

# Spin-Wave Relaxation in a Quantum Hall Ferromagnet

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We study spin wave relaxation in quantum Hall ferromagnet regimes. Spin-orbit coupling is considered as a factor determining spin nonconservation, and external random potential as a cause of energy dissipation making spin-flip processes irreversible. We compare this relaxation mechanism with other relaxation channels existing in a quantum Hall ferromagnet.

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1. Last years are characterized by growing interest in spin relaxation (SR) in low-dimension systems – first of all, in the relaxation in quantum dots studied within the projects aimed at development of a computer employing spin memory. Yet, the relaxation of an electron spin in lateral quantum dots manufactured on the basis of two-dimensional (2D) heterostructures, should be in many respects similar to the SR of electrons localized in the 2D layer in minima of a smooth random potential (SRP). In high magnetic fields this single-electron relaxation corresponds to the situation occurring at low Landau level (LL) filling:  $\nu \ll 1$  or  $|\nu - 2n| \ll 1$  ( $n$  is an integer) [1].

The SR at different filling factors,  $\nu \gtrsim 1$ , has quite different nature representing in this case a many-electron process. In particular, in a quantum Hall ferromagnet (QHF), i.e. at  $\nu = 1, 3, \dots$  or  $\nu = 1/3, 1/5, \dots$ , the SR reduces to the relaxation of lowest collective excitations, i.e. spin waves [2, 3]. The SR observation would thereby be a good tool to study fundamental collective properties of a strongly correlated 2D electron gas (2DEG). However, in spite of much recent interest in the SR in a 2DEG, up to now only a handful of experiments relevant to the SR in a QHF were performed: these are indirect results based on the linewidth measurements in the electron spin resonance [4], and a direct observation where the photoluminescence dynamics of spin-up and spin-down states was studied [5]. Meanwhile, availability of the new time-resolved technique of photon counting allows us to believe that new direct experiments on observation of excitations' relaxation in a 2DEG, in particular of the spin wave relaxation (SWR), will become available in the near future [6].

Theoretically the SWR in a QHF was studied in works [7, 8]. It is worth noting here that the SWR represents actually not spin dephasing but the energy

relaxation due to the spin-flip process. Indeed, any spin-flip means at least dissipation of the Zeeman energy  $\epsilon_Z = |g|\mu_B B$  ( $g \approx -0.44$  in a GaAs structure). The latter is a part of the spin-wave (spin exciton, SE) energy

$$E_{sw} = \epsilon_Z + \mathcal{E}_q, \quad (1)$$

where  $\mathcal{E}_q$  is the SE correlation energy depending on the 2D wave vector  $q$  [2, 3]. At variance with the relaxation channel of Ref. [7] where electron-phonon interaction was considered as the mechanism making the relaxation irreversible, and contrary to the case of Ref. [8] where the irreversibility was provided by an inter-spin-exciton interaction mechanism, we now study smooth disorder field as the reason causing the energy transform. The SRP thereby determines an alternative relaxation channel competing with the ones studied earlier. Another distinction of the present work from Refs. [7] and [8] consists in the study of not only the integer QHF (at  $\nu = 1, 3, \dots$ ) but also of the fractional one ( $\nu = 1/3, 1/5, \dots$ ) as well. At the same time we again consider the spin-orbit coupling (SO) as the cause mixing different spin states and therefore providing the spin nonconservation. Actually, various SWR channels coexist in parallel. We consider the total rate and find crossover regions of external parameters (magnetic field, temperature, etc.) where one relaxation channel ceases to be dominant and changes into another.

The SR channel due to SRP was already considered in the integer quantum Hall ferromagnetic case [1, 9]. However, studied in these works instead of the SWR was a specific SR when initially the total macroscopic spin  $\mathbf{S}$  of the system as a whole is turned away from the equilibrium direction parallel to  $\mathbf{B}$ . (Relaxation of this Goldstone mode microscopically reduces to annihilation processes of the so-called zero SEs, having exactly zero

momenta.) Contrary to this case, the spin perturbation determined by excitation of the spin waves (non-zero SEs) represents an initial deviation where  $\Delta S = \Delta S_z$ , so that  $\mathbf{S}$  is kept parallel to  $\mathbf{B}$  and the total symmetry of system remains unchanged.

