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Parton energy loss due to synchrotron-like gluon emission

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We develop a quasiclassical theory of the synchrotron-like gluon radiation. Our calculations show that the parton energy loss due to the synchrotron gluon emission may be important in the jet quenching phenomenon if the plasma instabilities generate a sufficiently strong chromomagnetic field. Our gluon spectrum disagrees with that obtained by Shuryak and Zahed within the Schwinger's proper time method.

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1. The hydrodynamic description [1] of the RHIC hadron spectra at low p_T gives strong evidence for the thermalization of the produced hot quark-gluon plasma (QGP) at the time scale $\tau_0 \sim 1\,\mathrm{fm}$. The early thermalization provides a serious challenge for the perturbative approach based on the Boltzmann equation with the binary and the $2 \leftrightarrow 3$ processes [2]. The fast thermalization can probably be explained if the plasma is strongly coupled [3]. A promising possibility of speeding the equilibration in a weakly coupled plasma scenario is connected with plasma instabilities [4 – 8]. One of the potentially important instabilities is the Weibel [9] instability which occurs for the anisotropic plasma distribution. In the QGP produced in AA-collisions the initial parton distribution has $p_z \ll p_T$ [2]. In this case the Weibel instability can generate considerable (predominantly transverse) chromomagnetic field which bends the parton trajectories and leads eventually to isotropization of the parton distribution [5, 7, 8]. Also, the generated chromomagnetic field may be important for explanation of the anomalously small plasma viscosity [10] and the longitudinal broadening of the jet cone (the ridge effect) [11].

In the scenario with generation of the chromomagnetic field an interesting question arises on the effect of the chromomagnetic field on the jet quenching due to the synchrotron-like gluon emission from fast partons. Evidently the available analyses of the jet quenching, based on the idea of the induced gluon emission due to multiple scattering [12 – 16], become inappropriate if the contribution of this mechanism to the parton energy loss is comparable to that from the ordinary multiple scattering. It is clear that in this case both these mechanism must be treated on an even footing. As a first step in understanding the role of the chromomagnetic field in the parton energy loss it would be interesting to calculate the purely synchrotron induced gluon radiation neglecting the interference between the two mechanisms. It requires a formalism for the synchrotron radiation in QCD.

In the present paper we develop a semiclassical approach to the synchrotron gluon radiation and give a qualitative estimate of the parton energy loss in the QGP produced in AA-collisions. For photon emission our result agrees with prediction of the quasiclassical operator approach [17]. For gluon emission our spectrum disagrees with that obtained by Shuryak and Zahed [18] within the Schwinger's proper time method. We give simple physical arguments that the spectrum obtained in [18] is incorrect.

2. We consider the synchrotron gluon radiation from a fast parton in the quasiclassical regime when for each parton (initial or final) the wavelength is much smaller than its Larmor radius, R_L. One can show that in this regime, similarly to the photon radiation in QED, the coherence length of the gluon emission, L_c, is small compared to the minimal R_L. It allows one to perform the calculation of the radiation rate per unit length by considering the case of a slab of chromomagnetic field of thickness L which is large compared to L_c, but small compared to the minimal R_L. In this case the transverse momenta of the final partons are small compared to their longitudinal momenta (we choose the z-axis along the initial parton momentum). We consider the case of a slab perpendicular to the z-axis with transverse chromomagnetic field, $\mathbf{H}_a$. It is enough to consider the chromomagnetic field with the only nonzero color components in the Cartan subalgebra, i.e., for $a = 3$ and $a = 8$ for the $SU(3)$ color group. The interaction of the gluons with the background chromomagnetic field is diagonalized by introducing the fields having definite color isospin, Q_A, and color hypercharge, Q_B, (we will describe the color charge by the two-dimensional vector $Q = (Q_A, Q_B)$). In terms of the usual gluon vector potential, G, the diagonal color gluon states read (the Lorentz indices are omitted) $X = (G_1 + iG_2)/\sqrt{2}$ ($Q =$

$= (-1,0))$, $Y = (G_4 + iG_5)/\sqrt{2}$ $(Q = (-1/2, -\sqrt{3}/2))$, $Z = (G_6 + iG_7)/\sqrt{2}$ $(Q = (1/2, -\sqrt{3}/2))$. The neutral gluons $A = G_3$ and $B = G_3$ with $Q = (0,0)$, to leading order in the coupling constant, do not interact with the background field, and the emission of these gluons are similar to the photon radiation in QED.

