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Anomalous mass dependence of radiative quark energy loss in a finite-size quark-gluon plasma

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We demonstrate that for a finite-size quark-gluon plasma the induced gluon radiation from heavy quarks is stronger than that for light quarks when the gluon formation length becomes comparable with (or exceeds) the size of the plasma. The effect is due to oscillations of the light-cone wave function for the in-medium $q \to gq$ transition. The dead cone model by Dokshitzer and Kharzeev neglecting quantum finite-size effects is not valid in this regime. The finite-size effects also enhance the photon emission from heavy quarks.

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1. It is widely believed that the observed at RHIC strong suppression of high-p_T hadrons (jet quenching) (for a review, see e.g. [1]) is due to radiative and collisional parton energy loss in the quark-gluon plasma (QGP) produced in the initial stage of AA-collision. The dominating contribution to the energy loss comes from the induced gluon emission (for reviews, see e.g. [2, 3]). The effect of collisional loss is relatively small [4 – 6]. One of the interesting questions, which is important for jet tomography of AA-collisions at RHIC and LHC, is the question on the difference between gluon emission from light and heavy quarks. Dokshitzer and Kharzeev [7] suggested that gluon radiation from heavy quarks should be suppressed due to the dead cone effect. However, the RHIC data on suppression of the non-photonic electrons [8] indicate that the energy loss of heavy quarks may be similar to that for light quarks. The theoretical calculations of the energy loss of one of us [5] within the light-cone path integral (LCPI) approach [9, 10] also are in contradiction with the predictions of [7] since for a finite-size (FS) QGP at high energies the radiative loss has an anomalous mass dependence, i.e., $\Delta E_{\text{heavy}} > \Delta E_{\text{light}}$.

In the present paper we give a physical interpretation of the anomalous mass dependence of the induced gluon emission. We show that the effect is related to the quantum FS effects which come into play when the gluon formation time becomes comparable with the size of the QGP. Physically the anomalous mass dependence of the induced gluon emission is due to oscillations of the light-cone wave function (LCWF) for the in-medium $q \to gq$ transition. The present paper restricts its attention to the physical nature of the effect. For this reason, to make the analysis more transparent we, similarly to [7], consider a QGP with a constant density. The mass dependence of the jet quenching parameter for expanding QGP will be addressed elsewhere.

2. We consider a fast quark with energy E produced in a QGP at $z = 0$ (we choose the z axis along the quark momentum). In the LCPI formalism [9, 10], which we use, the probability of the induced gluon emission can be written in terms of the Green's function for a two-dimensional Schrödinger equation with the Hamiltonian

$$H = -\frac{1}{2\mu}\left(\frac{\partial}{\partial \boldsymbol{\rho}}\right)^2 - \frac{in(z)\sigma_3(\rho, x)}{2} + \frac{1}{L_f}\,, \qquad (1)$$

where x is the gluon fractional momentum, $\mu = Ex(1 - x)$, n is the number medium density, σ_3 is the cross section of interaction with a plasma constituent of the $q\bar{q}g$ system, $L_f = 2\mu/\epsilon^2$, $\epsilon^2 = m_q^2 x^2 + m_g^2(1-x)$ (m_q and m_g are the quark and gluon quasiparticle masses). In the low density limit L_f gives the coherence/formation length of gluon emission in an infinite medium (the Bethe-Heitler regime). The three-body cross section entering (1) can be written as $\sigma_3(\rho, x) = \{9[\sigma_2(\rho) + \sigma_2((1 - x)\rho)] - \sigma_2(x\rho)\}/8$, where $\sigma_2(\rho)$ is the dipole cross section of interaction with color center of the color singlet $q\bar{q}$ system. The Hamiltonian (1) describes in-medium evolution of the LCWF of a fictitious $q\bar{q}g$ system.