Concerning the origin of SRP, one should note that it has in the 2D layer the “direct” component and the effective one. The former is the SRP determined by charged donors located outside the spacer. The latter is essential in some kinds of quantum wells, being determined by spatial fluctuations (in the plane of the layer) of quantum well width. These fluctuations lead to fluctuations of the size-quantization energy and may be presented as an SRP term in the single electron Hamiltonian. Both SRP components have approximately the same amplitude  $\Delta \sim 10$  K and correlation length  $\Lambda \sim 30 - 50$  nm.

2. The total Hamiltonian has form  $H_{\text{tot}} = \sum_j H_1^{(j)} + H_{\text{int}}$ , where  $j$  enumerates electrons,  $H_{\text{int}}$  is the  $e$ - $e$  interaction, and the single-electron operator is

$$H_1 = \hbar^2 \hat{\mathbf{q}}^2 / 2m_e^* - \epsilon_z \hat{\sigma}_z / 2 + H_{\text{SO}} + \varphi(\mathbf{r}). \quad (2)$$

In this equation  $\varphi(\mathbf{r})$  is the SRP field; the SO Hamiltonian is specified for the (001) GaAs plane,

$$H_{\text{SO}} = \alpha (\hat{\mathbf{q}} \times \hat{\boldsymbol{\sigma}})_z + \beta (\hat{q}_y \hat{\sigma}_y - \hat{q}_x \hat{\sigma}_x), \quad (3)$$

presenting a combination of the Rashba term and the crystalline anisotropy term [10] ( $\hat{\mathbf{q}} = -i\nabla + e\mathbf{A}/c\hbar$  is a 2D operator,  $\sigma_{x,y,z}$  are the Pauli matrices). If the SRP is assumed to be Gaussian, then it is defined by the correlator  $K(\mathbf{r}) = \langle \varphi(\mathbf{r})\varphi(0) \rangle$ . By choosing  $\langle \varphi(\mathbf{r}) \rangle = 0$ , in terms of the correlation length  $\Lambda$  and the LL width  $\Delta$  the correlator is

$$K(\mathbf{r}) = \Delta^2 \exp(-r^2/\Lambda^2). \quad (4)$$

We first find the bare single-electron basis diagonalizing the Hamiltonian (2) without the SRP field. To within the leading order in the  $H_{\text{SO}}$  terms we obtain

$$\begin{aligned} \Psi_{pa} &= \begin{pmatrix} \psi_{np} \\ v\sqrt{n+1}\psi_{n+1p} + iu\sqrt{n}\psi_{n-1,p} \end{pmatrix}, \\ \Psi_{pb} &= \begin{pmatrix} -v\sqrt{n}\psi_{n-1p} + iu\sqrt{n+1}\psi_{n+1p} \\ \psi_{np} \end{pmatrix}. \end{aligned} \quad (5)$$

Here  $\psi_{np}$  is the electron wave function in the Landau gauge,  $n$  is the number of the half-filled LL in the odd-integer quantum Hall regime, i.e. in the  $\nu = 2n + 1$  case. Otherwise, if  $\nu \leq 1$ , we set  $n = 0$ .  $u$  and  $v$  are small dimensionless parameters:  $u = \beta\sqrt{2}/l_B\hbar\omega_c$  and  $v = \alpha\sqrt{2}/l_B\hbar\omega_c$  ( $\omega_c$  and  $l_B$  are the cyclotron frequency and the magnetic length, respectively). The

single-electron states thus cease to be purely spin states but acquire a chirality  $a$  or  $b$ . The spin flip corresponds thereby to the  $a \rightarrow b$  process now.