The S-matrix element of the $q \to gq'$ synchrotron transition can be written as (we omit the color factors and indices)

$$\langle gq'|\hat{S}|q\rangle = -ig\int dy \bar{\psi}_{q'}(y)\gamma^{\mu}G^{*}_{\mu}(y)\psi_q(y), \quad (1)$$

where $\psi_{q,q'}$ are the wave functions of the initial quark and final quark, G is the wave function of the emitted gluon. We write each quark wave function in the form $\psi_i(y) = \exp[-iE_i(t-z)]\hat{u}_{\lambda}\phi_i(z,\boldsymbol{\rho})/\sqrt{2E_i}$ (hereafter the bold vectors denote the transverse vectors), where λ is quark helicity, $\hat{u}_{\lambda}$ is the Dirac spinor operator. The z-dependence of the transverse wave functions ϕ_i is governed by the two-dimensional Schrödinger equation

$$i\frac{\partial\phi_i(z,\boldsymbol{\rho})}{\partial z} = \\ = \left\{\frac{(\mathbf{p}-gQ_n\mathbf{G}_n)^2+m_q^2}{2E_i} + gQ_n(G_n^0 - G_n^3)\right\}\phi_i(z,\boldsymbol{\rho}), \quad (2)$$

where now G denotes the external vector potential (the superscripts are the Lorentz indexes and $n = 1,2$ correspond to the A and B color components in the Cartan subalgebra), Q_n is the quark color charge. The wave function of the emitted gluon can be represented in a similar way. We will assume that in the QGP for the usual parton masses one can use the corresponding quasiparticle masses.

We take the external vector potential in the form $G_n^3 = [\mathbf{H}_n \times \boldsymbol{\rho}]^3$, $\mathbf{G}_n = 0$, $G_n^0 = 0$ (we assume that chromoelectric field is absent, however, it can be included as well). For this choice of the vector potential the term $-gQ_nG_n^3$ in (2) can be viewed as the potential energy in the impact parameter plane $U_i = -\mathbf{F}_i\cdot\boldsymbol{\rho}$, where $\mathbf{F}_i$ is the corresponding Lorentz force. Then the solution of (2) can be taken in the form

$$\phi_i(z,\boldsymbol{\rho}) = \exp\left\{i\mathbf{p}_i(z)\boldsymbol{\rho} - \frac{i}{2E_i}\int_0^z dz'[\mathbf{p}_i^2(z') + m_q^2]\right\}. \quad (3)$$

Here the transverse momentum $\mathbf{p}_i(z)$ is the solution to the parton equation of motion in the impact parameter plane

$$\frac{d\mathbf{p}_i}{dz} = \mathbf{F}_i(z). \quad (4)$$

Below we denote the value of $\mathbf{p}_i(\pm\infty)$ as $\mathbf{p}_i^{\pm}$. By using (1), (3) one can obtain

$$\langle gq'|\hat{S}|q\rangle = -ig(2\pi)^3\delta(E_g + E_{q'} - E_q)\times \\ \times\int_{-\infty}^{\infty} dz V(z,\{\lambda\})\delta(\mathbf{p}_g(z) + \mathbf{p}_{q'}(z) - \mathbf{p}_q(z))\times \\ \times\exp\left\{-i\int_0^z dz'\left[\frac{\mathbf{p}_q^2(z') + m_q^2}{2E_q} - \frac{\mathbf{p}_g^2(z') + m_g^2}{2E_g} - \frac{\mathbf{p}_{q'}^2(z') + m_q^2}{2E_{q'}}\right]\right\}, \quad (5)$$

