Pis'ma v ZhETF, vol. 88, iss. 5, pp. 339 – 344

$f(R)$ cosmology from q–theory

F. R. Klinkhamer[#1)], *G. E. Volovik*[*+1)]

[#] *Institute for Theoretical Physics, University of Karlsruhe (TH), 76128 Karlsruhe, Germany*

[*] *Low Temperature Laboratory, Helsinki University of Technology, P.O.Box 5100, FIN-02015, HUT, Finland*

[+] *Landau Institute for Theoretical Physics RAS, Kosygina 2, 119334 Moscow, Russia*

Submitted 25 July 2008

From a macroscopic theory of the quantum vacuum in terms of conserved relativistic charges (generically denoted by $q^{(a)}$ with label a), we have obtained, in the low-energy limit, a particular type of $f(R)$ model relevant to cosmology. The macroscopic quantum-vacuum theory allows us to distinguish between different phenomenological $f(R)$ models on physical grounds.

PACS: 04.20.Cv, 11.30.Cp, 95.36.+x, 98.80.Jk

1. Introduction. The development of high-energy physics and cosmology over the last years has led to the realization that, most likely, Einstein's theory of gravity needs to be modified. At the moment, we are only able to calculate higher-order curvature corrections to the Einstein action coming from scalar, spinor, and vector quantum fields propagating over a fixed classical background (see, e.g., Refs. [1, 2]). The full theory of the modified-Einstein action requires the underlying microscopic theory – what is usually called *quantum gravity theory*. Since the latter theory is not yet established, phenomenological models are needed. Among these are the so-called $f(R)$ models (see, e.g., Refs. [2, 3] for two recent reviews) which have *ad hoc* powers of the curvature invariants added to the linear term of the Einstein–Hilbert action [4].

We have proposed another approach to modified Einstein gravity, which is based on the treatment of the Lorentz-invariant quantum vacuum as an extended self-sustained system characterized by a conserved relativistic charge q [5, 6]. Here, q is a microscopic variable describing the physics of the deep (ultraviolet) vacuum, but its thermodynamics and dynamics are described by macroscopic equations, because q is a conserved quantity. This quantity q is similar to the particle density in liquids, which describes a microscopic quantity – the density of atoms – but obeys the macroscopic equations of hydrodynamics, because of particle-number conservation. Different from known liquids, the quantum vacuum is Lorentz invariant. The quantity q must, therefore, be Lorentz invariant. This treatment has allowed us to discuss both the *thermodynamics* and the *dynamics* of a Lorentz-invariant quantum vacuum.

The thermodynamic approach [5] assumes that the quantum vacuum is a stable self-sustained equilibrium state, which is described by compressibility and other characteristics of the response to external perturbations. In this approach, we have found that the vacuum energy density appears in two forms.

First, there is the microscopic vacuum energy density which is characterized by an ultraviolet energy scale $E_{\rm UV}$, so that $\epsilon(q) \sim E_{\rm UV}^4$. Most likely, $E_{\rm UV}$ corresponds to the standard Planck energy scale defined in terms of Newton's gravitational constant $G_{\rm N}$, $E_{\rm Planck} \equiv \sqrt{\hbar c^5/G_{\rm N}} \approx 1.2 \times 10^{28}$ eV. But, here, we prefer to stay as general as possible and keep $E_{\rm UV}$ distinct from $E_{\rm Planck}$.

Second, there is the macroscopic vacuum energy density which is determined by a particular thermodynamic quantity, $\widetilde{\epsilon}_{\rm vac}(q) \equiv \epsilon - q\, d\epsilon/dq$, and it is this type of energy density which contributes to the effective gravitational field equations at low energies. For a self-sustained vacuum in full thermodynamic equilibrium and in the absence of matter, the effective (coarse-grained) vacuum energy density $\widetilde{\epsilon}_{\rm vac}(q)$ is automatically nullified (without fine tuning) by the spontaneous adjustment of the vacuum variable q to its equilibrium value q_0, so that $\widetilde{\epsilon}_{\rm vac}(q_0) = 0$. This implies that the effective cosmological constant Λ of a perfect quantum vacuum is strictly zero, which is consistent with the requirement of Lorentz invariance for zero external pressure.

