A}$ of the Dirac ›nopole.[4]

At $|z| \gg \xi \gg \rho = r\sin\theta$ and $z<0$, expression (2) goes over into the solution

$$\vec{\Psi} = \frac{1}{2}\mathrm{const}\,\rho^2 e^{i\phi}(-\mathbf{e}_\rho + i\mathbf{e}_\phi) = \frac{1}{2}\mathrm{const}\,\rho^2 e^{2i\phi}(-\mathbf{e}_x + i\mathbf{e}_y)$$

r an infinite vortex filament, the velocity circulation around which is equal to $\hbar/m$; this solution is analogous to the solution for a vortex in superfluid He^4, th two circulation quanta (see, e.g., [5]).

It is also seen from (2) that the vector $\mathbf{l}$ is directed along the radius ($\mathbf{l}=\mathbf{e}_r$). ıis recalls Polyakov's "porcupine",[6] but in contrast to[6] there is no herically symmetrical solution for $\mathbf{l}$ at $r \gtrsim \xi$. A vortex with two free ends rresponds to a Polyakov "porcupine—antiporcupine" pair. From dimensional- considerations, the energy of such a vortex is $\rho^s(\hbar/m)^2 Lf(L/\xi)$, where L is › length of the vortex. At large L this energy should go over into the energy a vortex in He^4

$$f(L/\xi) \to 2\pi \ln L/\xi \quad , \qquad L >> \xi .$$

At small $L \ll \xi$ we have $f \sim \xi / L$.

A vortex with two free ends can be stabilized by two like ions fastened to its ends. An estimate shows that the equilibrium length of such a complex, obtained by minimizing the energy

$$E = \frac{e^2}{L} + \rho^s \left(\frac{\hbar}{m}\right)^2 L f(L/\xi) ,$$

is less than ξ.

After this article was written, we noted that the just-published paper by Blaha[8] indicates a topological possibility of the existence of a vortex with a free end in the A phase of He^3.

[1]V. Ambegaokar, P.G. de Gennes, and D. Rainer, Phys. Rev. **A9**, 2676 (1974).

[2]N.D. Mermin and Tin-Lun Ho, Phys. Rev. Lett. **36**, 594 (1976).

[3]P. Wölfle, Phys. Lett. **47A**, 224 (1974); A.J. Leggett, Rev. Mod. Phys. **47** 331 (1975).

[4]P.A. Dirac, Proc. R. Soc. A **133**, 60 (1931).

[5]L.P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. **40**, 646 (1961) [Sov. Phys.-JETP **13** 451 (1961)].

[6]A.M. Polyakov, Zh. Eksp. Teor. Fiz. **68**, 1975 (1975) [Sov. Phys.-JETP **41** 988 (1975)].

[7]S. Blaha, Phys. Rev. Lett. **36**, 874 (1976).