where V is the spin vertex factor, $\{\lambda\}$ is the set of the parton helicities. For the transition conserving quark helicity $V = -iE_q\sqrt{1-x}[2\lambda_q x + (2 - x)\lambda_g][v_x(z) - i\lambda_g v_y(z)]/\sqrt{2}$, and for the spin-flip case $V = ixm_q(2\lambda_q\lambda_g + 1)/\sqrt{2(1-x)}$. Here $\mathbf{v}(z) = \mathbf{v}_g(z) - \mathbf{v}_q(z)$ is the relative transverse velocity in the final gq' parton system which can be written as $\mathbf{v}(z) = \mathbf{q}(z)/\mu$ with $\mathbf{q}(z) = \mathbf{p}_g(z)(1-x) - \mathbf{p}_{q'}(z)x$, $\mu = E_q x(1-x)$ (here x is the longitudinal gluon fractional momentum). Due to the color charge conservation $\mathbf{F}_q = \mathbf{F}_g + \mathbf{F}_{q'}$. For this reason, the argument of the second δ-function does not depend on z, and can be replaced by $\mathbf{p}_g^+ + \mathbf{p}_{q'}^+ - \mathbf{p}_q^+$. From (5) we obtain the gluon emission spectrum

$$\frac{dP}{dx} = \frac{1}{(2\pi)^2}\int d\mathbf{p}_g^+\int dz_1 dz_2 g(z_1,z_2)\times \\ \times\exp\left\{i\int_{z_1}^{z_2} dz\left[\frac{\mathbf{p}_q^2(z) + m_q^2}{2E_q} - \frac{\mathbf{p}_g^2(z) + m_g^2}{2E_g} - \frac{\mathbf{p}_{q'}^2(z) + m_q^2}{2E_{q'}}\right]\right\}, \quad (6)$$

where the vertex factor reads (we recover the vertex color factor C)

$$g(z_1,z_2) = \frac{C\alpha_s}{8E_q^2 x(1-x)}\times \\ \times\sum_{\{\lambda\}} V^*(z_2,\{\lambda\})V(z_1,\{\lambda\}) = g_1\mathbf{v}(z_2)\mathbf{v}(z_1) + g_2, \quad (7)$$

with $g_1 = C\alpha_s(1 - x + x^2/2)/x$ and $g_2 = C\alpha_s m_q^2 x^3/2\mu^2$ (the two terms in (7) correspond to the non-flip and spin-flip processes). The color factor reads $C = |\lambda_{fi}^a\chi_a^*/2|^2$, where i, f are the color indexes of the initial and final quarks, χ_a is the color wave function of the emitted gluon.

For a uniform external field we can write $\mathbf{v}(z_2)\mathbf{v}(z_1) = [\bar{\mathbf{q}}^2 - \mathbf{f}^2\tau^2/4]/\mu^2$, where $\bar{\mathbf{q}} = \mathbf{q}(\bar{z})$, $\bar{z} = (z_1 + z_2)/2)$, $\tau = z_2 - z_1$, and $\mathbf{f} = d\mathbf{q}/dz =$

$\mathbf{F}_g(1-x)-\mathbf{F}_{q'}x$. The argument of the exponential function in (6) can be rewritten as

$$\Phi(\tau,\bar{\mathbf{q}})=\frac{(\epsilon^2+\bar{\mathbf{q}}^2)\tau}{2\mu}+\frac{\mathbf{f}^2\tau^3}{24\mu}, \qquad (8)$$

with $\epsilon^2=m_q^2x^2+m_g^2(1-x)$. After replacing in (6) the integration over $\mathbf{p}_g^+$ by the integration over $\bar{\mathbf{q}}$ we obtain for the radiation rate per unit length

$$\frac{dP}{dLdx}=\frac{1}{(2\pi)^2}\int d\bar{\mathbf{q}}\int_{-\infty}^{\infty}d\tau\left[\frac{g_1}{\mu^2}\left(\bar{\mathbf{q}}^2-\frac{\mathbf{f}^2\tau^2}{4}\right)+g_2\right]\times \times\exp\left[-i\Phi(\tau,\bar{\mathbf{q}})\right]. \qquad (9)$$