The dynamic approach [6] demonstrates how, in a flat Friedmann–Robertson–Walker universe, the vacuum energy density $\widetilde{\epsilon}_{\rm vac}$ (effective cosmological "constant") relaxes from its natural Planck-scale value at

1) e-mail: frans.klinkhamer@physik.uni-karlsruhe.de; volovik@boojum.hut.fi

early Planckian times to a naturally small value at late times.

In this Letter, we show that the macroscopic theory of the quantum vacuum, when applied to cosmology, gives rise to a specific class of $f(R)$ models of modified gravity.

2. Gravity with F–fields and low-energy matter. We consider the general case of several conserved microscopic variables $q^{(a)}$, for $a = 1, \ldots, n$, and corresponding chemical potentials $\mu^{(a)}$ [7, 8]. As in Refs. [5, 6], each variable $q^{(a)}$ can be represented by a four-form field $F^{(a)}$:

$$(F^{(a)})^2 \equiv -\frac{1}{24} F^{(a)}_{\mu\nu\rho\sigma} F^{(a)\mu\nu\rho\sigma}, \quad F^{(a)}_{\mu\nu\rho\sigma} \equiv \nabla_{[\mu} A^{(a)}_{\nu\rho\sigma]} \,. \tag{1}$$

The action of the four-form fields $F^{(a)}(x)$, the matter field $\phi(x)$, and the gravitational field $g_{\mu\nu}(x)$ is given by

$$S[A^{(a)}, g, \phi] = -\int_{\mathbb{R}^4} d^4x \sqrt{-g} \left(K(F^{(a)})\, R + \epsilon(F^{(a)}, \phi) + \frac{1}{2} \partial_\mu\phi\, \partial^\mu\phi \right) . \tag{2}$$

For simplicity, low-energy matter is represented by a single real scalar field ϕ. The generalized potential $\epsilon(F^{(a)}, \phi)$ includes self-interactions and interactions between all fields $F^{(a)}$ and ϕ, but contains no derivatives of these fields and explicit factors of the metric field $g_{\mu\nu}$ or its inverse. The gravitational coupling parameter K is determined by ultraviolet physics and, therefore, depends on the microscopic vacuum variables $F^{(a)}$. Here, and in the following, we adopt the conventions of Ref. [4], in particular, those for the Riemann tensor and the metric signature $(-+++)$. Natural units with $\hbar = c = 1$ are used throughout.

The generalized Maxwell and Klein–Gordon equations from action (2) read

$$\nabla_\mu \left(\sqrt{-g}\, \frac{F^{(a)\mu\nu\rho\sigma}}{F^{(a)}} \left(\frac{\partial \epsilon}{\partial F^{(a)}} + R\, \frac{\partial K}{\partial F^{(a)}} \right) \right) = 0 \,, \tag{3a}$$

$$\Box\phi - \frac{\partial \epsilon}{\partial \phi} = 0 \,, \tag{3b}$$

where $\Box$ denotes the invariant d'Alembertian operator and the partial derivatives $\partial/\partial F^{(a)}$ and $\partial/\partial\phi$ stand for pointwise differentiation. The variation of (2) over the metric $g_{\mu\nu}$ gives the generalized Einstein equations:

$$2K \left(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} \right) + R\, g_{\mu\nu} \sum_{a=1}^{n} F^{(a)} \frac{\partial K}{\partial F^{(a)}} + 2\Big(\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box \Big) K - \widetilde{\epsilon}(F^{(a)}, \phi)\, g_{\mu\nu} + T^{\rm M}_{\mu\nu} = 0, \tag{4}$$

with effective potential

$$\widetilde{\epsilon}(F^{(a)}, \phi) \equiv \epsilon(F^{(a)}, \phi) - \sum_{a=1}^{n} F^{(a)} \frac{\partial \epsilon}{\partial F^{(a)}} \tag{5}$$

and scalar-field energy-momentum tensor

$$T^{\rm M}_{\mu\nu} = \partial_\mu\phi\, \partial_\nu\phi - \frac{1}{2} g_{\mu\nu}\, \partial_\rho\phi\, \partial^\rho\phi \,. \tag{6}$$

