

Failure of the collinear expansion in calculation of the induced gluon emission

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We demonstrate that the collinear expansion fails in the case of gluon emission from a fast quark produced in eA DIS. In this approximation the $N=1$ rescattering contribution to the gluon spectrum vanishes. We show that the higher-twist approach by Guo, Wang and Zhang [4, 5] based on the collinear expansion is wrong. The nonzero gluon spectrum obtained in [4, 5] is a consequence of unjustified neglecting some important terms in the collinear expansion.

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1. Since the early nineties considerable theoretical efforts have been made to study the induced gluon emission from fast partons due to multiple scattering in cold nuclear matter and hot quark-gluon plasma (QGP). There have been developed several approaches to this phenomenon. The well known BDMPS [1] and GLV [2] approaches are based on the time-ordered diagram technique. The BDMPS formalism is valid for massless partons in the limit of strong Landau-Pomeranchuk-Migdal (LPM) [3] suppression when the number of rescatterings $N \gg 1$. The GLV approach accounts for small number of rescatterings ($N \leq 3$) and applies only to thin plasmas. The higher-twist method due to Guo, Wang and Zhang (GWZ) [4, 5] is based on the Feynman diagram formalism and collinear expansion. It includes only $N=1$ rescattering. The formalism has originally been derived for the gluon emission from a fast quark produced in eA DIS. The light-cone path integral (LCPI) formalism [6] (see also [7, 8, 9]) is free from the restrictions of the approaches [1, 2, 4, 5]. It accurately treats the mass effects, and applies at arbitrary LPM suppression.

The LCPI [6] and BDMPS [1] approaches become equivalent at strong LPM suppression [8, 9]. The predictions of the GLV [2] formalism can be obtained in the LCPI approach [6] by expanding the gluon spectrum in the density of the medium. However, the status of the GWZ approach [4, 5] is not clear. The BDMPS-GLV-LCPI approaches in their original formulations neglect the quantum nonlocality in production of the fast partons. In the case of gluon emission in the QGP produced in AA -collisions this approximation is justified by

the fact that the quantum nonlocality is $\sim 1/E$ (E is the energy of the fast parton), and can be safely neglected at large E . In the GWZ approach to eA DIS the nonlocal fast quark production and gluon emission have been treated on even footing. However, as we will show below in the applicability region of the GWZ formalism the quantum nonlocality in the quark production in eA DIS, similarly to AA -collisions, is not important for gluon emission. For this reason one could expect that the GWZ gluon spectrum should coincide with $N=1$ contribution in the LCPI formalism. But the two spectra are not identical. Indeed, the GWZ gluon spectrum for the Gaussian nuclear density $n_A(r) \propto \exp(-r^2/2R_A^2)$ at $z \ll 1$ (hereafter $z = \omega/E$, where ω is the gluon energy and E is the struck quark energy) can be written as (up to unimportant factors)

$$\frac{dP_{GWZ}}{dz} \propto \alpha_s^2 n_A(0) R_A P_{Gq}(z) \times \int \frac{dp^2}{p^4} x G_N(x, p^2) \left\{ 1 - \exp \left[-\frac{p^4 R_A^2}{4E^2 z^2 (1-z)^2} \right] \right\}, \quad (1)$$

where $P_{Gq} = C_F [1 + (1-z)^2]/z$, $x \ll 1$, G_N is the nucleon gluon density. To leading order in α_s $x G_N(x, Q^2) \approx \frac{3\alpha_s C_F}{\pi} \ln(Q^2/\mu^2)$ ($Q^2 \sim p^2$, μ is infrared cutoff). On the other hand for the $N=1$ gluon spectrum in the LCPI approach one can obtain for massless partons

$$\frac{dP_{LCPI}}{dz} \propto \alpha_s^3 n_A(0) R_A P_{Gq}(z) \times \int \frac{dp^2}{p^2(p^2 + \mu^2)} \left\{ 1 - \exp \left[-\frac{p^4 R_A^2}{4E^2 z^2 (1-z)^2} \right] \right\}. \quad (2)$$

One can see that the integrand in (2) does not have any logarithmic factor which could be interpreted as the nucleon gluon density entering (1).

