

Particle decay in de Sitter spacetime via quantum tunneling

G. E. Volovik¹⁾

Low Temperature Laboratory, Helsinki University of Technology, FIN-02015, HUT, Finland

Landau Institute for Theoretical Physics RAS, 119334 Moscow, Russia

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The quantum tunneling process of decay of the composite particle in the de Sitter vacuum looks as thermal radiation with the effective temperature twice larger than the Hawking temperature associated with the cosmological horizon.

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1. *Introduction.* Both the black hole horizon and the cosmological horizon are described by the so-called “fluid” metric which is characterized by the “velocity” field \mathbf{v} [1]:

$$ds^2 = g_{\mu\nu} x^\mu x_\nu = -dt^2 + (d\mathbf{r} - \mathbf{v}dt)^2, \quad (1)$$

where we used the units with $c = 1$. The de Sitter spacetime is characterized by the radial velocity field

$$\mathbf{v}(\mathbf{r}) = v(r)\hat{\mathbf{r}}, \quad v(r) = Hr = r/r_H, \quad (2)$$

where $r_H = 1/H$ is the radius of cosmological horizon. The “fluid” metric for black hole at the end of the gravitational collapse is the Painleve–Gullstrand metric [2], which corresponds to the radial flow field in the form:

$$\mathbf{v}(\mathbf{r}) = v(r)\hat{\mathbf{r}}, \quad v(r) = -\sqrt{\frac{r_H}{r}}, \quad (3)$$

where r_H is the radius of the black hole horizon. The “fluid” metric is best suited for the derivation of the Hawking radiation using the semiclassical tunneling picture [3, 4], because this metric is stationary and thus the energy is well defined. The classical energy spectrum of a particle with mass m in the “fluid” spacetime is given by

$$E(\mathbf{p}, \mathbf{r}) = \sqrt{m^2 + p^2} + \mathbf{p} \cdot \mathbf{v}(\mathbf{r}), \quad (4)$$

where the first term is the spectrum in the frame “co-moving with the vacuum”, while the last term plays the role of the Doppler frequency shift. The tunneling probability is obtained from the imaginary part of the action along the semiclassical trajectory

$$w = w_0 \exp(-2\mathbf{Im} S). \quad (5)$$

The radial trajectory $p_r(r)$ is obtained from the energy conservation along the trajectory:

$$E(p_r, r) = \sqrt{m^2 + p_r^2} + p_r v(r) = E, \quad (6)$$

which gives the tunneling exponent

$$2\mathbf{Im} S = 2\mathbf{Im} \int dr p_r(r) = \frac{2\pi E}{|dv/dr|_{r=r_H}}. \quad (7)$$

The quantum tunneling thus simulates the thermal radiation from the horizon with Hawking temperature

$$T_H = \frac{\hbar}{2\pi} \left| \frac{dv}{dr} \right|_{r=r_H}. \quad (8)$$

For the de Sitter Universe with its $v(r) = Hr$, the corresponding temperature would be

$$T_H^{\text{dS}} = \hbar H / 2\pi. \quad (9)$$

However, in this semiclassical description the prefactor w_0 in (5) remains unknown, and there are arguments that the symmetry of the de Sitter background nullifies the prefactor [5, 6]. For a discussion of the controversies concerning the stability of de-Sitter vacuum towards the Hawking radiation, see e.g. Refs. [9–14].

While the de Sitter vacuum may be stable, the particles living in the de Sitter environment are certainly not [15]. This is because the mass of particle is not well defined in the de Sitter background. Calculations of the decay rate of the composite particles have been done in Refs. [16–18]. Let us stress, that contrary to conclusion made in Ref. [19], we argue that the possibility of massive free falling particles to radiate other massive particles does not mean that the de Sitter space cannot exist eternally. In the presence of an external body (detector or composite particle), the radiation occurs which takes the energy from the body. But the pure de Sitter vacuum (i.e. without any impurity) may be stable.

¹⁾e-mail: volovik@boo.jum.hut.fi

Here we use the semiclassical tunneling picture for the calculation of the decay rate and make comparison with the Hawking radiation.