By analogy with previous works [1, 7–9] (see also Ref. [11]) we define the SE creation operator

$$\mathcal{Q}_{ab\mathbf{q}}^\dagger = \frac{1}{\sqrt{N_\phi}} \sum_p e^{-iq_x p} b_{p+\frac{q_y}{2}}^\dagger a_{p-\frac{q_y}{2}}, \quad (6)$$

where  $a_p$  and  $b_p$  are the Fermi annihilation operators corresponding to states (5),  $N_\phi$  is the LL degeneracy number. In Eq. (6) and everywhere below we measure wave vector  $q$  in the  $1/l_B$  units. If the ratio  $r_c = (\alpha e^2/\kappa l_B)/\hbar\omega_c$  is considered to be small ( $\alpha < 1$  is the averaged formfactor which appears due to finiteness of the layer thickness), and the SRP and SO terms in Eq. (2) are ignored, then the operator (6) acting on the ground state in the *odd-integer quantum Hall regime* yields the *eigen state* of the total Hamiltonian: namely,  $[H_{\text{tot}}, \mathcal{Q}_{ab\mathbf{q}}^\dagger]|0\rangle = (\epsilon_z + \mathcal{E}_q)\mathcal{Q}_{ab\mathbf{q}}^\dagger|0\rangle$ , where

$|0\rangle = \overbrace{|\uparrow, \uparrow, \dots, \uparrow\rangle}^{N_\phi}$ . This basic property of the exciton state,  $\mathcal{Q}_{ab\mathbf{q}}^\dagger|0\rangle$ , is the asymptotically exact one to the first order in  $r_c$ .

Now consider corrections arising due to the  $H_{\text{SO}}$  terms. When presented in terms of basis states (5), spin operators  $\int \Psi^\dagger \hat{S}^2 \Psi d^2\mathbf{r}$  and  $\int \Psi^\dagger \hat{S}_z \Psi d^2\mathbf{r}$  [where  $\Psi = \sum_p (a_p \Psi_{pa} + b_p \Psi_{pb})$ ] preserve invariant form up to the second order in  $u$  and  $v$ . However, the interaction Hamiltonian  $H_{\text{int}} = \frac{1}{2} \int d\mathbf{r}_1 d\mathbf{r}_2 \Psi^\dagger(\mathbf{r}_2) \Psi^\dagger(\mathbf{r}_1) U(\mathbf{r}_1 - \mathbf{r}_2) \Psi(\mathbf{r}_1) \Psi(\mathbf{r}_2)$  acquires proportional to  $u$  and  $v$  terms which correspond to creation and annihilation of SEs in the system. These terms lead to the “coalescence” channel of the SWR [8]. In the present work we study another relaxation channel. Therefore, neglecting the SO corrections to  $\hat{H}_{\text{int}}$ , we focus on the SRP term. Calculating  $\int \Psi^\dagger \varphi(\mathbf{r}) \Psi d^2\mathbf{r}$ , we get the terms responsible for a spin-flip:

$$\hat{\varphi} = N_\phi^{1/2} l_B \sum_{\mathbf{q}} \overline{\varphi}(\mathbf{q}) (iuq_+ - vq_-) \mathcal{Q}_{\mathbf{q}} + \text{h.c.} \quad (7)$$

(it is assumed here that  $q \ll 1$ ).  $\overline{\varphi}(\mathbf{q})$  is the Fourier component [i.e.  $\varphi = \sum_{\mathbf{q}} \overline{\varphi}(\mathbf{q}) e^{i\mathbf{q}\mathbf{r}}$ ], and  $q_\pm = \mp i(q_x \pm iq_y)/\sqrt{2}$ .

At variance with integer QHF, the use of the excitonic basis  $\mathcal{Q}_{ab\mathbf{q}}^\dagger|0\rangle$  presents only a *model approach* in the case of *fractional quantum Hall regime*. Generally, spin-flip excitations within the same Landau level might be many-particle rather than two-particle excitations at fractional filling because the same change of the spin numbers  $\delta S = \delta S_z = -1$  may be achieved with participation of arbitrary number of intra-spin-sublevel excitations (charge-density waves). These waves are generated by the operator  $\mathcal{A}_{\mathbf{q}}^\dagger = N_\phi^{-1/2} \mathcal{Q}_{aa\mathbf{q}}^\dagger$  acting on the