Using $F^{(a)\mu\nu\rho\sigma}$ as given by (1), the Maxwell equations (3a) can be rewritten in the following form:

$$\partial_\mu \left(\frac{\partial \epsilon}{\partial F^{(a)}} + R\, \frac{\partial K}{\partial F^{(a)}} \right) = 0 \,. \tag{7}$$

The solution of these $4n$ equations is

$$\frac{\partial \epsilon}{\partial F^{(a)}} + R\, \frac{\partial K}{\partial F^{(a)}} = \mu^{(a)} \,, \tag{8}$$

where the $\mu^{(a)}$ are n integration constants. Eliminating $\partial K/\partial F^{(a)}$ from (4) and (8), one finds for the generalized Einstein equations

$$-2K\Big(R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu}\Big) - 2\Big(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\Big) K + \Big(\epsilon - \textstyle\sum_{a=1}^{n} \mu^{(a)} F^{(a)}\Big)\, g_{\mu\nu} = T^{\rm M}_{\mu\nu} \,. \tag{9}$$

Equations (8) and (9) can also be obtained if we use, instead of the original action, the following effective action:

$$S_{\rm eff}[A^{(a)}, \mu^{(a)}, g, \phi] = -\int_{\mathbb{R}^4} d^4x \sqrt{-g} \left(K\, R + \epsilon - \sum_{a=1}^{n} \mu^{(a)} F^{(a)} + \frac{1}{2} \partial_\mu\phi\, \partial^\mu\phi \right) . \tag{10}$$

The $\mu^{(a)} F^{(a)}$ terms in this action do not contribute to the equations of motion (3a), because they are total derivatives,

$$\int_{\mathbb{R}^4} d^4x\, \sqrt{|g|}\, \mu^{(a)}\, F^{(a)} = -\frac{\mu^{(a)}}{24}\, e^{\kappa\lambda\mu\nu} \int_{\mathbb{R}^4} d^4x\, F^{(a)}_{\kappa\lambda\mu\nu} \,. \tag{11}$$

The constant $\mu^{(a)}$ is seen to play the role of a Lagrange multiplier related to the conservation of the vacuum "charge" $F^{(a)}$.

Instead of the large microscopic energy density $\epsilon(F^{(a)}, \phi)$ in the original action (2), a potentially smaller macroscopic vacuum energy density enters the effective action (10), namely,

$$\rho_{\rm V} \equiv \epsilon(F^{(a)}, \phi) - \sum_{a=1}^{n} \mu^{(a)} F^{(a)} \,. \tag{12}$$

Precisely this macroscopic vacuum energy density gravitates and determines the cosmological term in the gravitational field equations (9).

3. Equilibrium vacua and stability conditions. The main goal of our approach is to describe the thermodynamics of the equilibrium vacuum [5] and to consider cosmology as the dynamics of relaxation towards an equilibrium state [6, 9, 10]. That is why, in what follows, we assume that the universe is close to equilibrium and that all its parameters, including the fields $F^{(a)}$ and the chemical potentials $\mu^{(a)}$, are close to their equilibrium values. A static homogeneous equilibrium vacuum, in the absence of thermal matter, corresponds to a stationary point $\left(F_0^{(a)}, \phi_0\right)$ of Eqs. (3b), (8), and (9) for $R_{\mu\nu} = T^{\mathrm{M}}_{\mu\nu} = 0$:

$$\frac{\partial \epsilon}{\partial \phi} = 0\,, \quad \frac{\partial \epsilon}{\partial F^{(a)}} = \mu^{(a)}\,, \quad \epsilon - \sum_{a=1}^{n} \mu^{(a)} F^{(a)} = 0\,, \tag{13}$$

where the last equation demonstrates that the vacuum energy (12) is zero in an equilibrium vacuum, $\rho_{\mathrm{V}}|_{\mathrm{eq}} = 0$.

