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The possibility of Z(4430) resonance structure description in $\pi\psi'$ reaction

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The possible description of Z(4430) as a pseudoresonance structure in $\pi\psi'$ reaction, is considered. The analysis is performed with single-scattering contribution to $\pi\psi'$ elastic scattering via $D^*D_1(2420)$ intermediate energy.

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1. Introduction. The resonance structure with mass $M = 4433 \pm 4 \pm 2\,\text{MeV}$ and width $\Gamma = 45^{+18+30}_{-13-13}$ MeV in the charged quarkonium system $\pi^{\pm}\psi'$ was found by the Belle Collaboration [1]. On the other hand, BABAR Collaboration [2] did not see significant evidence for a $Z(4430)^-$ signal in any of the processes investigated, neither in the total $J/\psi\pi^-$ or $\psi(2S)\pi^-$ mass distribution, nor in the corresponding distributions for the regions of $K\pi^-$ mass for which observation of the $Z(4430)^-$ signal was reported. Several mechanisms have been proposed to explain the properties of the new resonance [3 – 6]. In particular, Rosner [3] pointed out to the close by threshold of $D_1(2420)\bar{D}^*(2010)$ state and suggested a mechanism of production of $\pi\psi'$ in the decay $B \to KZ(4430)$, $Z(4430) \to \pi^+\psi'$. The proximity of the threshold invokes a possible near-threshold singularity, either due to a pole of the amplitude (virtual or real loosely coupled bound state of D_1D^*) [3 – 5] or else due to the threshold cusp [6].

In this letter we are trying to understand whether the Z(4430) resonance can be due to pseudoresonance mechanism known for πd system [7]. We analyze the structure of the scattering amplitude for the reaction $\pi\psi' \to \pi\psi'$ near the D_1D^* threshold in the same way as was done for πd system near the $\triangle N$ resonance. It is well known that the peak in the cross section for pion-nucleon (πN) scattering around $T_\pi = 180\,\text{MeV}$ is associated with the $\triangle(1232)$. An analogous peak is observed in the cross section for pion-deuteron (πd) scattering near $\triangle N$ threshold shifted slightly in position and broadened with respect to the πN peak (see Fig.1). Therefore, one can not exclude that the Z(4430) resonance, which lies near D_1D^* threshold could be connected to the $D_1(2420)$ resonance, as it takes place in the $\triangle(1232)$. The $D_1(2420)$ state with mass $M = 2420^{+1+2}_{-2-2}$ MeV and width $\Gamma = 20^{+6+3}_{-5-3}$ MeV was observed in $D^{*\pm}(2010)\pi^{\mp}$ invariant distribution. Therefore the dynamical picture of the pion charmonium scattering in our approach is: the p-wave off-energy-shell charmonium decay to $D^*\bar{D}^*$, then in the πD^* scattering the creation of $D_1(2420)$ resonance. The diagram corresponding to this reaction is shown in Fig.2.

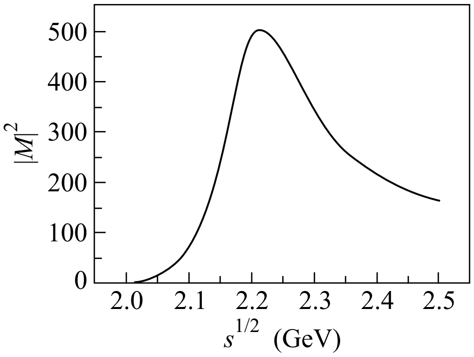

Fig.1. The squared πd scattering amplitude

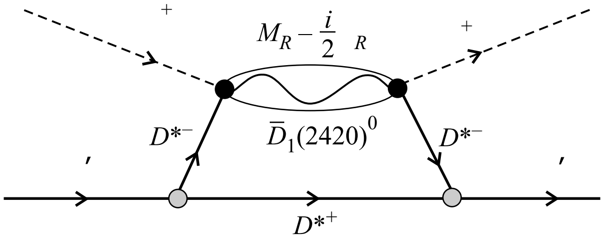

Fig.2. Representation of the single scattering $\pi\psi'$ diagram

In our paper, firstly, we calculate the scattering amplitude for πd system using a single Breit-Wigner resonance for $\triangle(1232)$ and obtain a good description of $\triangle N$ resonance. Then we apply the same formulas to

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$\pi\psi'$ scattering in which the vertex of the $\psi' \to D^*\bar{D}^*$ decay is calculated in the many channel formalism developed in [8]. For simplicity reasons we didn't include rescattering terms which slightly shift the peak in the πd case.