2. Ionization rate and Hawking temperature.

We consider two examples of the radiation caused by the presence of external object in the de Sitter vacuum: ionization of an atom caused by the de Sitter expansion discussed in [5], and the decay of the composite particle into two particles in the de Sitter background discussed in Refs. [16–18]. The atom (or any other composite or massive particle) plays two roles: it serves as the detector of radiation; and it violates the de Sitter symmetry and provides the nonzero matrix element for the radiation, since as we argue the pure de Sitter vacuum is not radiating due its symmetry.

Let us start with an atom [5], which is at rest in the comoving reference frame. In the reference frame of the atom its position is at the origin, $r = 0$. The electron bounded to an atom absorbs the energy from the gravitational field of the de Sitter background, which is sufficient to escape from the electric potential barrier that originally confined it. If the electron is originally sitting at the energy level E_n , then the ionization potential $\epsilon_0 = -E_n$. If the ionization potential is much smaller than the electron mass, $\epsilon_0 \ll m$, one can use the non-relativistic quantum mechanics to estimate the tunneling rate through the barrier. The corresponding radial trajectory $p_r(r)$ is obtained from the classical equation $E(p_r, r) = -\epsilon_0$, which in the non-relativistic approximation reads

$$-\epsilon_0 = p_r^2(r)/2m + p_r(r)Hr. \quad (10)$$

Here p_r is the radial momentum of electron, and the last term is the Doppler shift $p_r v(r)$ in Eq. (4) provided by the de Sitter expansion (2). This gives the following radial trajectory of electron:

$$p_r(r) = -mHr + \sqrt{m^2 H^2 r^2 - 2m\epsilon_0}. \quad (11)$$

The sign in front of the square root is chosen such that it corresponds to the flux from the center, i.e. the radial velocity of the particle $u_r = dE/dp_r = p_r/m + Hr$ is positive in the classically allowed region $r > r_0$, where

$$r_0^2 = 2\epsilon_0/mH^2. \quad (12)$$

The momentum p_r is imaginary in the classically forbidden region $0 < r < r_0$, which demonstrates that there is an energy barrier between the position of the electron in the atom, i.e. at $r = 0$, and the position of the free electron with the same energy at $r = r_0$. Since we assume that $\epsilon_0 \ll m$, one has $r_0 \ll r_H = 1/H$, which means that tunneling occurs well within the horizon. We

also assume that $H \ll \epsilon_0(\epsilon_0/m)^{1/2}\alpha^{-1}$, which allows us to neglect the region close to the origin where the contribution of the Coulomb potential $-\alpha/r$ to Eq.(10) is important.

The imaginary part of the action

$$\mathbf{Im} \int dr p_r(r) = mH \int_0^{r_0} dr \sqrt{r_0^2 - r^2} = \frac{\pi\epsilon_0}{2H}, \quad (13)$$

gives the probability of ionization

$$w \propto \exp(-2\mathbf{Im} S) = \exp(-\pi\epsilon_0/H). \quad (14)$$

The quantum tunneling of the electron in the gravitational field of the de Sitter spacetime thus simulates the thermal activation of an atom by a heat bath with effective temperature T , which is twice larger than the corresponding Hawking temperature in (9):

$$w \propto \exp(-\epsilon_0/T), \quad T = \hbar H/\pi = 2T_H^{\text{dS}}. \quad (15)$$

3. Decay rate of composite particle and Hawking temperature. The same result is obtained in Refs. [16–18], whose authors considered the decay of a composite particle with mass m_0 into two particles, each with mass $m_1 > m_0/2$. Such decay is energetically forbidden in the Minkowski spacetime, but is allowed in the de Sitter background. It is instructive to derive the results of Refs. [16–18] using also the semiclassical tunneling picture. The trajectory of each of the two particles with mass m_1 moving in the opposite radial directions from the origin at $r = 0$ is obtained from equation

$$E(p_r, r) = \sqrt{p_r^2 + m_1^2} + p_r Hr = \frac{m_0}{2}. \quad (16)$$

We took into account that each of the two particles carries the one half of the energy of the original particle, i.e. $E = m_0/2$. The momentum along the trajectory is

$$p_r(r) = \frac{1}{1 - H^2 r^2} \left[-\frac{m_0}{2} Hr + \sqrt{\frac{m_0^2}{4} - m_1^2 + m_1^2 H^2 r^2} \right]. \quad (17)$$