With the help of integration by parts one can rewrite (9) as

$$\frac{dP}{dLdx}=-\frac{1}{(2\pi)^2}\int d\bar{\mathbf{q}}\int_{-\infty}^{\infty}d\tau\left[\frac{g_1}{\mu^2}\left(\epsilon^2+\frac{\mathbf{f}^2\tau^2}{2}\right)-g_2\right]\times \times\exp\left[-i\Phi(\tau,\bar{\mathbf{q}})\right]. \qquad (10)$$

After integrating over $\bar{\mathbf{q}}$ (10) takes the form

$$\frac{dP}{dLdx}=\frac{i\mu}{2\pi}\int_{-\infty}^{\infty}\frac{d\tau}{\tau}\left[\frac{g_1}{\mu^2}\left(\epsilon^2+\frac{\mathbf{f}^2\tau^2}{2}\right)-g_2\right]\times \times\exp\left\{-i\left[\frac{\epsilon^2\tau}{2\mu}+\frac{\mathbf{f}^2\tau^3}{24\mu}\right]\right\}. \qquad (11)$$

Note that in (9)-(11) it is assumed that τ has a small negative imaginary part. One can easily show that in (11) the integral around the lower semicircle near the pole at $\tau=0$ plays the role of the $\mathbf{f}=0$ subtraction term. Expressing the integrals along the real axis in terms of the Airy function $\mathrm{Ai}(z)=\frac{1}{\pi}\sqrt{\frac{z}{3}}K_{1/3}(2z^{3/2}/3)$ (here $K_{1/3}$ is the Bessel function) (11) can be written as

$$\frac{dP}{dLdx}=\frac{a}{\kappa}\mathrm{Ai}'(\kappa)+b\int_{\kappa}^{\infty}dy\,\mathrm{Ai}(y), \qquad (12)$$

where $a=-2\epsilon^2g_1/\mu$, $b=\mu g_2-\epsilon^2g_1/\mu$, $\kappa==\epsilon^2/(\mu^2\mathbf{f}^2)^{1/3}$.

From (11) one can obtain for the coherence length of the gluon emission $L_c\sim\min(L_1,L_2)$, where $L_1=2\mu/\epsilon^2$ and $L_2=(24\mu/\mathbf{f}^2)^{1/3}$. From this estimate one can easily show that for radiation of a charged gluon the condition $L_c/R_{g,L}\ll 1$ (which is necessary for validity of our small angle approximation) is really satisfied in the quasiclassical regime when $E_gR_{g,L}\gg 1$ and $E_g\gg m_g$. In the interesting to us region $F_{q'}\lesssim m_g^2$ (hereafter $F_i=|\mathbf{F}_i|$) for emission of a neutral gluon the condition $L_c/R_{q,L}\ll 1$ is also satisfied.

Our spectrum for neutral gluons when $F_g=0$ for $m_g=0$ agrees with the photon spectrum obtained in the quasiclassical operator approach [17][1)], and similarly to the photon emission (12) gives $dP/dLdx\propto x^{-2/3}$ at $x\ll 1$. The nonzero gluon mass leads to the Ter-Mikaelian suppression at $x\lesssim(m_g^3/E_qF_{q'})^{1/2}$. In this region the parameter κ becomes larger than unity and the spectrum $dP/dLdx\propto\exp\left[-2m_g^3/3E_qF_{q'}x^2\right]/x$. For the charged gluons from (12) one can obtain in the massless limit $dP/dLdx\propto x^{-4/3}$ at $x\ll 1$. The Ter-Mikaelian mass effect suppresses the gluon spectrum at $x\lesssim m_g^3/E_qF_g$. In this region $dP/dLdx\propto\exp\left[-2m_g^3/3xE_qF_g\right]/x^{3/2}$ at $x\ll 1$. Note that in general if one neglects the parton masses (or in the strong field limit) the spectrum (12) takes a simple form

$$\frac{dP}{dLdx}\approx\frac{\alpha_sC\Gamma(2/3)[1-x+x^2/2](9\mathbf{f}^2)^{1/3}}{\pi\sqrt{3}x[E_qx(1-x)]^{1/3}}. \qquad (13)$$