ground state  $|0\rangle = \overbrace{|\uparrow, \dots, \uparrow, \dots, \uparrow\rangle}^{\nu N_\phi}$  [12]. It is trivial in the case of integer  $\nu$  ( $\mathcal{A}_{\mathbf{q}}^\dagger|0\rangle = \delta_{\mathbf{q},0}|0\rangle$ ); however, states of the  $\mathcal{Q}_{ab\mathbf{q}_1}^\dagger \mathcal{A}_{\mathbf{q}_2}^\dagger \mathcal{A}_{\mathbf{q}_3}^\dagger \dots |0\rangle$  type might constitute a basis set if one studies a spin-flip at fractional  $\nu$ . On the other hand, a comprehensive phenomenological analysis [3, 12] suggests that even the spin-flip basis reduced to single-mode (single-exciton) states would be quite appropriate, at least for lowest-energy excitations in the case of fractional QHF. This single-mode approach is indirectly substantiated by the fact that the charge-density wave has a Coulomb gap [12] which is well larger than the Zeeman gap  $\epsilon_Z$ . Hence for a fractional QHF, just as in Ref. [3], we will consider the only state  $\mathcal{Q}_{ab\mathbf{q}}^\dagger|0\rangle$  to describe the spin-flip excitation. The commutation algebra for operators  $\mathcal{Q}_{ab\mathbf{q}}^\dagger$ ,  $\mathcal{A}_{\mathbf{q}}^\dagger$ , and  $\mathcal{B}_{\mathbf{q}'}^\dagger = N_\phi^{-1/2} \mathcal{Q}_{bb\mathbf{q}'}^\dagger$  is certainly the same as for integer filling [7–9]. A difference arises in the calculation of expectation  $\langle 0|\mathcal{A}_{\mathbf{q}}\mathcal{A}_{\mathbf{q}'}^\dagger|0\rangle$  which is needful for the following. This value is simply  $\delta_{\mathbf{q},0}\delta_{\mathbf{q}',0}$  at integer filling, but at  $\nu < 1$  it is expressed in terms of the two-particle correlation function  $g(r)$  calculated for the ground state:

$$\langle 0|\mathcal{A}_{\mathbf{q}}\mathcal{A}_{\mathbf{q}'}^\dagger|0\rangle = \frac{\nu}{N_\phi} \left[ 2\pi\nu\bar{g}(q)e^{q^2/2} + 1 \right] \delta_{\mathbf{q},\mathbf{q}'} \quad (8)$$

Here  $\bar{g}(q) = \frac{1}{(2\pi)^2} \int g(r)e^{-i\mathbf{q}r}d^2r$  is the Fourier component. Function  $g(r)$  is well known, e.g., in the case of Laughlin's state [12, 13]. If the ground state is presented in terms of the Hartree-Fock model, we get the expression  $2\pi\bar{g} = \left( N_\phi\delta_{q,0} - e^{-q^2/2} \right)$  which does not depend on  $\nu$ . Besides, at odd-integer filling factors this Hartree-Fock expression becomes Fourier component of the exact correlation function. In the latter case one should also formally set  $\nu = 1$  in Eq. (8).

**3.** The operator (7) obviously does not conserve the number of SEs. However, if the SWR is governed by this operator, the corresponding problem can not be solved in terms of a single-exciton study. Indeed, the SE interaction with the SRP incorporates the energy  $U_{x\text{-SRP}} \sim ql_B\Delta/\Lambda$  (the SE possesses the dipole momentum  $el_B[\mathbf{q} \times \hat{z}]$ ) [2]. The SE momentum is estimated from the condition  $\mathcal{E}_{\mathbf{q}} \lesssim T$ , and we therefore find that  $U_{x\text{-SRP}} \ll \epsilon_Z, T$ . Due to this inequality, the energy of annihilating exciton can not be transformed to anywhere. By analogy with Ref. [8], we study a coalescence process where initial double-exciton state  $|i\rangle = \mathcal{Q}_{ab\mathbf{q}_1}^\dagger \mathcal{Q}_{ab\mathbf{q}_2}^\dagger|0\rangle$  transforms to final single-exciton state  $|f\rangle = \mathcal{Q}_{ab\mathbf{q}'}^\dagger|0\rangle$  having the combined energy:

$$\epsilon_Z + \mathcal{E}_{\mathbf{q}'} = 2\epsilon_Z + \mathcal{E}_{\mathbf{q}_1} + \mathcal{E}_{\mathbf{q}_2} \quad (9)$$