The gluon spectrum for a FS medium can also be written in the form [11, 12]

$$\frac{dP}{dx} = \int\limits_0^L dz\, n(z) \frac{d\sigma_{eff}^{BH}(x, z)}{dx}\,, \qquad (2)$$

$$\frac{d\sigma_{eff}^{BH}(x, z)}{dx} = \mathrm{Re}\int d\boldsymbol{\rho}\, \psi^*(\boldsymbol{\rho}, x)\sigma_3(\rho, x)\psi(\boldsymbol{\rho}, x, z)\,, \quad (3)$$

where L is the quark pathlength in the medium, $\psi(\boldsymbol{\rho},x)$ is the ordinary LCWF for $q\to gq$ transition in vacuum, and $\psi(\boldsymbol{\rho},x,z)$ is the in-medium LCWF at the longitudinal coordinate z (hereafter we drop spin and color indices). The latter can be written as

$$\psi(x,\boldsymbol{\rho},z)=\hat{U}\int\limits_0^z dz\mathcal{K}(\boldsymbol{\rho},z|\boldsymbol{\rho}',0)\Bigg|_{\boldsymbol{\rho}'=0}\,, \tag{4}$$

where $\mathcal{K}$ is the Green's function for the Hamiltonian (1), and $\hat{U}$ is the spin vertex operator (its specific form can be found e.g. in [10, 13]). In the low density limit $\psi(\boldsymbol{\rho},x,z)$ goes to $\psi(\boldsymbol{\rho},x)$ at $z/L_f\gg 1$. In this limit (3) reduces to the ordinary Bethe-Heitler cross section, $d\sigma^{BH}/dx$, in the form derived in [14].

3. Let us consider the qualitative pattern of gluon emission. The integral (4) is saturated at $z\gtrsim \bar{L}_f$, where $\bar{L}_f$ is the effective (with the LPM effect) gluon formation length in an infinite medium. The typical transverse size of the $q\bar{q}g$ system for gluon emission in this regime, $\bar{\rho}$, is related to $\bar{L}_f$ by the Schrödinger diffusion relation $\bar{\rho}^2\sim \bar{L}_f/\mu$. In a FS medium the dynamics of the gluon emission depends crucially on the ratio $\xi=L/\bar{L}_f$. For $\xi\gg 1$ $\psi(\boldsymbol{\rho},x,z)$ is very close to $\psi(\boldsymbol{\rho},x,\infty)$. In this regime the FS effects become negligible and the spectrum is close to that for an infinite medium (we call this situation the infinite medium regime). At $\xi\lesssim 1$ the effective Bethe-Heitler cross section is chiefly controlled by the FS effects. In this regime (as in [12] we call it the diffusion regime) the dominating contribution comes from $N=1$ rescattering [12, 15, 16] which gives the effective Bethe-Heitler cross section of the order of $\xi d\sigma^{BH}/dx$.

In the infinite medium regime the Coulomb effects are not very important and the gluon yield can be estimated in the oscillator approximation (OA) which corresponds to the parametrization $\sigma_2(\rho)=C_2\rho^2$. Then the Hamiltonian takes the oscillator form with the oscillator frequency $\Omega=\sqrt{-inC_3}/\mu$ with $C_3=\frac{1}{8}\left\{9[1+(1-x)^2]-x^2\right\}C_2$. Note that in terms of the well known BDMPS transport coefficient $\hat{q}$ [17, 2] $C_2=\hat{q}C_F/2nC_A$. In the OA the probability of gluon emission per unit length reads

$$\frac{dP}{dxdL}=n\frac{d\sigma_{OA}^{BH}}{dx}S(\eta)\,, \tag{5}$$

where $d\sigma_{OA}^{BH}/dx=2\alpha_s C_3 P_{gq}(x)/3\pi\epsilon^2$, is the Bethe-Heitler cross section ($P_{gq}(x)$ is the ordinary $q\to g$ splitting function), and $S(\eta)$ is the LPM suppression factor given by [10]

$$S(\eta)=\frac{3}{\eta\sqrt{2}}\int\limits_0^\infty dy\left(\frac{1}{y^2}-\frac{1}{\mathrm{sh}^2 y}\right)\exp\left(-\frac{y}{\eta\sqrt{2}}\right)\times \times\left[\cos\left(\frac{y}{\eta\sqrt{2}}\right)+\sin\left(\frac{y}{\eta\sqrt{2}}\right)\right], \tag{6}$$

with $\eta=L_f|\Omega|=\sqrt{4nC_3\mu}/\epsilon^2$. At $\eta\gg 1$ when the LPM suppression becomes strong from (6) one can obtain

$$S(\eta)\approx\frac{3}{\eta\sqrt{2}}\left(1-\frac{\pi}{\eta 2\sqrt{2}}\right).$$