One can see the difference between the conventional matter field ϕ and the conserved vacuum fields $F^{(a)}$, as only the fields $F^{(a)}$ provide the integration constants $\mu^{(a)}$ which arise dynamically from the solution (8) of the generalized Maxwell equations (7). (The field equations of generic matter fields do not give rise to such integration constants.) These integration constants $\mu^{(a)}$ play the role of chemical potentials in thermodynamics and are thermodynamically conjugate to the density of the conserved quantities $F^{(a)}$. With appropriate nonzero chemical potentials, the large vacuum energy $\epsilon(F^{(a)}, \phi)$ is reduced to $\rho_{\mathrm{V}} = 0$ in a homogeneous equilibrium vacuum state according to (13). Specifically, two large quantities, $\epsilon(F^{(a)}, \phi)$ and $\sum_{a=1}^{n} \mu^{(a)} F^{(a)}$, each of order E^4_{UV}, cancel each other due to the self-tuning mechanism [5]. This is the main property of a self-sustained vacuum.

For the case of a single vacuum variable F, the chemical potential μ in equilibrium is completely fixed by the constraint $\rho_{\mathrm{V}} = 0$. But, for the case of several variables $F^{(a)}$, there are $n-1$ degrees of freedom, since the equation $\rho_{\mathrm{V}} = 0$ gives only a single constraint on the n chemical potentials $\mu^{(a)}$. This allows for the existence of many different equilibrium vacua and also for the coexistence of several vacua [7, 8]. This last observation may give microscopic support to the multiple point principle which postulates the existence of a number of phases with the same energy density (see, e.g., Ref. [11] and references therein).

The stationary point $(F_0^{(a)}, \phi_0)$ of the thermodynamic potential $W \equiv \epsilon(F^{(a)}, \phi) - \sum_a \mu^{(a)} F^{(a)}$ [i.e., the solution of (13)] should correspond to a minimum, which can be local or global. In particular, the vacuum compressibility introduced in Ref. [5] must be positive:

$$\chi_0 \equiv \left[\sum_{a,b=1}^{n} F^{(a)} F^{(b)} \; \frac{\partial^2 \epsilon}{\partial F^{(a)} \partial F^{(b)}} \right]^{-1}_{F^{(a)} = F_0^{(a)},\; \phi = \phi_0} > 0\,. \tag{14}$$

Furthermore, the effective Newton's constant must be positive in an equilibrium vacuum:

$$G_{\mathrm{N}} \equiv \frac{1}{16\pi K(F_0^{(a)}, \phi_0)} > 0\,, \tag{15}$$

in order to have a physically consistent description of an attractive gravitational force. More specifically, negative K gives the wrong sign of the kinetic term for the graviton (which becomes ghostlike) and the quantum vacuum is unstable. As q–theory is based on a stable self-sustaining vacuum, having $K > 0$ is a necessary condition for stability of the vacuum, together with the compressibility condition (14).

4. $f(R)$ model from q–theory. Modified-gravity $f(R)$ models appeared already in the 1960s (see, e.g., Refs. [12, 13]) and were used to construct an inflationary model of the early universe in the 1980s [14]. More recently, these $f(R)$ models have received renewed attention as a way to explain the inferred cosmic "dark energy" by attributing it to a modification of Einstein gravity (see, e.g., Refs. [2, 3, 15, 16] and references therein). These models are, in fact, purely phenomenological models, which, in their simplest form, replace the linear function of the Ricci scalar R from the Einstein–Hilbert action term by a more general function $f(R)$. This function $f(R)$ can, in principle, be adjusted to fit the astronomical observations and to produce a viable cosmological model.

To obtain $f(R)$ from q–theory [6], one can express $F^{(a)}$ in terms of R, ϕ, and $\mu^{(a)}$ by use of the equation system (8) and substitute the resulting functions $F^{(a)}(R)$ into (9). This can be done in a general way (see Ref. [3]), but, since we consider the relaxation to an equilibrium vacuum, we are only interested in the simpler situation of a system already close to equilibrium. In addition, we will restrict ourselves to a single F-field (the generalization to n fields is straightforward) and we also omit the explicit matter ϕ-field, keeping only the general matter energy-momentum tensor $T^{\mathrm{M}}_{\mu\nu}$.