(c.f. also the Auger magnetoplasma relaxation considered in Ref. [14]). At the same time, contrary to Ref. [8],

there is no momentum conservation in this SWR channel. Thus the phase volume where the  $X_{\mathbf{q}_1} + X_{\mathbf{q}_2} \rightarrow X_{\mathbf{q}'}$  transition is possible turns out to be much larger than that in the coalescence process of Ref. [8]. This transition is governed by the Fermi golden rule probability:  $w_{fi} = (2\pi/\hbar)|\mathcal{M}_{fi}|^2\delta(E_f - E_i)$ , and our immediate task is to calculate the matrix element  $\mathcal{M}_{fi} = \nu^{-3/2}\langle f|\hat{\varphi}|i\rangle$ . (The factor  $\nu^{-3/2}$  appears due to the normalization since norms of the  $|i\rangle$  and  $|f\rangle$  states are  $\nu^2$  and  $\nu$ , respectively.) The latter reduces to calculation of the expectation  $\langle 0|\mathcal{Q}_{ab\mathbf{q}'}\mathcal{Q}_{ab\mathbf{q}_1}\mathcal{Q}_{ab\mathbf{q}_2}^\dagger\mathcal{Q}_{ab\mathbf{q}_2}^\dagger|0\rangle$  [15].

We perform the  $\mathcal{M}_{fi}$  calculation for relevant values of momenta  $q_1, q_2, q' \ll 1$  which satisfy the conditions  $\mathcal{E}_{q_1}, \mathcal{E}_{q_2} \lesssim T \lesssim 1\text{K}$ . (These inequalities correspond to  $q_1, q_2, q' \ll 1/l_B$  in usual dimensional units.) By employing exciton-operators' commutation rules [7], Eq. (8) and evident identities  $\mathcal{Q}_{ab\mathbf{q}}|0\rangle \equiv \mathcal{B}_{\mathbf{q}}|0\rangle \equiv 0$  and  $\langle 0|\mathcal{A}_{\mathbf{q}}|0\rangle \equiv \nu\delta_{\mathbf{q},0}$ , we obtain with the help of Eqs. (7), (8) that

$$\begin{aligned} \mathcal{M}_{fi}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}') &\approx \frac{2\pi\nu^{1/2}}{N_\phi^{1/2}} [\bar{g}(|\mathbf{q}_1 - \mathbf{q}'|) + \bar{g}(|\mathbf{q}_2 - \mathbf{q}'|)] \times \\ &\times \sum_{\mathbf{q}} \bar{\varphi}(\mathbf{q}) (iuq_+ - \nu q_-) \delta_{\mathbf{q}_1 + \mathbf{q}_2, \mathbf{q} + \mathbf{q}'}. \end{aligned} \quad (10)$$

Besides, within our approximation,  $\bar{g}(q)$  should be replaced with  $\bar{g}(q)|_{q \rightarrow 0}$ . The latter quantity is equal to  $-1/2\pi$  in the Hartree-Fock approach or  $-1/2\pi\nu$  when calculated in the case of Laughlin's ground state describing the fractional QHF [12, 13]. So, for  $\nu = 1, 1/3, 1/5, \dots$  we obtain a simple result:

$$|\mathcal{M}_{fi}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}')|^2 = 4\pi\bar{K}(q) \frac{q^2(u^2 + v^2)}{\nu N_\phi^2} \Big|_{\mathbf{q} = \mathbf{q}_1 + \mathbf{q}_2 - \mathbf{q}'} \quad (11)$$

It is used here that the squared modulus of  $\bar{\varphi}(\mathbf{q})$  at  $\mathbf{q} \neq 0$  is expressed in terms of Fourier component of the correlator (4):  $|\bar{\varphi}(\mathbf{q})|^2 = 2\pi K(q)/N_\phi$ . In the Hartree-Fock model the expression (11) should be multiplied by  $\nu^2$ ; hence the calculated relaxation rate would be by a factor of  $\nu^2$  slower. Notice also that if  $\nu = 3, 5, \dots$ , one should formally set  $\nu = 1$  in Eqs. (10), (11).