Then from (5) one obtains

$$\frac{dP}{dxdL}\approx\frac{\alpha_s P_{gq}(x)}{2\pi}\sqrt{\frac{2nC_3}{\mu}}\left[1-\frac{\pi\epsilon^2}{4\sqrt{\mu nC_3}}\right]. \tag{7}$$

Using (7) we obtain the heavy-to-light mass suppression K-factor:

$$K\approx 1-\frac{\pi(M_Q^2-m_q^2)x^{3/2}}{4\sqrt{E(1-x)nC_3}}\,, \tag{8}$$

where M_Q is the heavy quark mass. Thus, we see that, similarly to the ordinary Bethe-Heitler cross section, in the infinite medium regime with strong LPM suppression the gluon yield falls with quark mass. Note that the analysis [7] is supposed to be applicable namely to the infinite medium regime. In our notation the mass K-factor obtained in [7] reads (in [7] the light quark and gluon are massless)

$$K_{DK}=\left[1+\frac{M_Q^2x^{3/2}}{3\sqrt{EnC_2/2}}\right]^{-2}. \tag{9}$$

Both (8) and (9) are obtained for strong LPM suppression using the OA. Some difference between the two expressions is not surprising since our formula, contrary to that of [7], is obtained with accurate treatment of the mass dependence of the oscillator Green's function.

Let us now consider the diffusion situation. For a given L the boundary x for onset of this regime can be obtained using the OA estimate $\bar{L}_f\sim\min(L_f,|\Omega|^{-1})$ [12]. Assuming that the LPM effect is strong in the infinite medium regime (it means that $\bar{L}_f\sim|\Omega|^{-1}$) one obtains that the diffusion situation corresponds to $x\gtrsim nC_3L^2/E$ (we assume that x is small). Of course, it is only a crude estimate, and as will be seen from our numerical results in reality the FS effects becomes important at much smaller x.

Neglecting a small contribution from $N\geq 2$ rescatterings the effective cross section (3) for Debye potential with screening mass m_D can be written in momentum space as [12, 18]

$$\frac{d\sigma_{\text{eff}}^{BH}(x,z)}{dx} = \\ = \frac{\alpha_s^3 C_T P_{gq}(x)}{\pi^2 C_F}\left[F(1,z) + F(1-x,z) - F(x,z)/9\right], \quad (10)$$

$$F(y,z) = \int \frac{d\mathbf{k}d\mathbf{p}}{(k^2+m_D^2)^2} H(y\mathbf{k},\mathbf{p}) \times \\ \times \left[1 - \cos\left((p^2+\epsilon^2)\rho_d^2(z)\right)\right], \quad (11)$$

$$H(\mathbf{k},\mathbf{p}) = \frac{\mathbf{p}^2}{(\mathbf{p}^2+\epsilon^2)^2} - \frac{(\mathbf{p}-\mathbf{k})\mathbf{p}}{(\mathbf{p}^2+\epsilon^2)((\mathbf{p}-\mathbf{k})^2+\epsilon^2)}, \quad (12)$$

where $C_{T,F}$ are the plasma constituent and quark Casimirs, $\rho_d(z) = \sqrt{z/2\mu}$ is the diffusion radius. We represent F as a sum $F = F_0 + \delta F$, where $F_0 = F(\epsilon = 0)$, and δF is the mass correction. In the massless limit the momentum integration in (11) gives $F_0(y,z) = \pi^3 y^2 \rho_d^2(z)/2$. This leads to the spectrum [12]

$$\left.\frac{dP_{N=1}}{dx}\right|_{\epsilon=0} = \frac{\pi n L^2 \alpha_s^3 C_T P_{gq}(x)[1+(1-x)^2 - x^2/9]}{8 C_F E x(1-x)}. \quad (13)$$