For a single F-field, (8) gives

$$\frac{\partial \epsilon(F)}{\partial F} - \mu = -R\, \frac{\partial K(F)}{\partial F}\,. \tag{16}$$

Close to the equilibrium state determined by (13), one can expand the microscopic variables as follows:

$$F = F_0 + \delta F\,, \quad \mu = \mu_0 + \delta\mu. \tag{17}$$

Expressing δF in terms of R and $\delta\mu$ and excluding δF from the Einstein equations (9), one obtains:

$$-2K\Big(R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu}\Big) + 2\,\widetilde{\chi}\,\Big(\nabla_\mu\nabla_\nu\, R - g_{\mu\nu}\,\Box\, R + \frac{1}{4}\,R^2\, g_{\mu\nu}\Big) - F_0\,\delta\mu\, g_{\mu\nu} = T^{\rm M}_{\mu\nu}\,, \tag{18}$$

in terms of the new dimensionless parameter

$$\widetilde{\chi} \equiv \chi_0\, \left(F\,\frac{\partial K}{\partial F}\right)^2_{F=F_0}. \tag{19}$$

In (18), we have omitted the $(\delta\mu)^2$ term and kept only the leading term containing $\delta\mu$. Expanding K in the first term of (18), $K = (16\pi G_{\rm N})^{-1} + \delta F\;\partial K/\partial F$, one obtains the following modified Einstein equations:

$$-\frac{1}{8\pi G_{\rm N}}\Big(R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu}\Big) + 2\,\widetilde{\chi}\Big(\nabla_\mu\nabla_\nu\, R - g_{\mu\nu}\,\Box\, R - \frac{1}{4}\,R^2\, g_{\mu\nu} + R\,R_{\mu\nu}\Big) - F_0\,\delta\mu\, g_{\mu\nu} = T^{\rm M}_{\mu\nu}\,, \tag{20}$$

where we have kept only the leading term $F_0\,\delta\mu$ and omitted terms $R\,\delta\mu$ and $\delta\mu^2$.

The field equations (20) correspond to the following phenomenological model:

$$S_{\rm phenom} = \int_{\mathbb{R}^4} d^4x\,\sqrt{-g}\,\left(\frac{1}{16\pi G_{\rm N}}\,\widetilde{f}(R) + \mathcal{L}^{\rm M}\right)\,, \tag{21a}$$

$$\widetilde{f}(R) = -R + 16\pi G_{\rm N}\,\widetilde{\chi}\,R^2 - 2\,\lambda\,, \tag{21b}$$

with $\mathcal{L}^{\rm M}$ the standard matter Lagrange density. The function $\widetilde{f}(R)$ found belongs to the class of models $f(R) \sim -R + R^2/(6M^2)$, where a bare cosmological constant may or may not be added and $M \equiv (3\,d^2f/dR^2)^{-1/2}$ is the scalaron mass (see Refs. [14, 15] and references therein; note that our sign convention for R is opposite to that in Ref. [15]). From q–theory, the scalaron mass square is given by

$$M^2 = 1/\big(\,96\pi G_{\rm N}\,\widetilde{\chi}\,\big)\,, \tag{22}$$

with $\widetilde{\chi}$ defined by (19) for a single "charge" F and by (25) below for multiple "charges" $F^{(a)}$. At this moment, we have two parenthetical remarks. First, the effective action for (20) does not contain $R_{\mu\nu}R^{\mu\nu}$ or $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ terms in addition to the R^2 term of (21b). Second, a purely phenomenological connection between generalized-equation-of-state models and $f(R)$ modified-gravity models has also been noted in App. A of Ref. [17].