Our derivation is valid for the $g\to gg$ transition as well. In this case a and b have the same form but with $\epsilon^2=m_g^2(1-x+x^2)$, $g_1=C\alpha_s[1+x^4+(1-x)^4]/4x(1-x)$ and $g_2=0$. The gluon vertex color factor reads $C=|\chi_a\chi_b^*\chi_c^*f_{abc}|^2$, where the index a corresponds to the initial gluon, and b, c to the final gluons.

3. In our formula for the spectrum for $a\to bc$ transition all the properties of the final bc state are only accumulated in $\mathbf{f}^2=\mathbf{F}_b^2x_b^2-2\mathbf{F}_b\mathbf{F}_cx_bx_c+\mathbf{F}_c^2x_c^2$ (except for the trivial vertex factor). The $\mathbf{f}^2$ crucially depends on the relation between the forces acting on the final partons. The value of $\mathbf{f}^2$ characterizes the difference in bending of the trajectories of the b and c partons in the external field which is responsible for the synchrotron radiation. For this reason the spectrum vanishes if at some x $\mathbf{f}=0$. This, for example, occurs for the $g_X\to g_Yg_{\bar{Z}}$ gluon process at $x=0.5$ for the external field in the color state A.

Note, that the spectrum obtained in the present paper can also be derived making use the light-cone path integral formalism [13] for gluon emission due to multiple scattering. In [13] the spectrum was expressed in terms of the Green's function for the Schrödinger equation describing the relative motion in the $\bar{q}q'g$ system. In the absence of the external field the corresponding Hamiltonian has a purely imaginary potential $U(\boldsymbol{\rho})=-in\sigma_{\bar{q}q'g}(|\boldsymbol{\rho}|)/2$ (here $\sigma_{\bar{q}q'g}$ is the cross section for the $\bar{q}q'g$ state, and n is the number density of the medium). In the case of the synchrotron radiation (without multiple scattering) this potential should be replaced

[1)] Note that for the photon emission by an electron the integrand in the exponential term in (6) can also be written as $E_ek^{\mu}x_{\mu}(t)/(E_e-E_{\gamma})$, where k^{μ} is the photon four momentum and $x_{\mu}(t)$ is the classical electron trajectory. This gives precisely the spectrum in the form obtained by Baier and Katkov [17].