In the present paper we clarify the situation with the discrepancy between the predictions of the GWZ and LCPI approaches. We demonstrate that the approximations used in [4, 5] really lead to a disagreement with the LCPI approach. However, contrary to the results of [4, 5] the consistent use of the method of [4, 5] gives a vanishing gluon spectrum. This fact is a consequence of failure of the collinear expansion. We show that the authors of Refs. [4, 5] obtained the nonzero spectrum just due to unjustified neglecting some important terms in the collinear expansion.

2. As in [4, 5], we consider eA DIS (we will discuss the case when the virtual photon strikes out a quark with charge e_q). As usual we choose the virtual photon momentum in the negative z direction, and describe the 0 and 3 components of the four-vectors in terms of the light-cone variables $y^\pm = (y^0 \pm y^3)/\sqrt{2}$. In the laboratory frame the photon energy reads $\nu = Q^2/2m_N x_B$ (x_B is the Bjorken variable). The p^- momentum of the struck quark equals $\sqrt{2}\nu(1+x_B/2m_N\nu)$. The transverse momentum integrated distribution for the gq final state in eA DIS can be described in terms of the semi-inclusive nuclear hadronic tensor $dW_A^{\mu\nu}/dz$. To leading order in ν the spin effects in the final-state rescatterings of fast partons can be neglected. It ensures that the spin structure of $dW_A^{\mu\nu}/dz$ is the same as for the usual hadronic tensor $W_N^{\mu\nu}$ in eN DIS. It allows one to write the semi-inclusive nuclear hadronic tensor as $dW_A^{\mu\nu}/dz = -e_q^2 g_T^{\mu\nu} df_A/dz$, where df_A/dz is the semi-inclusive quark distribution of the target nucleus. Neglecting the EMC and shadowing effects it can be written as (we suppress the arguments x_B , Q and z)

$$\frac{df_A}{dz} = \int d\mathbf{r} n_A(\mathbf{r}) \frac{df_N(\mathbf{r})}{dz}, \quad (3)$$

where $df_N(\mathbf{r})/dz$ is the in-medium semi-inclusive quark distribution for a nucleon located at \mathbf{r} , and $n_A(\mathbf{r})$ is the nucleus number density.

Let us first discuss the evaluation of the in-medium semi-inclusive quark distribution in the LCPI formalism. We will only concentrate on the aspects which are important for comparison with the higher-twist method. Particularly interesting is the question on the quantum nonlocality of the fast quark production in eA DIS. Our consideration will be physical and diagrammatic. A more formal analysis will be given in further detailed publication. In the LCPI approach [6] the matrix element of the $q \rightarrow gq'$ in-medium transition is written in terms of the wave functions of the initial quark and final

quark and gluon in the nucleus color field (we omit the color factors and indices)

$$\langle gq' | \hat{S} | q \rangle = ig \int dy \bar{\psi}_{q'}(y) \gamma^\mu A_\mu^*(y) \psi_q(y). \quad (4)$$

Each quark wave function in (4) is written as $\psi(y) = \exp(-ip^- y^+) \hat{u}_\lambda \phi(y^-, \mathbf{y}_T) / \sqrt{2p^-}$, where λ is quark helicity, \hat{u}_λ is the Dirac spinor operator. The y^- dependence of the transverse wave functions ϕ is governed by the two-dimensional Schrödinger equation

$$i \frac{\partial \phi(y^-, \mathbf{y}_T)}{\partial y^-} = \left\{ \frac{[(\mathbf{p}_T - g\mathbf{A}_T)^2 + m_q^2]}{2\mu} + gA^+ \right\} \phi(y^-, \mathbf{y}_T) \quad (5)$$

with the Schrödinger “mass” $\mu = p^-$. The wave function of the emitted gluon can be represented in a similar way.