Here again we choose the sign in front of the square root, which in the classically allowed region at $r > r_0$, where $r_0 = \sqrt{1 - m_0^2/4m_1^2}/H$, corresponds to the classical motion from the center. The momentum is imaginary in the classically forbidden region $r < r_0$, which gives the imaginary contribution to the action:

$$\mathbf{Im} \int dr p_r(r) = \frac{m_1}{H} \int_0^{r_0} dr \frac{\sqrt{r_0^2 - r^2}}{r_H^2 - r^2} = \frac{\pi}{4}(2m_1 - m_0), \quad (18)$$

where as before $r_H = 1/H$ is the position of the de Sitter horizon. We must take into account that due to momentum conservation, the two particles tunnel simultaneously in opposite directions (which is called co-tunneling). This adds extra factor two in the exponent. As a result one obtains the decay rate:

$$w \propto \exp(-4\text{Im } S) = \exp\left(-\frac{\pi(2m_1 - m_0)}{H}\right). \quad (19)$$

This looks as thermal activation by a heat bath with the temperature which is again twice larger than the corresponding Hawking temperature in (9):

$$w \propto \exp\left(-\frac{\pi\Delta m}{H}\right) = \exp\left(-\frac{\Delta m}{T}\right), \quad T = \frac{\hbar H}{\pi} = 2T_H^{\text{dS}}. \quad (20)$$

Here Δm is the mass deficit. It is the analog of the ionization potential ϵ_0 in (14). In the case of the decay of the particle with mass m_0 into two particles with masses m_1 , the mass deficit is $\Delta m = 2m_1 - m_0 > 0$.

4. Discussion. The decay rate calculated using the semiclassical method reproduces exact result obtained in Refs. [16–18] (note that there is misprint in Eq.(16) of Ref. [18]: the factor π has been omitted in the exponent). Both approaches demonstrate that the effective temperature which characterizes the decay rate of composite particles in de Sitter space is twice larger than the Hawking temperature of the de Sitter horizon. The controversies concerning the factor of 2 for the Hawking and Unruh temperatures can be found in Refs. [20–23] and references therein. However, the same semiclassical method applied to the black hole radiation [3] gives rise to the correct factor in the Hawking temperature. In the case of the Unruh effect [24], the tunneling approach is different because of the time dependent potential [25], but it also gives the correct factor for the Unruh temperature.

It is important that the effective temperature (15) has nothing to do with the existence of the cosmological horizon, since both for the atom and for the decaying particle the energy barrier is situated within the horizon: $r_0 < r_H = 1/H$. Moreover, the residue of the pole at $r = r_H$ in Eq. (17) vanishes. That is why the possible subtleties, which may influence the semiclassical tunneling approach in the presence of horizon and restore the ‘correct’ factor [20–23], are irrelevant here. The extra factor of 2 appears in some calculations of the Hawking temperature, when people use the Schwarzschild static coordinates in the tunneling method (see also discussion in Ref. [26]). The Schwarzschild coordinates are not well suited for calculations, since they have coordinate

singularity at the horizon and do not describe the interior of the black hole. The Painlevé-Gullstrand ‘fluid’ metric used in Refs. [3, 4] and in the present paper does not suffer from such drawbacks. That is why the extra factor 2 which appears in the decay of composite particle is not the artefact of the wrong coordinates.

It is interesting that the ‘correct’ factor in Eq. (20) may be restored in the limit of the vanishingly small mass of the decaying particle: $m_0 \ll m_1$. In this case, the equation (19) becomes

$$w \propto \exp\left(-\frac{2\pi m_1}{H}\right) = \exp\left(-\frac{m_1}{T_H^{\text{dS}}}\right). \quad (21)$$

The presence of the cosmological horizon does become important in this case, since in the limit $m_0/m_1 \rightarrow 0$ the position r_0 , to which the particle with mass m_1 is tunneling, approaches the horizon: $r_0 \rightarrow r_H$ when $m_0/m_1 \rightarrow 0$. One may argue that the limit, when the mass m_0 of the decaying particle approaches zero, formally corresponds to the creation of the pair of particles with mass m_1 from the vacuum; and this corresponds to the Hawking radiation from the de Sitter vacuum.