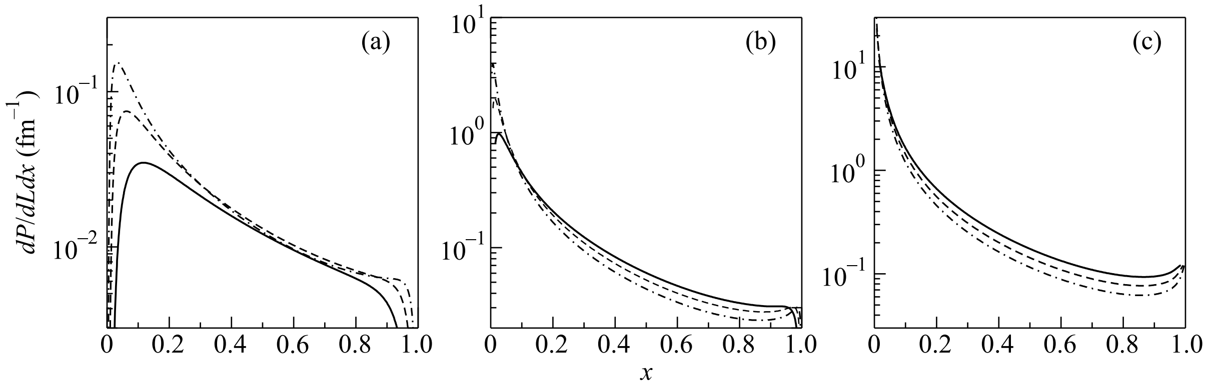

Fig.1. The spectrum for the $q \to gq$ process in the chromomagnetic field in the color state A for $\alpha_s = 0.3$, $gH_A/m_D^2 = 0.05$ (a), 0.25 (b) and 1 (c), for the initial quark energies $E_q = 20$ GeV (solid line), $E_q = 40$ GeV (dashed line), $E_q = 80$ GeV (dot-dashed line)

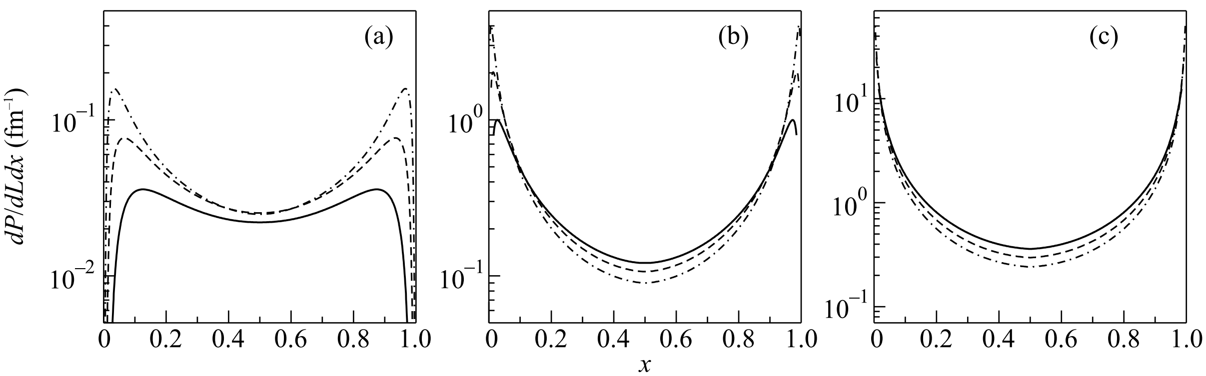

Fig.2. The same as in Fig.1 but for the $g \to gg$ process

by the real potential $U(\boldsymbol{\rho}) = -\mathbf{f}\cdot\boldsymbol{\rho}$. The advantage of the path integral approach is that it allows one to treat the gluon emission including both the multiple scattering and bending of the trajectories in the external field. This analysis will be given in further publications.

Our formula (11) disagrees with that obtained by Shuryak and Zahed in the soft gluon limit within the Schwinger's proper time method. [18]. In the spectrum derived in [18] (Eq. (20) of [18]) the argument of the exponential contains (we use our notation) $\mathbf{F}_{q'}^2 x_g^2 + \mathbf{F}_g^2$ instead of our $\mathbf{f}^2$. Also, in the pre-exponential factor instead of $\mathbf{f}^2$ there appears $\mathbf{F}_{q'}^2 x_g^2$. Thus, our result at $x_g \ll 1$ agrees with that of [18] only for the QED like processes (emission of the neutral gluons). For a real QCD process, with charged gluon and quark, due to the absence of the interference term the spectrum of [18] is insensitive to the relation between the signs of the color charges of the final partons. It is strange enough, since the difference in the bending of the final parton trajectories (which is responsible for the synchrotron radiation) is sensitive to the relation between the color charges of the final partons. Also, Eq. (20) of [18] gives clearly wrong prediction that in the massless limit the spectrum of the $q_1 \to g_Z q_3$ transition for the chromomagnetic field in the color state A vanishes (since in this case $\mathbf{F}_{q'} = 0$). Indeed, this process except for the spin effects is analogous to the synchrotron radiation in QED, and there is no physical reason why it should vanish. Note also that since in [18] the pre-exponential factor contains the Lorentz forces acting on the final partons in non-symmetric form it is clear that for the $g \to gg$ process the method of [18] should give the spectrum with incorrect permutation properties Thus, one sees that the formula obtained in [18] clearly leads to physically absurd predictions. Unfortunately, the details of the calculations have not been given in [18]. For this reason it is difficult to understand what is really wrong in the analysis [18].