In q–theory, the cosmological constant λ in (21b) is induced by the deviation of the chemical potential μ from its equilibrium value μ_0: $\lambda \equiv 8\pi G_{\rm N}\,\rho_{\rm V} = -8\pi G_{\rm N} F_0 \delta\mu$. For the case of n charges $F^{(a)}$, the cosmological constant λ is given by

$$\lambda \equiv 8\pi G_{\rm N}\,\rho_{\rm V} = -8\pi G_{\rm N}\,\sum_{a=1}^{n} F_0^{(a)}\,\delta\mu^{(a)}\,. \tag{23}$$

For the general case of a quantum vacuum described by conserved vacuum variables $q^{(a)}$, one has:

$$\Lambda \equiv \frac{\lambda}{8\pi G_{\rm N}} \equiv \rho_{\rm V} = -\sum_{a=1}^{n} q^{(a)}\delta\mu^{(a)}\,. \tag{24}$$

This last equation follows directly from the zero-temperature Gibbs–Duhem relation [5] applied to the thermodynamic system characterized by several conserved variables: $\mu^{(a)}$ is the variable thermodynamically conjugate to the conserved variable $q^{(a)}$.

Note that $f(R)$ phenomenology adds a pair of thermodynamically conjugate variables, R and K. This follows from, e.g., the thermodynamic potential $\epsilon + K\,R - \sum \mu^{(a)}F^{(a)}$ in (10). The corresponding thermodynamic identities can be used to obtain, for example, the dimensionless parameter $\widetilde{\chi}$ in (21b), which can be interpreted as $-\partial K/\partial R$ evaluated at $R=0$. One then has, at $R=0$,

$$\widetilde{\chi} = -\frac{\partial K}{\partial R} = -\sum_a \frac{\partial K}{\partial F^{(a)}}\,\frac{\partial F^{(a)}}{\partial R} = \sum_a \frac{\partial K}{\partial F^{(a)}}\,\frac{\partial K}{\partial \mu^{(a)}} = \sum_{a,b} \frac{\partial K}{\partial F^{(a)}}\,\frac{\partial K}{\partial F^{(b)}}\,\frac{\partial F^{(b)}}{\partial \mu^{(a)}} = \sum_{a,b} \frac{\partial K}{\partial F^{(a)}}\,\frac{\partial K}{\partial F^{(b)}}\,\left(\frac{\partial^2\epsilon}{\partial F^{(a)}\partial F^{(b)}}\right)^{-1}\,, \tag{25}$$

which gives, using definition (14), precisely (19) for the case of a single charge F. However, this thermodynamic description is only applicable to $f(R)$ phenomenology, since it does not take into account non-$f(R)$ terms in the action such as $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ which appear due to quantum corrections [1, 2].

For q–theory of the type considered in (2), there are two stability conditions, namely, condition (15) for the effective Newton's constant $G_{\rm N}$ and condition (14) for the zero-temperature vacuum compressibility χ_0. These two conditions of q–theory correspond, respectively, to the following stability conditions of $f(R)$ models (see, e.g., Eq. (7) of Ref. [15]):

$$-f'(R)\Big|_{R=0} > 0\,, \quad f''(R)\Big|_{R=0} > 0\,, \tag{26}$$

where the prime denotes differentiation with respect to R and the extra minus sign traces back to our conventions, as mentioned already a few lines under (21b). These conditions ensure the stability of the empty flat universe, which is a basic assumption for q–theory as it concerns the thermodynamics of the equilibrium vacuum. In $f(R)$ models, the second condition of (26), i.e., the positive mass squared of the scalaron, $M^2 > 0$, implies the stability of the Minkowski vacuum, whereas for the negative mass squared, $M^2 < 0$, the Minkowski vacuum experiences the scalaron instability [15].

For the case of $M^2 > 0$, the relaxation of the universe to equilibrium is accompanied by oscillations of R with frequency M (see Ref. [15] for the $f(R)$–model and Ref. [6] for the q–theory which also has δF oscillations). In particular, the vacuum energy in q–theory has been found to relax as follows [6]:

$$\rho_{\rm V} \sim \frac{M^2}{t^2}\, \sin^2 Mt\,, \tag{27}$$

for cosmic time $t \gg 1/M$. While the q–field itself has an equation of state parameter $w = -1$, corresponding to vacuum energy density and a cosmological constant, the decay of the vacuum energy density in (27) simulates the evolution of cold dark matter with $w = 0$. The effective equation of state $w = 0$ is induced by the interaction of the q–field with gravity [6].