The SWR rate is defined as the difference between the fluxes of annihilating and created SEs. We assume that the thermodynamic equilibrium in the system of spin waves is established much faster than the spin-flip processes occur, so that the rate is

$$\begin{aligned} \frac{dN_{\mathbf{x}}}{dt} &= \frac{1}{2} \sum_{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}'} \frac{2\pi}{\hbar} |\mathcal{M}_{fi}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}')|^2 \delta(E_1 + E_2 - E') \times \\ &\times [n_1 n_2 (1 + n') - n' (1 + n_1) (1 + n_2)]. \end{aligned} \quad (12)$$

The notations used here are  $E_i = \epsilon_Z + \mathcal{E}_{q_i}$ ,  $n_i = n(E_i)$  ( $i = 1, 2$ ) and  $E' = \epsilon_Z + \mathcal{E}_{q'}$ ,  $n' = n(E')$ , where the Bose distribution function is  $n(E) = 1/(e^{(E-\mu)/T} - 1)$ . The total number of SWs in the system is  $N_x = \sum_{\mathbf{q}} n(\epsilon_Z + \mathcal{E}_{\mathbf{q}})$ . Ratio of the SW number and the total spin is  $N_x(\mu) = \nu N_\phi/2 - S$ . This thus determines chemical potential  $\mu$  as a function of the nonequilibrium value  $S$ . In our case, when temperature is rather low, we can certainly use quadratic approximation for the “kinetic” exciton energy:  $\mathcal{E}_q \approx q^2/2M_x$ . As a result, the rate  $dN_x/dt$  is completely determined by Eqs. (11), (12) and is a function of parameters  $B$ ,  $T$  and nonequilibrium spin number  $S = S_z$ . Calculating the quantity  $N_x^{(0)} = N_x|_{\mu=0}$ , one obtains the equilibrium number of excitons. We will find the rate at the final stage of the relaxation process where  $N_x - N_x^{(0)} \ll N_x^{(0)}$ . So, by employing the quadratic approximation for the SE kinetic energy, and changing in Eqs. (11), (12) from summations to integrations we obtain  $dN_x/dt = (N_x - N_x^{(0)})/\tau_{\text{SRP}}$ , where

$$1/\tau_{\text{SRP}} = \frac{(u^2 + v^2)M_x^3}{2\nu\pi\hbar} \left( \frac{\Delta\Lambda T}{l_B} \right)^2 \times \\ \times \left( e^{-\epsilon_Z/T} - e^{-2\epsilon_Z/T} \right) F_{\text{SRP}}(\Lambda^2 M_x T/l_B^2, \epsilon_Z/T). \quad (13)$$

Here  $F_{\text{SRP}}(\alpha, \beta)$  is a dimensionless function arising as a result of integrations over  $q_1$  and  $q_2$  and averaging over angles  $\theta_1 = \mathbf{q}_1 \wedge \mathbf{q}'$  and  $\theta_2 = \mathbf{q}_2 \wedge \mathbf{q}'$ :

$$F_{\text{SRP}}(\alpha, \beta) = \int_0^\infty \int_0^\infty \frac{e^{-x-y} dx dy}{(1-e^{-x-\beta})(1-e^{-y-\beta})(1-e^{-x-y-2\beta})} \\ \times \int_{-\pi}^\pi d\theta_1 \int_{-\pi}^\pi d\theta_2 r(x, y, \theta_1, \theta_2) \exp[-\alpha r(x, y, \theta_1, \theta_2)],$$

where  $r(x, y, \theta_1, \theta_2) = x + y + \beta/2 - \sqrt{x+y+\beta} \times (\sqrt{x} \cos \theta_1 + \sqrt{y} \cos \theta_2) + \sqrt{xy} \cos(\theta_1 - \theta_2)$ .