An exact analytical calculation of δF is impossible. We have performed approximate calculation of δF for $\xi \gg 1$. Keeping only the terms with large logarithms one can obtain in this limit

$$\delta F(y,z) \approx \frac{\pi^2\epsilon^2\rho_d^4(z)y^2}{2} \times \\ \times \left\{2\log^2\left(\frac{1}{\epsilon^2\rho_d^2(z)}\right) + \log\left(\frac{1}{\epsilon^2\rho_d^2(z)}\right)\log\left(\frac{\epsilon^2}{y^4 m_D^4 \rho_d^2}\right) - \\ -3\log\left(\frac{1}{\epsilon^2\rho_d^2(z)}\right) - \frac{y^2 m_D^2}{\epsilon^2}\log\left(\frac{1}{\epsilon^2\rho_d^2(z)}\right)\right\}. \quad (14)$$

Then, neglecting in (14) the linear subleading logarithms, we obtain for the mass correction to the spectrum

$$\delta\frac{dP_{N=1}}{dx} \approx \frac{\alpha_s^3 C_T P_{gq}(x)[1+(1-x)^2-x^2/9] L n \epsilon^2 \rho_d^4(L)}{2C_F} \\ \times \log^2\left(\frac{1}{\epsilon^2\rho_d^2(L)}\right). \quad (15)$$

To calculate the $N=1$ term in the OA one should replace in (11) $1/(\mathbf{k}^2+m_D^2)^2$ by $2\hat{q}\delta(\mathbf{k})/n\alpha_s^2 C_A C_T$. In this case the $N=1$ contribution vanishes in the massless limit [12, 15, 16], and all the contribution comes from the mass correction

$$\delta\frac{dP_{N=1}^{OA}}{dx} \approx \frac{4\hat{q} L \alpha_s P_{gq}(x)[1+(1-x)^2-x^2/9]\epsilon^2\rho_d^4(L)}{6\pi C_A C_F} \\ \times \log\left(\frac{1}{\epsilon^2\rho_d^2(L)}\right). \quad (16)$$

Thus, one sees that in the deep diffusion regime the gluon yield has anomalous mass dependence both for the realistic Hamiltonian and in the OA.

In terms of the representation (3) the different mass dependence in the diffusion and infinite medium regimes is due to qualitatively different character of the ρ-dependence of the in-medium LCWF in these two situations. In the infinite medium regime $\psi(x,\boldsymbol{\rho},z)$, like $\psi(x,\boldsymbol{\rho})$, is a smooth exponentially decreasing function of ρ. In this case the probability of gluon emission decreases with quark mass due to a reduction of the dominating ρ scale in (3). Naively one could expect that the same mechanism should suppress the gluon emission from heavy quarks in the diffusion case. However, this is not true since in the diffusion regime the in-medium LCWF entering the integrand of (3) becomes oscillating in ρ. It is well seen from Fig.1 where we plot the leading order radial in-medium LCWF for different values of the

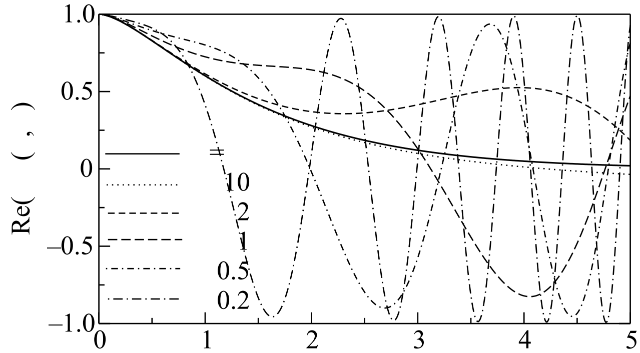

Fig.1. The leading order radial LCWF for in-medium $q \to gq$ transition as a function of ρ in units of $1/\epsilon$ for different values of the dimensionless longitudinal coordinate $\xi = z/L_f$

dimensionless parameter $\xi = L/L_f$. One can see that the FS effects becomes small only at $L/L_f \gtrsim 10$. The oscillations of the in-medium LCWF suppress the probability of gluon emission. This suppression is weaker for heavy quarks (since the ratio L/L_f is smaller). If the effect of the FS oscillations overshoots the mass suppression of the integrand in (3) which arises from the ordinary LCWF, the gluon spectrum rises with quark mass. Our calculations above demonstrate that namely this occurs in the deep diffusion regime.