5. Comparison with phenomenological $f(R)$ models. As explained above and, more briefly, in Ref. [6], F–theory (or, more generally, q–theory) may give a microscopic justification for the phenomenological $f(R)$ models used in theoretical cosmology and may allow for a choice between different classes of model functions $f(R)$ based on fundamental physics. Close to equilibrium, the effective $f(R)$ model emerging from the q–theory of quantum vacuum belongs to the class of $f(R) \sim -R + R^2/M^2$ models. This rules out so-called $1/R$ models, i.e., models with $f(R) \sim -R + M^4/R$ (see, e.g., Refs. [2, 3, 16] and references therein).

Here, we have assumed that the physics of the vacuum variable in q–theory [5, 6] is determined by a unique microscopic energy scale $E_{\rm UV}$. This implies the following orders of magnitude:

$$|F_0^{(a)}| \sim |\mu_0^{(a)}| \sim E_{\rm UV}^2\,, \tag{28a}$$

$$|F_0^{(a)}\partial K/\partial F_0^{(a)}| \sim G_{\rm N}^{-1} \sim E_{\rm UV}^2\,, \tag{28b}$$

$$|\epsilon(F_0^{(a)})| \sim E_{\rm UV}^4\,, \tag{28c}$$

$$\chi_0 \sim E_{\rm UV}^{-4}\,. \tag{28d}$$

As a result, the dimensionless parameter $\widetilde{\chi}$ in (19) can be expected to be of order unity. If the assumption of a single fundamental energy scale holds true, one then obtains that the mass parameter (22) of the induced $f(R) = -R + R^2/(6M^2)$ model is of the order of the ultraviolet scale, $M \sim E_{\rm UV}$. In this case, the R^2 correction is of the order of the quantum R^2 corrections to the Einstein action [1, 2]. Moreover, as q–theory starts from the assumption that the vacuum is a stable self-sustained medium, one has $M^2 > 0$, as follows explicitly from (14), (19), and (21b).

All this agrees with the simplest $f(R) \sim -R + R^2/M^2$ model having a Planck-scale mass M [14], but disagrees with certain other models which use more complicated phenomenological functions $f(R)$. For the particular models suggested in Ref. [15], the functions $f(R)$ expanded around $R = 0$ have negative M^2 (making Minkowski spacetime unstable) and $-M^2$ of the order of $\lambda_{\rm now}$, where $\lambda_{\rm now} = 8\pi G_{\rm N}\, \rho_{\rm V,\,now} \approx (10^{-33}\ {\rm eV})^2$ is the present (positive) cosmological constant as determined by observational cosmology [18]. These cosmologically desirable $f(R)$ models can, in principle, be obtained by using special choices of $\epsilon(F^{(a)})$ and $K(F^{(a)})$. But such choices require a careful tuning of the parameters, which is unnatural from our point-of-view on the properties of the self-sustained quantum vacuum.

6. Conclusion. In q–theory [5, 6] with chemical potentials $\mu^{(a)}$ at their equilibrium values $\mu_0^{(a)}$, the vacuum energy density $\rho_{\rm V}$ has been found to relax, according to (27), from its natural Planck-scale value at early times when the system is far from equilibrium to a naturally small value at late times when the system is close to equilibrium. This solves the main cosmological problem: the present cosmological constant is small compared to Planck-scale values simply because the universe happens to be old compared to Planck-scale times. The remaining problem is to understand why the cosmological constant does not completely relax to zero as $t \to \infty$ or, in other words, to determine the origin of the small residual part of the vacuum energy which remains (almost) constant during the present epoch.

The suggested $f(R)$ model in which the cosmological constant appears just at the latest stage [15] does not follow from the macroscopic quantum-vacuum approach. That is why one needs to find another explanation for the observed value of Λ. In order to produce a nonzero Λ, one must find, according to (23)–(24), processes which shift the chemical potentials $\mu^{(a)}$ away from the equilibrium mean-field values $\mu_0^{(a)}$.