4. Now we calculate the numerical value of  $1/\tau_{\text{SRP}}$  at typical SRP parameters and compare it with inverse relaxation times  $1/\tau_{e-e}$  and  $1/\tau_{ph}$  governed by the inter-SEs' interaction mechanism [8] and the SE-acoustic-phonon coupling [7]. We carry out this analysis for the  $\nu = 1$  QHF assuming that  $\Delta = 10$  K and  $\Lambda = 40$  nm. The Zeeman splitting at  $g = -0.44$  is  $\epsilon_Z = 0.295BK$  ( $B$  is everywhere in Teslas), and the combination of SO parameters is estimated as  $u^2 + v^2 = 10^{-3}/B$ . The SE mass  $M_x$  might be calculated theoretically by using general expressions for  $\mathcal{E}_q$  [2, 3]. Yet, the result depends on specific formfactor inherent in a given heterostructure due to finite thickness and it is therefore more convenient to extract  $M_x$  immediately from experiments. According to recent data available for currently used wide quantum wells [16], we estimate that

$1/M_x = 9.24\sqrt{B}$  K. Using Eq. (13), we thus calculate  $1/\tau_{\text{SRP}}$  as a function of temperature  $T$  at given field  $B$ . The results are presented in Fig.1 by dash curves. The dot and dash-dot curves correspond to the  $1/\tau_{e-e}$  and  $1/\tau_{ph}$  values given by formulas

$$1/\tau_{e-e} = \frac{2}{\hbar}(u^2 + v^2)T \left( e^{-\epsilon_Z/T} - e^{-2\epsilon_Z/T} \right) F_{e-e}(\epsilon_Z/T), \quad (14)$$

where

$$F_{e-e}(\beta) = \iint_{xy > \beta^2/4} \frac{dx dy (x+y+\beta)e^{-x-y}}{(xy-\beta^2/4)^{1/2}(1-e^{-\beta-x})(1-e^{-\beta-y})(1-e^{-2\beta-x-\beta})},$$

and

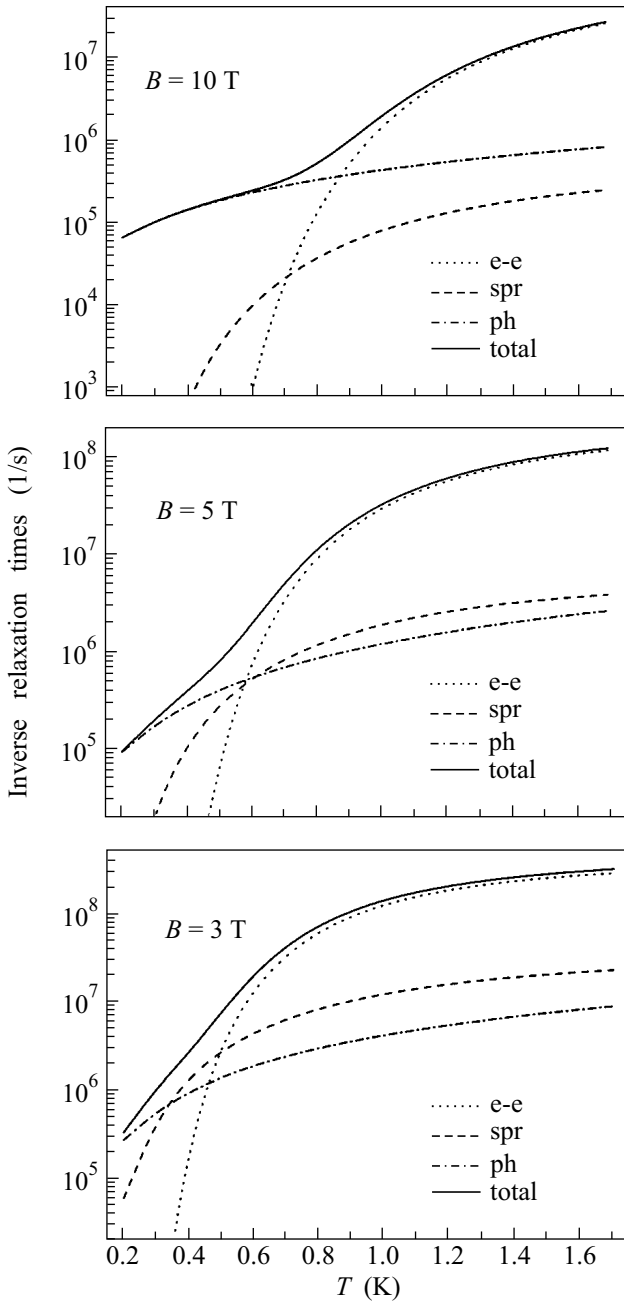
$$\tau_{ph}^{-1} = \frac{MT\epsilon_Z(u^2 + v^2)}{\hbar c_s p_0^2 l_B^2} \left[ \frac{\gamma_1(\epsilon_Z/T)}{\tau_D} + 10 \frac{MT}{\tau_P} \left( \frac{\hbar c_s}{\epsilon_Z} \right)^4 \left( \frac{p_0}{l_B} \right)^2 \gamma_2(\epsilon_Z/T) \right], \quad (15)$$

where

$$\gamma_k(\beta) = (e^{2\beta} - e^\beta) \int_0^\infty \frac{e^x x^k dx}{(e^{\beta+x} - 1)^2}, \quad k = 1, 2.$$