We pay a special attention to the influence of different properties of deuteron and charmonium family. First of all its different size: the deuteron is a large object with size $R_d \sim 4.3$ fm while charmonium ψ' state has only $R_{\psi'} \sim 0.5$ fm. The analysis of the results and discussion are given in the last section.

2. The amplitude for πd system. For the sake of simplicity we neglect any spin dependence and write the single-scattering non-relativistic term for the πd amplitude:

$$M(\mathbf{k}',\mathbf{k}) = \\ = \int \frac{d^3p}{(2\pi)^3}\, \phi^*(\mathbf{p} - \frac{1}{2}\mathbf{k}'), M_{\pi N}(\mathbf{x}',\mathbf{x},W_1)\, \phi(\mathbf{p} - \frac{1}{2}\mathbf{k}), \quad (1)$$

where $\sqrt{s} = \sqrt{\mathbf{k}^2 + m_\pi^2} + \sqrt{\mathbf{k}^2 + m_d^2}$ is the total invariant energy of the πd system. In (1) the πN amplitude depends on

$$\mathbf{x} = \mathbf{k} - \eta(p)\mathbf{p} \quad \mathbf{x}' = \mathbf{k}' - \eta(p)\mathbf{p},$$
$$\eta(p) = \frac{\sqrt{\mathbf{p}^2 + m_\pi^2}}{\sqrt{\mathbf{p}^2 + m_\pi^2} + m_N},$$

and on the total invariant πN energy

$$W_1 = \sqrt{\left(\sqrt{s} - \sqrt{\mathbf{p}^2 + m_N^2}\right)^2 - \mathbf{p}^2}. \quad (2)$$

The πN amplitude will be truncated to include only the dominant resonance p wave in the following way:

$$M_{\pi N} = \frac{64}{3}\pi W_1 \mathbf{x}'\,\mathbf{x}\left(-\frac{\Gamma_R}{2q}\right)\frac{1}{W_1 - M_R + \frac{1}{2}i\Gamma_R}, \quad (3)$$

with momentum q of the πN system

$$q = \sqrt{\frac{(W_1^2 - (m_N^2 + m_\pi^2)^2)(W_1^2 - (m_N^2 - m_\pi^2)^2)}{4W_1^2}}. \quad (4)$$

The deuteron wave function contains the deuteron pole:

$$\phi(\mathbf{p}) = \frac{\sqrt{\alpha}}{(p^2 + \alpha^2)(p^2 + c^2)}, \quad (5)$$

with $\alpha = \sqrt{m_N \varepsilon_D}$, ε_D being the deuteron binding energy and $c \approx 0.4\,\text{GeV}$.

One can see in Fig.1 that the forward scattering ($k = k'$) amplitude has a quite good resonance form which agrees with the experiment result (see, for example [9]).

3. The amplitude for $\pi\psi'$ system. The $\phi(\mathbf{p})$ in (1) for $\pi\psi'$ system includes the propogator and overlapped integral of the process $\psi' \to D^*\bar{D}^*$:

$$\phi(\mathbf{p}) = \frac{J(\mathbf{p})M_\omega}{E_{\psi'} - E_{D^*} - E_{\bar{D}^*}} = \frac{J(\mathbf{p})M_\omega}{M_{\psi'} - 2M_{D^*} - p^2/M_{D^*}}, \quad (6)$$

where $J(\mathbf{p})$ is an overlapped matrix element between wave functions $\Psi(nS)$ of the n-th charmonium state and $\psi(1S)$ of $D^*(\bar{D}^*)$ mesons states, which were derived in the framework of many-channel formalism with decay channel coupling [8]:

$$J(\mathbf{p}) = \int \bar{y}_{123}\frac{d^3q}{(2\pi)^3}\Psi(nS; c\mathbf{p} + \mathbf{q})\psi(1S;\mathbf{q})\psi(1S;\mathbf{q}) \quad (7)$$

here $c = \Omega/(\Omega + \omega)$ (Ω, ω is the energy of heavy and light quarks in D^* meson); $\bar{y}_{123}$ is defined by the Dirac traces of the amplitude given in appendix. In eq. (7) $\Psi(nS), \psi(1S)$ are a series of oscillator wave functions which are fitted to realistic wave functions. We obtain them from the solution of the Relativistic String Hamiltonian, described in [10, 11].

Fig.3 shows the squared $\pi\psi'$ scattering amplitude averaging over vector polarization $\frac{1}{3}\sum_{ii'}|M|^2$. As can be seen, the structure has a too large width and a peak located near energy $\sqrt{s} \sim 4.7\,\text{GeV}$ and cannot be associated with Z(4430).

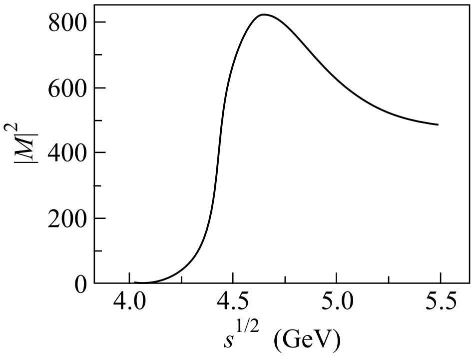

Fig.3. The squared $\pi\psi'$ scattering amplitude

4. Discussion. An important distinction between πd and $\pi\psi'$ is the difference in the deuteron and charmonium sizes, p wave decay ψ' to $D^*\bar{D}^*$ is also significant. It is interesting that we can obtain a desirable resonance structure if ψ' has admixture of the near-threshold state with size $R \sim 5$ fm due to the coupling to the $D^*\bar{D}^*$ channel. In this case the width turns out to be smaller $\Gamma \sim 60$ MeV and the peak is shifted to the position $\sqrt{s} \sim 4.5\,\text{GeV}$. This result is shown in Fig.4.

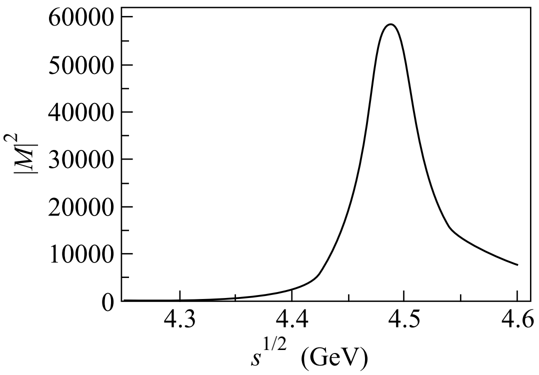

Fig.4. The squared $\pi\psi'$ scattering amplitude in case of large charmonium size

In our paper we have used dynamical picture of pion interaction with heavy quarkonia corresponding to the diagram in Figure 2. Our analysis shows that there is no resonance near $\sqrt{s} \sim 4430$ energy in the $\psi'\pi$ system, unless an admixture of large size near-the-threshold state is taken into account.