There has been the suggestion to relate the present value of Λ to quantum fluctuations of the vacuum energy, for the case that the vacuum energy itself is nullified (see, e.g., Ref. [19]). This approach could be helpful in q–theory, where the vacuum energy is necessarily zero in full equilibrium. The observed nonzero value of Λ

would then correspond to quantum or thermodynamic fluctuations of the chemical potentials $\mu^{(a)}$ compared to the equilibrium mean-field values $\mu_0^{(a)}$. Still, there could be other processes which are able to shift $\mu^{(a)}$ somewhat away from the mean-field values $\mu_0^{(a)}$, one example being the backreaction of matter and another example being matter production by the rapid oscillations which accompany the relaxation of the vacuum energy (27). It remains to determine which of these processes, if any, is the dominant one for the inferred small but nonzero value of the vacuum energy density from observational cosmology.

It is a pleasure to thank A.A. Starobinsky for helpful discussions. GEV is supported in part by the Russian Foundation for Basic Research (grant 06–02–16002–a) and the Khalatnikov–Starobinsky leading scientific school (grant 4899.2008.2)

1. N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
2. E. J. Copeland, M. Sami, and S. Tsujikawa, "Dynamics of dark energy". Int. J. Mod. Phys. D **15**, 1753 (2006), arXiv:hep-th/0603057.
3. T. P. Sotiriou and V. Faraoni, "$f(R)$ theories of gravity," arXiv:0805.1726 [gr-qc].
4. S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
5. F. R. Klinkhamer and G. E. Volovik, "Self-tuning vacuum variable and cosmological constant," Phys. Rev. D **77**, 085015 (2008), arXiv:0711.3170 [gr-qc].
6. F. R. Klinkhamer and G. E. Volovik, "Dynamic vacuum variable and equilibrium approach in cosmology," to appear in Phys. Rev. D, arXiv:0806.2805 [gr-qc].
7. G. E. Volovik, "Coexistence of different vacua in the effective quantum field theory and multiple point principle", JETP Lett. **79**, 101 (2004), arXiv:hep-ph/0309144.
8. F. R. Klinkhamer and G. E. Volovik, "Coexisting vacua and effective gravity," Phys. Lett. A **347**, 8 (2005), arXiv:gr-qc/0503090.
9. C. Barcelo, "Cosmology as a search for overall equilibrium," JETP Lett. **84**, 635 (2007), arXiv:gr-qc/0611090.
10. F. R. Klinkhamer, "Equilibrium boundary conditions, dynamic vacuum energy, and the Big Bang," arXiv:0803.0281 [gr-qc].
11. C. D. Froggatt, R. Nevzorov, H. B. Nielsen, and D. Thompson, "On the origin of $Z(2)$ symmetry in the two–Higgs doublet extension of the SM," arXiv:0806.3190 [hep-ph].
12. T. V. Ruzmaikina and A. A. Ruzmaikin, "Quadratic corrections to the Lagrangian gravitational field density and singularity," Zh. Eksp. Teor. Fiz. **57**, 680 (1969); Sov. Phys. JETP **30**, 372 (1970).
13. B. N. Breizman, V. Ts. Gurovich, and V. P. Sokolov, "Possibility of setting up regular cosmological soutions," Zh. Eksp. Teor. Fiz. **59**, 288 (1970); Sov. Phys. JETP **32**, 155 (1971).
14. A. A. Starobinsky, "A new type of isotropic cosmological models without singularity," Phys. Lett. **B 91**, 99 (1980).
15. A. A. Starobinsky, "Disappearing cosmological constant in $f(R)$ gravity," JETP Lett. **86**, 157 (2007), arXiv:0706.2041 [astro-ph].
16. T. Kobayashi and K. Maeda, "Relativistic stars in $f(R)$ gravity, and absence thereof", arXiv:0807.2503 [astro-ph].
17. S. Nojiri and S. D. Odintsov, "Inhomogeneous equation of state of the universe: Phantom era, future singularity and crossing the phantom barrier," Phys. Rev. D **72**, 023003 (2005), arXiv:hep-th/0505215.
18. E. Komatsu et al., "Five-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Cosmological interpretation," arXiv:0803.0547v1 [astro-ph].
19. T. Padmanabhan, "Dark energy and its implications for gravity", arXiv:0807.2356 [gr-qc]