The y^- evolution of the transverse wave functions can be written in terms of the Green’s function for the Schrödinger equation (5)

$$\phi(y_2^-, \mathbf{y}_{T,2}) = \int d\mathbf{y}_{T,1} \mathcal{K}(\mathbf{y}_{T,2}, \mathbf{y}_2^- | \mathbf{y}_{T,1}, \mathbf{y}_1^-) \phi(y_1^-, \mathbf{y}_{T,1}). \quad (6)$$

It allows one to represent the matrix element in terms of the transverse Green’s functions \mathcal{K} . In the LCPI approach [6] the path integral representation for the Green’s functions is a crucial step in the final stage of calculations. It allows to perform averaging over the medium potential at the level of the integrand in the double path integral representation of the gluon spectrum. This method turns out to be very convenient in accounting for arbitrary number of rescatterings. But for comparison with the higher-twist method we need only the $N=1$ rescattering contribution. It can be analyzed without using the path integral representation of the Green’s functions. For calculation of the $N=1$ term it is enough to have the perturbative expansion of the Green’s functions to the second order in the external potential. One can show that for gauges with potential vanishing at large distances (say, covariant gauges, or Coulomb gauge) one can ignore the transverse component \mathbf{A}_T . Then, the second order expansion of \mathcal{K} can be written as

$$\begin{aligned} \mathcal{K}(\mathbf{y}_{T,2}, \mathbf{y}_2^- | \mathbf{y}_{T,1}, \mathbf{y}_1^-) &= K(\mathbf{y}_{T,2}, \mathbf{y}_2^- | \mathbf{y}_{T,1}, \mathbf{y}_1^-) - \\ &- ig \int dy^- dy_T K(\mathbf{y}_{T,2}, \mathbf{y}_2^- | \mathbf{y}_T, y^-) A^+(\mathbf{y}_T, y^-) \times \end{aligned}$$

$$\begin{aligned}
& \times K(\mathbf{y}_T, y^- | \mathbf{y}_{T,1}, y_1^-) - g^2 \int_{y^- > z^-} dy^- dy_T \times \\
& \times \int dz^- dz_T K(\mathbf{y}_{T,2}, y_2^- | \mathbf{y}_T, y^-) A^+(\mathbf{y}_T, y^-) \times \\
& \times K(\mathbf{y}_T, y^- | \mathbf{z}_T, z^-) A^+(\mathbf{z}_T, z^-) K(\mathbf{z}_T, z^- | \mathbf{y}_{T,1}, y_1^-), \quad (7)
\end{aligned}$$

where K is the free Green's function when the external field is absent. The expansion (7) allows one to represent diagrammatically the $N = 1$ induced gluon emission in eA DIS by the set of diagrams like shown in Fig. 1 in which the horizontal solid line corresponds to K (\rightarrow) and K^* (\leftarrow), the gluon line shows the correlator $\langle A^+(y_1) A^+(y_2) \rangle$ in the nucleus, and the vertical dashed line shows the transverse density matrices of the final partons at very large y^- . The graphs like Fig.1a describe the real induced $q \rightarrow gq$ transition, and the vir-

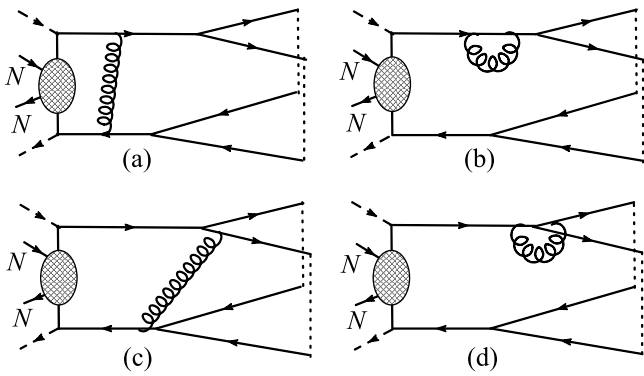


Fig.1

tual graphs like Fig. 1b, coming from the interference of the second and zeroth order terms on the right-hand side of (7), correspond to the unitarity correction due to interference of the induced and vacuum gluon emission.