(See Ref. [7, 8, 17]; the used material parameters characterizing the electron-phonon coupling are  $c_s = 5.14 \cdot 10^5$  cm/s,  $\tau_D = 0.8 \cdot 10^{-12}$  s $^{-1}$ ,  $\tau_P = 35 \cdot 10^{-12}$  s $^{-1}$ , and  $p_0 = 2.52 \cdot 10^6$  cm $^{-1}$ ; both kinds of  $e-ph$  interaction, deformation and polarization ones, are taken into account.)

It is seen from Figure that the SRP relaxation channel actually competes with other mechanisms in the experimentally relevant range of parameters: namely, at fields  $B \leq 5$  and temperatures  $T \sim 0.3 - 0.5$  K. We have indicated above that the basic advantage of the SRP channel, as compared to the  $e-e$  one, consist in the absence of momentum conservation in the coalescence process. On the other hand, the SRP mechanisms is also determined by effective SE-SE collisions. Therefore the inverse relaxation time is proportional to the SE concentration and drops exponentially as  $\sim \exp(-\epsilon_Z/T)$  with vanishing  $T$  [rather than as  $\sim \exp(-2\epsilon_Z/T)$  which occurs for the  $e-e$  mechanism due to the SEs' momentum conservation]. The phonon mechanism of SWR dominates at low temperatures due to its weak temperature dependence ( $\sim T$ ), in spite of small value of the electron-phonon coupling constant in GaAs. The dependence on the filling factor in the case of integer QHF is only determined by the SE mass  $M_x$  because  $\nu$  in Eq. (13) is formally set equal to unit. For fractional QHF there are both direct and indirect (through the mass  $M_x$ ) dependences on  $\nu$ .



Inverse SWR times against  $T$  calculated by using formulas (13)-(15) at  $B = 3, 5, 10$  T. Specific material parameters are given in the text. Dash, dot, and dash-dot lines are for  $1/\tau_{\text{srp}}$ ,  $1/\tau_{e-e}$  and  $1/\tau_{ph}$ , respectively. Solid lines present the result of calculation of the combined inverse time (16)

Finally we calculate the combined inverse relaxation time determined by the SO coupling:

$$1/\tau_{\text{tot}} = 1/\tau_{\text{srp}} + 1/\tau_{e-e} + 1/\tau_{ph}. \quad (16)$$

The result is presented by solid curves in Figure. It is worth mentioning that it demonstrates a good agreement with the measured value  $\tau_{\text{tot}} \simeq 10$  ns of Ref. [5]

when calculated for parameters  $B$  and  $T$  corresponding to the experiment.

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15. If  $Q$ -operators were usual Bose operators, this expectation would simply be equal to  $\delta_{\mathbf{q}', \mathbf{q}_1} \delta_{\mathbf{q}, \mathbf{q}_2} + \delta_{\mathbf{q}', \mathbf{q}_2} \delta_{\mathbf{q}, \mathbf{q}_1}$  resulting in violation of the energy conservation condition (9). Hence the studied relaxation channel exists only due to non-Bose quantum nature of spin excitons: the  $Q$ -operators obey their own specific commutation rules which are different from both Bose and Fermi commutation rules [7–9, 14]. The SRP thus provides a *kinematic* SE-SE interaction determined by specific commutation rules but irrelevant to the real interaction of spin waves considered in Ref. [8]. On the other hand, SEs still obey the Bose statistics because a large number of excitons may occupy the same state, i.e. may have the same momentum.
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17. The formula for inverse relaxation time in Ref. [8] contains a misprint. Now we present the corrected result in Eq. (14).