However, this is not exactly true. The presence of the original particle is necessary for the radiation, otherwise the matrix element $\langle m_0 | \mathcal{H} | m_1, m_1 \rangle$, which is needed for the decay of the original particle [16, 17, 18], drops out. The presence of the original particle, even with zero mass, violates the symmetry of the de Sitter vacuum, and the radiation becomes possible (see discussion in Ref. [5]). Also one should not forget that the factor 2π in (21) appears simply because two particles with masses m_1 tunnel simultaneously, that is why the effective activation temperature, which appears in the decay of the composite particle in de Sitter background is $T = 2T_H^{\text{dS}}$. It is still unclear whether there is a deep physics in this relation or this is just a coincidence.

Hawking radiation from the black hole can be also considered as the decay of the black hole with initial mass M_i concentrated in the singularity into a particle with the mass m radiated away and the black hole with the smaller mass $M_f < M_i$. The corresponding trajectory of the radiated particle is

$$E(p_r, r) = \sqrt{m^2 + p_r^2} + p_r v(r) = M_i - M_f. \quad (22)$$

If the black hole is immersed in the Minkowski spacetime, the energy conservation prescribes $M_i = M_f + m$ for particle radiated with zero momentum at infinity. Then the straightforward application of the semiclassical tunneling approach [3] gives the radiation rate

$$w \propto \exp(-m/T_H^{\text{bh}}), \quad (23)$$

with the correct Hawking temperature T_H^{bh} for the black hole [27]. This demonstrates that the black hole immersed in the Minkowski spacetime is decaying, while the Minkowski vacuum itself remains stable. In the same manner the body (composite particle, atom, black hole or other object which serves as detector of radiation) immersed in the de Sitter vacuum is decaying, while the de Sitter vacuum may remain stable towards the Hawking radiation.

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6. In simulation of the dynamics of the cosmological constant in Ref. [7] within the so-called q -theory of quantum vacuum, it was found that the initial large cosmological constant (vacuum energy) relaxes either to zero (and thus the universe relaxes to Minkowski vacuum), or to a finite value (and thus universe relaxes to de Sitter vacuum). The remnant vacuum energy is determined by the chemical potential μ , which emerges due to the conservation law for the vacuum variable q and enters the solution of dynamical equations as the integration constant. Thus in the q -theory, de Sitter space-time may serve as attractor in the vacuum dynamics in the same manner as Minkowski vacuum. This distinguishes the de Sitter space-time from the other space-times with non-zero space-time curvature. In principle, it is possible that quantum effects and the screening of the remnant cosmological constant in the infrared [13] may lead to relaxation of the integration constant μ and thus to the further decay of the vacuum energy; however, the details of this mechanism remain unclear. In condensed matter the analogous phenomenon is probably the relaxation of the superconducting order parameter to its equilibrium value after abrupt kick, see e.g. Ref. [8] and references therein. The energy of a superconductor relaxes towards the equilibrium ground state with rapid

oscillations, similar to what occurs with the vacuum energy in the q -theory approach in Ref. [7].

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27. If the black hole is immersed into the de Sitter vacuum, the radiation with the mass deficit, $M_i < M_f + m$, becomes possible. For this case the “fluid” metric in the form of Eq.(1) does not exist because of different sign of the frame dragging velocity $v(r)$ in (2) and in (3). Nevertheless, if the black hole and cosmological horizons are far apart, $T_H^{\text{dS}} \ll T_H^{\text{bh}}$, one may calculate the tunneling exponents separately. Then in the limit of small mass deficit, $m + M_f - M_i \ll m$, one obtains $w(m, M_i - M_f) \propto \exp\left(-\frac{m}{T_H^{\text{bh}}} - \frac{m + M_f - M_i}{2T_H^{\text{dS}}}\right)$.