The typical difference in the coordinate y^- for the upper and lower γ^*qq vertices in Fig. 1 (which gives the scale of the quantum nonlocality of the fast quark production) is given by the well known Ioffe length $L_I = 1/m_N x_B$. For the usual nucleon quark distribution L_I is the dominating scale in the Collins-Soper formula [10]

$$f_N = \frac{1}{4\pi} \int dy^- e^{i x_B P^+ y^-} \langle N | \bar{\psi}(-y^-/2) \gamma^+ \psi(y^-/2) | N \rangle.$$

Contrary to the final state with single quark for the gq final state the integration over the y^- coordinate of the γ^*qq vertex is now affected by the integration over the positions of rescatterings and the $q \rightarrow gq$ splitting. But for moderate x_B when $L_I \ll R_A$ one can neglect the effect of rescatterings on the integration over y^- . One can easily show that for production of the final gq states

with $M_{gq}^2 \ll Q^2$ the restriction on y^- from the splitting point can also be ignored. Indeed, the typical scale in integrating over the splitting points is given by the gluon formation length $L_f \sim \nu/M_{gq}^2$ which is much bigger than L_I at $M_{gq}^2 \ll Q^2$ (this is true for both the vacuum DGLAP and the induced gluon emission). Also, at $L_I \ll L_f$ one can take for the lower limit of the integration over the splitting points for the upper and lower parts of the diagrams in Fig. 1 the position of the struck nucleon. After this simplifications the quark production and gluon emission become mutually independent, and the $df_N(\mathbf{r})/dz$ can be approximated by the factorized form

$$\frac{df_N(\mathbf{r})}{dz} \approx f_N \frac{dP(\mathbf{r})}{dz}, \quad (8)$$

where the quark distribution f_N stems from the left parts of the diagrams, and the gluon spectrum dP/dz is described by the right parts of the diagrams evaluated neglecting the quantum nonlocality of the fast quark production.

One can make some more simplifications. If one neglects the multi-quark configurations in the nucleus both the vector potentials in the gluon correlators should belong to the same nucleon. For this reason the typical separation of the arguments in the correlators is of the order of the nucleon radius, R_N . The restriction $|y_1^- - y_2^-| \lesssim R_N$ allows one to replace the fast parton propagators between the gluon fields in the graphs like Fig.1b by δ functions in impact parameter space. This approximation is valid for parton energy $\gg 1/R_N$. It follows from the Schrödinger diffusion relation for the parton transverse motion $\rho^2 \sim L/E$. Also, the smallness of the fast parton diffusion radius at the longitudinal scale $\sim R_N$ allows one to replace in other transverse Green's functions the y^- coordinates by the mean values of the arguments of the vector potentials in the gluon correlators. This approximation corresponds to a picture with rescatterings of fast partons on zero thickness scattering centers (nucleons). The inequality $\omega \gg 1/R_N$ for the emitted gluon is equivalent to $R_N \ll L_f$. For this reason, in the picture of thin nucleons the contribution of the graphs like Fig. 1c,d with gluon correlators connecting the initial quark and final quark or gluon can be neglected since they are suppressed by the small factor R_N/L_f . These approximations have been used in the original formulation of the LCPI approach [6] (the BDMPs [1] and GLV [2] approaches use them as well). Note that (similarly to the case of QGP [11]) each gluon correlator appears only in the form of an integral over $\Delta y^- = y_2^- - y_1^-$ and at $y_2^+ = y_1^+$. One can easily show that this ensures gauge invariance of the result (to leading order in α_s).