It is not surprising that the dead cone arguments of [7] fail in the diffusion regime since they are probabilistic in nature, while the oscillations of the in-medium LCWF leading to the anomalous mass dependence is a purely quantum effect. It is worth noting that the upper bound on the gluon momentum up to which the derivation of the dead cone suppression is supposed to be valid has

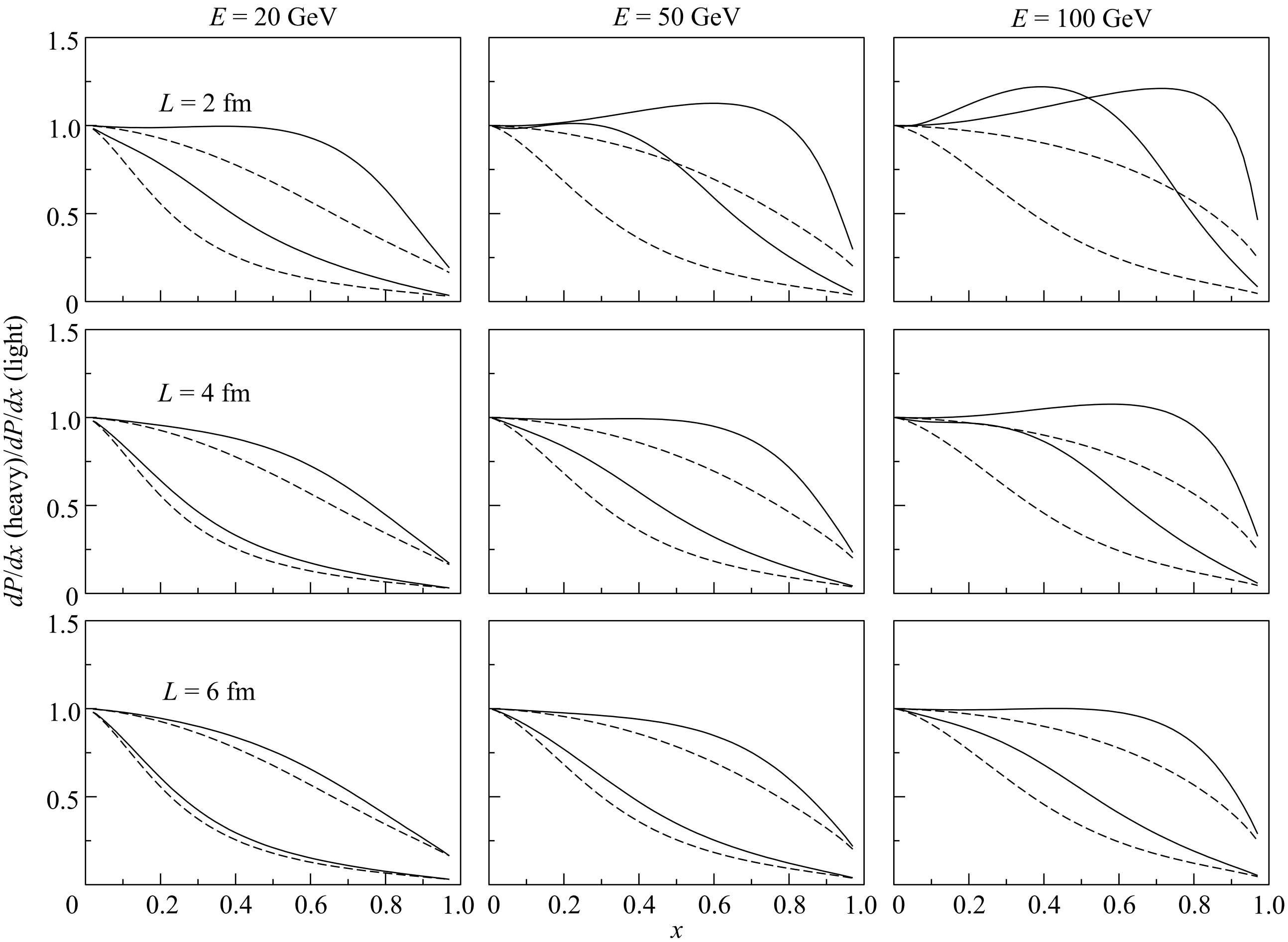

Fig.2. The ratio of the gluon spectra for heavy and light quarks evaluated with the dipole cross section for the Debye-screened potential. The thin curves are for c-quark, the thick curves are for b-quark. The solid and dashed curves show the results with and without the FS effects. All the results are for $m_g = 0.4$ GeV

been obtained in [7] from a crude estimate. In reality, as will be seen from our numerical results, the applicability region of the infinite medium approximation turns out to be considerably narrower, and in a broad kinematical range of gluon energy instead of the dead cone suppression we have an enhancement of the gluon yield.

Note that, similarly to the gluon emission, the photon radiation from massive quarks is also enhanced in the diffusion regime. The $N = 1$ contribution in the massless limit has been calculated in [19] (similarly to the gluon spectrum it is $\propto L^2$). The mass correction reads