4. We use the quasiparticle masses obtained from the analysis of the lattice data within the quasiparticle model [19]. For the plasma temperature $T \sim (1-3)T_c$ which is relevant to the RHIC and LHC conditions the analysis [19] gives $m_q \approx 0.3$ and $m_g \approx 0.4$ GeV. In Figs.1, 2 we present the averaged over the color states gluon spectra for $q \to gq'$ and $g \to gg$ processes for the chromomagnetic field in the color state A for different initial parton

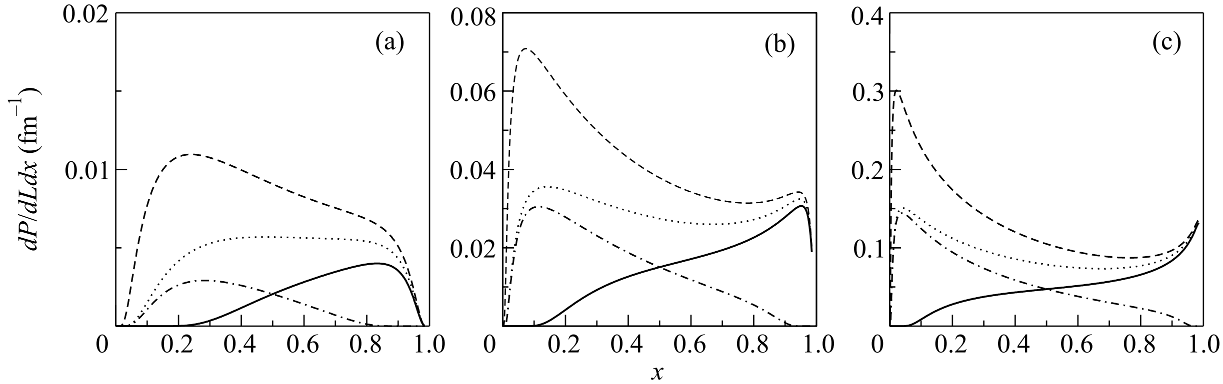

Fig.3. The spectrum for the $q \to gq$ process for specific color states. The curves correspond to $q_1 \to g_A q_1$ (solid line), $q_1 \to g_{\bar{X}} q_2$ (dashed line), $q_1 \to g_Z q_3$ (dot-dashed line), $q_3 \to g_Y q_1$ (dotted line). In all the processes the vertex color factor C in (7) is replaced by unity. The computations are performed for $\alpha_s = 0.3$, and $E_q = 20$ GeV. The chromomagnetic field is in the color state A with $gH_A/m_D^2 = 0.05$ (a), 0.25 (b) and 1 (c)

energies. The computations are performed for $\alpha_s = 0.3$ and $gH_A/m_D^2 = 0.05$, 0.25 and 1, where m_D is the Debye mass (we assume that as for an isotropic weakly coupled plasma $m_D^2 = 2m_g^2$). The results for the chromomagnetic field in the color state B are very close to that shown in Figs.1, 2, and we do not show them. The decrease of the spectra at $x \to 0$ (and $x \to 1$ for $g \to gg$ process) which is well seen for the smallest value of the field is due to the Ter-Mikaelian mass effect. This suppression decreases with increase of the chromomagnetic field.

In Fig.3 we show the spectra for the $q \to gq'$ process for different quark and gluon color states. To illustrate better namely the dependence on the color indexes in all the spectra we replace the color vertex factor C by unity. One can see that the spectra grow strongly in the region of small x with the gluon color charge. For this reason the averaged over color spectra shown in Figs.1, 2 are dominated by the processes with charged gluons.