The starting point of the higher-twist approach [4, 5] to the gluon emission in eA DIS is the representation of the nuclear hadronic tensor for the $\gamma^*A \rightarrow gqX$ transition in terms of the diagrams like shown in Fig.2. The

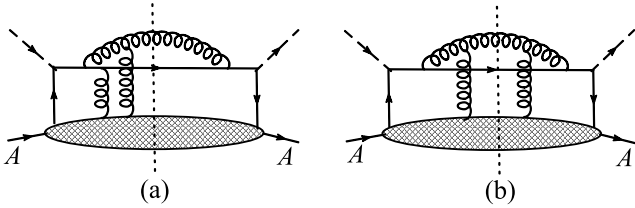


Fig.2

lower soft part is expressed in terms of the matrix element $\langle A|\bar{\psi}(0)A^+(y_1)A^+(y_2)\psi(y)|A\rangle$, and the upper hard parts are calculated perturbatively. In the calculation of the hard parts to leading order in the struck quark energy the integrations over the y^+ coordinates of the vertices give conservations of the large p^- momentum components of fast partons. At the same time the integration over the p^- momentum components of the final partons ensures that to leading order in the struck quark energy all the y^+ coordinates in the soft part can be set to zero. Due to conservation of the large p^- momenta of fast partons in the Feynman propagators only the Fourier components with $p^- > 0$ are important. It means that the Feynman propagators are effectively reduced to the retarded (in y^- coordinate) ones.

One can easily demonstrate that the Feynman diagram treatment of [4, 5] is equivalent to that in terms of the transverse Green's functions. Indeed, using the representation

$$K(\mathbf{y}_{T,2}, y_2^- | \mathbf{y}_{T,1}, y_1^-) = i \int \frac{d\mathbf{p}_T^+ d\mathbf{p}_T}{(2\pi)^3} \frac{\exp[-ip^+(y_2^- - y_1^-) + i\mathbf{p}_T(\mathbf{y}_{T,2} - \mathbf{y}_{T,1})]}{p^+ - \frac{\mathbf{p}_T^2 + m^2}{2p^-} + i0} \quad (9)$$

one can write the retarded quark propagator as

$$G_r(y_2 - y_1) = \frac{1}{4\pi} \int_0^\infty \frac{dp^-}{p^-} e^{-ip^-(y_2^+ - y_1^+)} \times \left[\sum_\lambda \hat{u}_\lambda \bar{\hat{u}}_\lambda K(\mathbf{y}_{T,2}, y_2^- | \mathbf{y}_{T,1}, y_1^-) + i\gamma^- \delta(y_2^- - y_1^-) \delta(\mathbf{y}_{T,2} - \mathbf{y}_{T,1}) \right]. \quad (10)$$

Here \hat{u}_λ and $\bar{\hat{u}}_\lambda$ act on the variables with indices 2 and 1, respectively. The last term in (10) is the so-called contact term. It does not propagate in y^- and can be omitted in calculating the nuclear final-state interaction effects for fast partons. One can easily verify that using

(10) and a similar representation for the gluon propagator the hard parts in the higher-twist method can be represented in terms of the transverse Green's functions as it is done in the LCPI treatment. Thus, the hard parts in the approach [4, 5] should agree with that of the LCPI formalism.

3. The calculation of the diagrams like shown in Fig.1 is simplified by noting that the free transverse Green's function for a parton with mass m can be written as

$$K(\mathbf{y}_{T,2}, y_2^- | \mathbf{y}_{T,1}, y_1^-) = \theta(y_2^- - y_1^-) \sum_{p_T} \phi_{p_T}(\mathbf{y}_{T,2}, y_2^-) \phi_{p_T}^*(\mathbf{y}_{T,1}, y_1^-), \quad (11)$$

where $\phi_{p_T}(\mathbf{y}_T, y^-) = \exp[i\mathbf{p}_T \mathbf{y}_T - iy^-(\mathbf{p}_T^2 + m^2)/2p^-]$ is the plane wave solution to the Schrödinger equation for $A^\mu = 0$ with the transverse momentum \mathbf{p}_T . It allows one to represent the upper and lower parts of the diagrams shown in Fig.1 in the form $\int dy^- d\mathbf{y}_T \phi_q^*(\mathbf{y}_T, y^-) \phi_g^*(\mathbf{y}_T, y^-) \phi_q(\mathbf{y}_T, y^-)$ where the outgoing and incoming wave functions have the form of the plane waves with sharp change of the transverse momentum at the points of interactions with the external gluon fields. This method has previously been used in [12] for investigation of the kinematical effects. If one ignores the finite kinematical limits it reproduces the first order in the density term of the full LCPI expression for the induced gluon spectrum obtained in [13]. We have checked that all the hard parts evaluated with the help of the plane waves agree with that obtained in Refs. [4, 5].