$$\delta\frac{dP^{q\to\gamma}_{N=1}}{dx}\approx\frac{e_q^2\alpha_{em}\alpha_s^2C_TC_FnL^3m_q^2x[1+(1-x)^2]}{16E^2(1-x)^2}\times\log^2\left(\frac{1}{\epsilon^2\rho_d^2(L)}\right), \tag{17}$$

where now $\epsilon^2 = m_q^2x^2$. Due to large cross section of the charm production the photon radiation from c-quark may become an important mechanism of the photon production at LHC energies. Since the radiated photon has large momentum ($x \sim 1$) measurements of the photon tagged charm production would be especially interesting.

4. The above analysis is very qualitative, and is not valid in the intermediate region $L/\bar{L}_f \sim 1$. For an accurate evaluation of the mass dependence numerical calculations are necessary. We have performed computations (with any number of rescatterings) using the method suggested in [4]. It reduces calculation of the spectrum to solving the Schrödinger equation for the Hamiltonian (1) with a smooth boundary condition. As in [5] for light quark we use the quasiparticle mass $m_q = 0.3$ and for gluon $m_g = 0.4$ GeV obtained in [20] from the lattice data within the quasiparticle model. For the Debye mass we take $m_D = \sqrt{2}m_g$. For heavy quarks we take $m_c = 1.5$ and $m_b = 4.5$ GeV. We have evaluated the ratio of the gluon spectra for heavy and light quarks for plasma temperature $T = 250$ MeV and $\alpha_s = 0.4$. In Fig.2 we plot the results for $E = 20$, 50 and 100 GeV,