For understanding the potential role of the synchrotron-like radiation in the jet quenching in AA-collisions it is interesting to estimate the energy loss due to the synchrotron gluon emission. Of course, a realistic estimate of this effect requires detailed information on the time evolution of the QGP instabilities. Also, as we said in the introduction, an accurate analysis should treat the synchrotron radiation and usual bremsstrahlung due to multiple scattering on an even footing. This is, however, beyond the scope of the present analysis. In the present paper we can give only a crude estimate of the effect. To estimate the chromomagnetic field we rely on the idea that the small viscosity of the QGP observed at RHIC is due to parton rescatterings in the turbulent magnetic field [10]. In this model the viscosity/entropy ratio $\eta/s \sim 1/g^2\xi^{3/2}$ [10], where ξ is the anisotropy parameter of the initial plasma distribution. It is expected that the generated magnetic field is saturated at $g^2\langle H^2\rangle \sim \xi^2 m_D^2$. To obtain a realistic η/s ratio one should assume that $\xi \sim 1$. Probably, the value of the magnetic field obtained in this way may be viewed as an upper bound. Indeed, in this case the ratio of the magnetic energy to the thermal parton energy is ~ 0.3, and a higher fraction of the magnetic energy looks unrealistic. Making use the estimated magnetic field for RHIC conditions for $\alpha_s = 0.3$ we obtained $\Delta E/E \sim 0.1 - 0.2$ for quarks and $\Delta E/E \sim 0.2 - 0.4$ for gluons at $E \sim 10 - 20\,\mathrm{GeV}$ (for $\alpha_s = 0.5$ the results are about two times bigger). These estimates have been obtained assuming that the parton path length in the magnetic field is $\sim 2 - 4$ fm. Also, we neglected any finite-size effects. These effects may be important if L_c and L are of the same order. For RHIC conditions the dominating contribution to the energy loss comes from the soft gluon emission where $L_c \sim 1 - 2\,\mathrm{fm}$. In this situation the finite-size effects may suppress the energy loss by a factor ~ 0.5. One more suppression mechanism may be connected with the finite coherence length of the turbulent magnetic field, L_m. However, if for the unstable magnetic field modes the wave vector $k^2 \lesssim \xi m_D^2$ [10], for RHIC conditions this suppression should not be very strong since we have $L_m/L_c \gtrsim 1$. In this regime the turbulent effects should not suppress strongly the energy loss, and as a plausible estimate one can take the turbulent suppression factor ~ 0.5. Even with these suppression factors the synchrotron energy loss turns out to be comparable with that due to ordinary bremsstrahlung, and its relative contribution may be larger than that from the collisional energy loss [20]. Of course, the above estimates are very crude. Nevertheless they demonstrate that the synchrotron radiation can be important in jet quenching and deserves further more accurate investigations. Of course, the synchrotron radiation and bremsstrahlung are not additive, since magnetic bending of the parton trajectories will suppress bremsstrahlung due to multiple scattering and vice versa multiple scattering will suppress the synchrotron radiation. This, in principle, can mask the effect of the synchrotron emission in the energy loss. However, the synchrotron radiation can reveal itself in the longitudinal broadening of the jet cone. Note that for the synchrotron emission this effect appears already at the level of the gluon emission itself contrary to the mechanism of Ref. [11] where it is a purely final-state interaction effect.

5. In summary, we have developed a quasiclassical theory of the synchrotron-like gluon radiation. In the QGP the gluon spectrum is dominated by the processes with emission of the charged gluons, the effect of the neutral gluons is relatively small. Our calculations show that the parton energy loss due to the synchrotron radiation may be important in the jet quenching if the QGP instabilities generate magnetic field $\langle H^2\rangle \sim m_D^2/g^2$. Our gluon spectrum disagrees with that obtained by Shuryak and Zahed [18]. We give simple physical arguments that the spectrum derived in [18] is incorrect.

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