The sum of the complete set of the diagrams contributing to the $N=1$ spectrum can be written as [13, 12]

$$\frac{dP(\mathbf{r})}{dz} = \int_{r_3}^\infty d\xi n_A(\mathbf{r}_T, r_3 + \xi) \frac{d\sigma(z, \xi)}{dz}. \quad (12)$$

Here $d\sigma(z, \xi)/dz$ is the cross section of gluon emission from a fast quark produced at distance ξ from the scattering nucleon. At $z \ll 1$ (we consider the soft gluon emission just to simplify the formulas) for massless partons it reads [13, 12]

$$\frac{d\sigma(z, \xi)}{dz} = \frac{2\alpha_s^2 P_{Gq}(z)}{C_F} \times \int d\mathbf{p}_T \frac{d\mathbf{k}_T}{\mathbf{k}_T^2} \frac{xdG(k_T^2, x)}{d\mathbf{k}_T} H(\mathbf{p}_T, \mathbf{k}_T, z, \xi), \quad (13)$$

$$H(\mathbf{p}_T, \mathbf{k}_T, z, \xi) = \left[\frac{1}{\mathbf{p}_T^2} - \frac{(\mathbf{p}_T - \mathbf{k}_T)\mathbf{p}_T}{\mathbf{p}_T^2(\mathbf{p}_T - \mathbf{k}_T)^2} \right] \times \left[1 - \cos\left(\frac{i\mathbf{p}_T^2 \xi}{2Ez(1-z)}\right) \right]. \quad (14)$$

Here the limit $x \rightarrow 0$ is implicit, $dG(k_T^2, x)/d\mathbf{k}_T$ is the unintegrated nucleon gluon density which in leading order in α_s at $x \ll 1$ reads

$$\frac{dG(k_T^2, x)}{d\mathbf{k}_T} = \frac{1}{4\pi^3 x} \int d\rho d\mathbf{y}_T \times \exp(-i\mathbf{k}_T \rho) \langle \Psi_N | \nabla_{\mathbf{y}_T} W^a(\mathbf{y}_T + \rho) \nabla_{\mathbf{y}_T} W^a(\mathbf{y}_T) | \Psi_N \rangle. \quad (15)$$

Here $W^a(\mathbf{y}_T) = \int dy^- A^{+a}(y^-, \mathbf{y}_T)$ (the color index, a , is shown explicitly), and Ψ_N is the internal nucleon wave function normalized to unity. One can show that the formula (15) being rewritten through the nucleon wave functions with relativistic normalization $\langle N' | N \rangle = 2p^+ (2\pi)^3 \delta(p'^+ - p^+) \delta(\mathbf{p}' - \mathbf{p}_T)$ is reduced to the Collins-Soper definition [10] of the gluon density. Eq. (15) can also be written as

$$\frac{dG(k_T^2, x)}{d\mathbf{k}_T} = \frac{N_c^2 - 1}{x 32\pi^4 \alpha_s C_F} \int d\rho \exp(-i\mathbf{k}_T \rho) \nabla^2 \sigma(\rho), \quad (16)$$

where $\sigma(\rho)$ is the well known dipole cross section given by

$$\sigma(\rho) = \frac{8\pi\alpha_s^2 C_F}{N_c^2 - 1} \int d\mathbf{y}_T \langle \Psi_N | W^a(\mathbf{y}_T) W^a(\mathbf{y}_T) - W^a(\mathbf{y}_T + \rho) W^a(\mathbf{y}_T) | \Psi_N \rangle. \quad (17)$$

Using (12)–(15) with the phase W^a calculated in the Born approximation one can obtain (2).