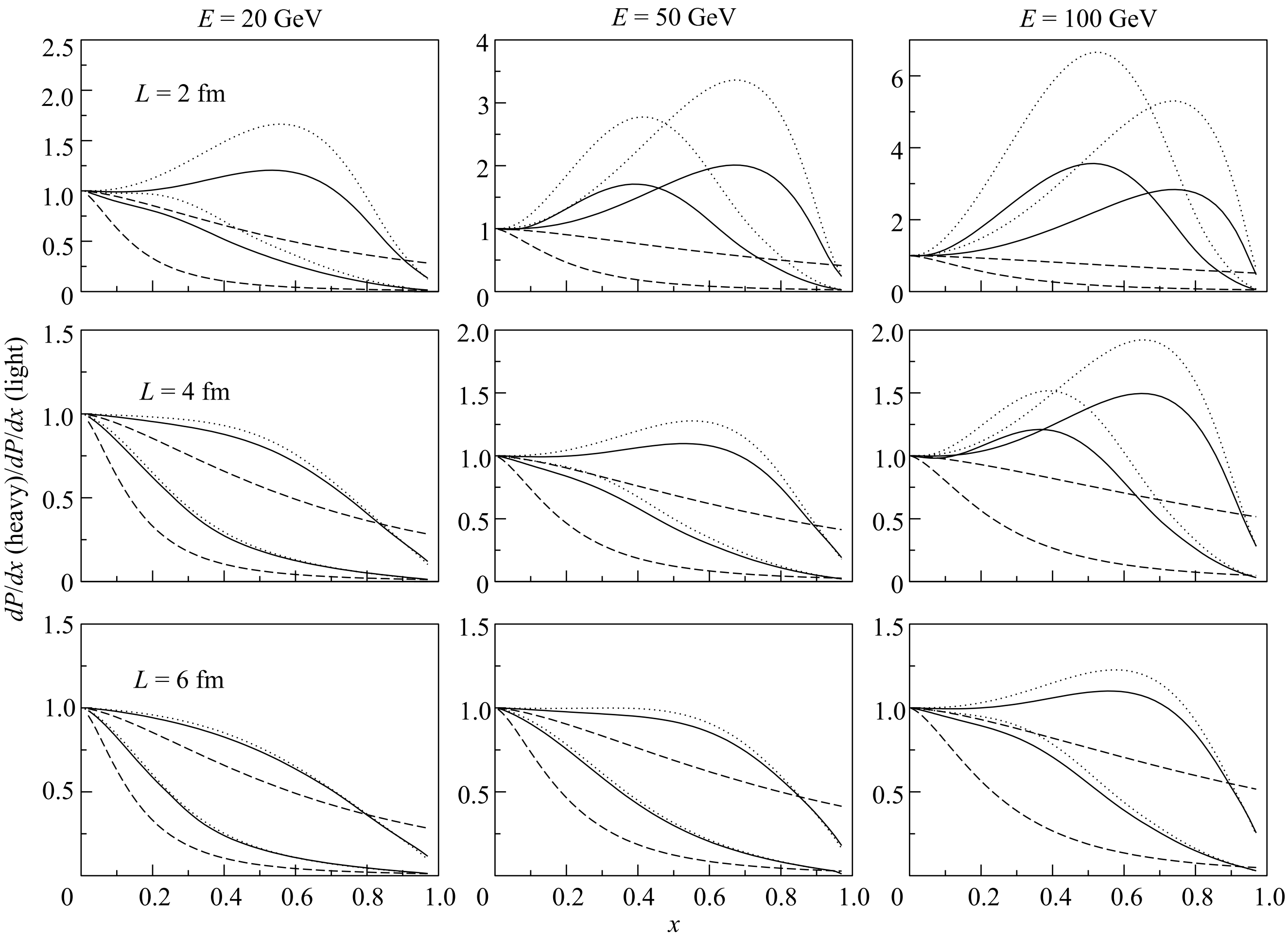

Fig.3. The ratio of the gluon spectra for heavy and light quarks evaluated in the OA. The thin curves are for c-quark, the thick curves are for b-quark. The solid curves show our results with FS effects for $m_g = 0.4\,\mathrm{GeV}$, and the light quark mass $m_q = 0.3\,\mathrm{GeV}$, and the dotted curves show our results for massless gluon and light quark as in [7]. The dashed curves show the Dokshitzer-Kharzeev dead cone suppression factor (9)

and for the plasma thicknesses $L = 2$, 4 and 6 fm. The value $L = 2\,\mathrm{fm}$ approximately corresponds to the typical opacity of the plasma produced at RHIC energies. The solid and dashed curves show the results with and without the FS effects. One can see that at $E \gtrsim 50\,\mathrm{GeV}$ the FS effects are very important at $L \sim 2-4\,\mathrm{fm}$. From the point of view of the jet quenching the soft region $x \lesssim 0.5$ is especially important. Fig. 2 shows that in this region for $L \sim 2\,\mathrm{fm}$ and $E \sim 100\,\mathrm{GeV}$ the FS effects increase considerably gluon emission from heavy quarks. The FS effects become small only at $E \sim 20\,\mathrm{GeV}$ for $L \gtrsim 4\,\mathrm{fm}$. Thus, Fig.2 clearly demonstrates importance of the FS effects in the mass dependence of the gluon yield at $L \lesssim 4\,\mathrm{fm}$.