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Appendix

The vertex factor $\bar{y}_{123} = \bar{Z}/\sqrt{\bar{Z}_1\bar{Z}_2\bar{Z}_3}$ is calculated in the same way as in [8], namely from the Dirac trace of the projection operators for the decay process, in our case this is $\psi(nS) \to D^*\bar{D}^*$. Identify the creation operators as $\bar{\psi}_c\gamma_i\psi_c$, $\bar{\psi}_c\gamma_j\psi_d$, $\bar{\psi}_c\gamma_k\psi_d$ one has for the decay process

$$\bar{Z} = tr(\gamma_i\Lambda^+\gamma_j\Lambda^-\Lambda^+\gamma_k\Lambda^-) \tag{8}$$

with the projection operators $\Lambda^{\pm} = (m_k \pm \omega_k\gamma_4 \mp \mp ip_i^{(k)}\gamma_i)/2\omega_k$, $k = c, d$. Here ω_k is the average energy of quark in given meson ($\Omega = 1.5\,\mathrm{GeV}$, $\omega = 0.55\,\mathrm{GeV}$), m_k is the pole mass of c and d quarks ($m_c = 1.4$, $m_d \approx 0$). One can identifying the momenta of $q, \bar{q}, Q, \bar{Q}$ as in [8]:

$$\mathbf{p}_{\bar{q}} = -\mathbf{q}_1 + \frac{\omega}{\omega+\Omega}\mathbf{p}, \quad \mathbf{p}_q = -\mathbf{q}_2 - \frac{\omega}{\omega+\Omega}\mathbf{p}, \tag{9}$$
$$\mathbf{p}_Q = \mathbf{p} - \mathbf{p}_{\bar{q}}, \quad \mathbf{p}_{\bar{Q}} = -\mathbf{p} - \mathbf{p}_q.$$

Finally one obtains from (8), taking into account that $\mathbf{q}_2 = -\mathbf{q}_1 \equiv -\mathbf{q}$

$$\bar{Z} = \frac{8im_Q}{16\omega^2\Omega^2}\left\{2\omega\Omega\left(\frac{p_k\omega}{\omega+\Omega} - q_k\right)\delta_{\mathbf{ij}} + \right.$$
$$+ 2\omega\Omega\left(\frac{p_j\omega}{\omega+\Omega} - q_j\right)\delta_{\mathbf{ik}} +$$
$$+\left(p_i\left(-\frac{\Omega\omega^2}{\omega+\Omega} - \frac{p^2\Omega\omega^2}{(\omega+\Omega)^3} + \frac{2\Omega(p\cdot q)\omega}{(\omega+\Omega)^2} - \frac{q^2\Omega}{\omega+\Omega}\right) + \right.$$
$$\left.\left. + q_i\left(\omega^2 + 2\omega\Omega - q^2 + \frac{2\omega p\cdot q}{\omega+\Omega} - \frac{p^2\omega^2}{(\omega+\Omega)^2}\right)\right)\delta_{\mathbf{jk}}\right\}. \tag{10}$$

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1. K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. **100**, 142001 (2008); arXiv:0708.1790 [hep-ex].
2. BABAR Collaboration, arXiv:0811.0564 [hep-ex].
3. J.L. Rosner, Phys. Rev. D **76**, 114002 (2007); arXiv:0708.3496 [hep-ph].
4. L. Maiani, A.D. Polosa, and V. Riquer, arXiv:0708.3997 [hep-ph].
5. C. Meng and K.T. Chao, arXiv:0708.4222 [hep-ph].
6. D.V. Bugg, arXiv:0709.1254 [hep-ph].
7. Yu.A. Simonov and M. van der Velde, J. Phys. G **5**, 493 (1979).
8. Yu.A. Simonov and A.I. Veselov, arXiv:0804.4635 [hep-ph] (to be published in Phys. Rev. D); Yu.A. Simonov, Phys. Atom. Nucl. **71**, 1048 (2008); arXiv:0711.3626 [hep-ph].
9. C.H. Oh, R.A. Arndt, I.I. Strakovsky, and R.L. Workman, Phys. Rev. C **56**, 635 (1997); arXiv:nucl-th/9702006; A.W. Thomas and R.H. Landau, Phys. Rept. **58**, 121 (1980).
10. A.Y. Dubin, A.B. Kaidalov, and Yu.A. Simonov, Phys. Atom. Nucl. **56**, 1745 (1993) [Yad. Fiz. **56**, 213 (1993)].
11. A.M. Badalian and I.V. Danilkin, arXiv:0801.1614 [hep-ph](to be published in Yad. Fiz.).