The collinear expansion corresponds to replacement of the hard part by its second order expansion in \mathbf{k}_T (we suppress all the arguments except for \mathbf{k}_T)

$$H(\mathbf{k}_T) \approx H(\mathbf{k}_T = 0) + \left. \frac{\partial H}{\partial k_T^\alpha} \right|_{\mathbf{k}_T=0} k_T^\alpha + \left. \frac{\partial^2 H}{\partial k_T^\alpha \partial k_T^\beta} \right|_{\mathbf{k}_T=0} \frac{k_T^\alpha k_T^\beta}{2}. \quad (18)$$

Then, to logarithmic accuracy

$$d\sigma(z, \xi)/dz \propto \int d\mathbf{p}_T x G(p_T^2, x) \nabla_{\mathbf{k}_T}^2 H(\mathbf{p}_T, \mathbf{k}_T, z, \xi)|_{\mathbf{k}_T=0}.$$

But from (14) one can easily obtain $\nabla_{\mathbf{k}_T}^2 H|_{\mathbf{k}_T=0} = 0$. It is also seen from averaging of the hard part over the azimuthal angle of \mathbf{k}_T which gives explicitly

$$\frac{1}{2\pi} \int d\phi_{\mathbf{k}_T} H(\mathbf{p}_T, \mathbf{k}_T, z, \xi) = \frac{\theta(k_T - p_T)}{p_T^2} \left[1 - \cos\left(\frac{i\mathbf{p}_T^2 \xi}{2Ez(1-z)}\right) \right]. \quad (19)$$

Thus, contrary to the expected in the collinear approximation dominance of the region $k_T \lesssim p_T$ only the region $k_T > p_T$ contributes to the gluon emission, and formal

use of the collinear expansion gives completely wrong result with zero gluon spectrum.

The fact that the $N = 1$ gluon spectrum vanishes in the collinear approximation agrees with prediction of the harmonic oscillator approximation in the LCPI [6] and BDMPS [1] approaches. The full gluon spectrum in these approaches is expressed in terms of the Green's function of the Schrödinger equation with an imaginary potential which is proportional to the dipole cross section. The corresponding Hamiltonian takes the harmonic oscillator form for a quadratic parametrization $\sigma(\rho) = C\rho^2$. The $N = 1$ contribution to the gluon spectrum in the oscillator approximation should coincide with prediction of the collinear expansion in the GWZ treatment [4, 5] since the oscillator approximation in the LCPI and BDMPS approaches is equivalent to the collinear approximation of [4, 5]. Indeed, the quadratic form of the dipole cross section corresponds to the vector potential approximated by the linear expansion $A^+(y^-, \mathbf{y}_T + \rho) \approx A^+(y^-, \mathbf{y}_T) + \rho \nabla_{\mathbf{y}_T} A^+(y^-, \mathbf{y}_T)$ which can be traced back to the collinear expansion in momentum space. For a target of thickness L in the oscillator approximation the BDMPS and LCPI approaches for massless partons give the spectrum [1, 8, 13]

$$\frac{dP_{OA}}{dz} = \frac{\alpha_s P_{Gq}(z)}{\pi} \ln |\cos \Omega L|, \quad (20)$$

where $\Omega = \sqrt{-iC_3 n/z(1-z)E}$, $C_3 = CC_A/C_F$. Keeping only the first term in the expansion of the spectrum (20) in the density one gets [13]

$$\frac{dP_{OA}}{dz} \approx \frac{\alpha_s P_{Gq}(z) C_3^2 n^2 L^4}{16\pi E^2 z^2 (1-z)^2}. \quad (21)$$

Since the right-hand side of (21) $\propto n^2$ it corresponds to the $N = 2$ rescatterings [13], and the contribution of $N = 1$ rescattering is absent. Note that from the point of view of the representation (13) the zero $N = 1$ gluon spectrum in the oscillator approximation is a consequence of the fact that in this case $dG/d\mathbf{k}_T \propto \delta(\mathbf{k}_T)$ (as one sees from (16)).

4. Let us now discuss why the calculations of Refs. [4, 5] give nonzero gluon spectrum. In [4, 5] the nonvanishing second derivative of the hard part comes from the graph shown in Fig. 2b (at $z \ll 1$). The authors use for the integration variable in the hard part of this graph the transverse momentum of the final gluon, \mathbf{l}_T . The \mathbf{l}_T -integrated hard part obtained in [5] (Eq. 15 of [5]) reads (up to an unimportant factor)

$$H(\mathbf{k}_T) \propto \int \frac{d\mathbf{l}_T}{(\mathbf{l}_T - \mathbf{k}_T)^2} R(y^-, y_1^-, y_2^-, \mathbf{l}_T, \mathbf{k}_T), \quad (22)$$

where

$$\begin{aligned}
R(y^-, y_1^-, y_2^-, \mathbf{l}_T, \mathbf{k}_T) &= \frac{1}{2} \times \\
\exp \left[i \frac{y^-(\mathbf{l}_T - \mathbf{k}_T)^2 - (1-z)(y_1^- - y_2^-)(\mathbf{k}_T^2 - 2\mathbf{l}_T \mathbf{k}_T)}{2q^- z(1-z)} \right] \\
&\times \left[1 - \exp \left(i \frac{(y_1^- - y^-)(\mathbf{l}_T - \mathbf{k}_T)^2}{2q^- z(1-z)} \right) \right] \times \\
&\times \left[1 - \exp \left(-i \frac{y_2^-(\mathbf{l}_T - \mathbf{k}_T)^2}{2q^- z(1-z)} \right) \right] \quad (23)
\end{aligned}$$

is an analog of the last factor in the square brackets in (14) (our z corresponds to $1-z$ in [4, 5]), the coordinates $y^-, y_{1,2}^-$ correspond to the quark interactions with the virtual photon and t -channel gluons. In calculating $\nabla_{\mathbf{k}_T}^2 H$ the authors differentiate only the singular factor $1/(\mathbf{l}_T - \mathbf{k}_T)^2$. But the omitted terms from differentiating the factor R are important. After the \mathbf{l}_T integration they almost completely cancel the contribution from the $1/(\mathbf{l}_T - \mathbf{k}_T)^2$ term. Indeed, after putting $y_1^- = y_2^-$ and changing the integration variable $\mathbf{l}_T \rightarrow (\mathbf{l}_T + \mathbf{k}_T)$ the right-hand part of (22) does not depend on \mathbf{k}_T at all. If one does not put $y_1^- = y_2^-$, there will be some nonzero contribution to $\nabla_{\mathbf{k}_T}^2 H|_{\mathbf{k}_T=0}$ which, however, is suppressed by the small factor $(R_N/L_f)^2$. Keeping such contributions does not make any sense since they are clearly beyond predictive accuracy of the approximations used in [4, 5]¹⁾.

5. In summary, we have demonstrated that the collinear expansion fails in the case of gluon emission from a fast massless quark produced in eA DIS. It gives a zero $N=1$ rescattering contribution to the gluon spectrum. This agrees with vanishing $N=1$ contribution to the gluon spectrum in the BDMPs [1] and LCPI [6] approaches in the harmonic oscillator approximation which corresponds to the collinear expansion in momentum

space. The nonzero gluon spectrum obtained in [4, 5] is a consequence of unjustified neglecting some important terms in the collinear expansion. The established facts demonstrate that the GWZ approach [4, 5] is completely wrong. Its predictions for eA DIS and jet quenching in AA collisions do not make sense.

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¹⁾ Keeping the nonzero y^- in the \mathbf{l}_T -, \mathbf{k}_T -dependent terms, as it is done in [4, 5], also does not make sense under the approximations of [4, 5].