For comparison with [7] we have also performed numerical calculations in the OA when the rescatterings can be characterized by the transport coefficient $\hat{q}$. We take $\hat{q} = 0.3\ \mathrm{GeV}^3$. At this value of $\hat{q}$ the gluon spectrum in the Bethe-Heitler limit $x \to 0$ agrees with that calculated with accurate three-body cross section. In Fig.3 we present the results for two sets of the parton masses. The solid curves correspond to the same masses as in Fig.2, and the dotted ones are for massless gluon and light quark as in [7]. The dashed curves show the Dokshitzer-Kharzeev [7] dead cone suppression factor. Comparison of the dashed and dotted curves show that the results of the accurate calculations differ drastically from that predicted in the dead cone model. The results for massive and massless gluon in Fig.3 show that the nonzero gluon mass (the Ter-Mikaelian effect) reduces the anomalous mass dependence. The anomalous mass dependence in Fig.3 is stronger than in Fig.2. It is quite natural since in the OA the leading $N = 1$ rescattering term simply vanishes for massless partons, contrary to the case of realistic Hamiltonian when due to the Coulomb effects the single scattering contributes even in the massless limit.

From Fig.3 one can conclude that the applicability region of the infinite medium approximation is considerably narrower than it was assumed in [7]. The authors of [7] assume that their model is valid for the gluon energy $\omega \lesssim \hat{q}L^2$. This means that for $L \sim 5\,\mathrm{fm}$ the mass dependence can be described in terms of the dead cone suppression in the whole kinematical range of x at $E \lesssim 100\,\mathrm{GeV}$. Our results shown in Fig.3 demonstrate that this is not true. Indeed, one can see that the FS effects come into play very early. As a result for $x \lesssim 0.5$, $E \sim 100\,\mathrm{GeV}$ there is no the dead cone suppression at all.

Our results indicate that there must not be a considerable difference between the jet quenching for c and light quarks at $E \sim 20 - 50\,\mathrm{GeV}$. This energy interval is interesting from the point of view of the nuclear modification factor R_{AA} for the non-photonic electrons measured at RHIC [8]. If charm dominates in the region $p_T \lesssim 8$ GeV studied in [8], then the observed suppression of the non-photonic electrons is not surprising. Even if at $p_T \sim 5 - 8$ GeV about 50% of the electrons come from bottom quark [21, 22] it is hardly possible to speak of a serious contradiction of the theory and experiment. Indeed, in this scenario one can obtain the electron suppression about 0.3-0.5 while the data [8] give $R_{AA} \sim 0.1 - 0.55$ with large error bars. More accurate comparison of the energy loss models with experiment will be possible after appearance of the data on the open charm production that are expected soon [23].

Note that numerical calculations within the path integral approach performed in [24] also indicated that in some parameter range the average energy loss grows with quark mass. The authors interpreted this fact as an unphysical effect related to inapplicability of the high-energy approximation. However, the situation with applicability of the LCPI formalism in the kinematical region dominating the anomalous mass dependence is in fact quite good. Indeed, the LCPI formalism [9] is derived assuming that the longitudinal momenta are large compared to the transverse ones and the parton masses. Both these conditions are well satisfied in the diffusion kinematical region related to the anomalous mass dependence. In this regime the situation with the validity of the LCPI approach is even better than in the infinite medium regime (which gives the mass suppressed gluon yield) where the radiated gluons are softer.

5. In summary, we have shown that for a FS QGP the induced gluon radiation from heavy quarks becomes stronger than that for light quarks when the gluon formation length becomes comparable with or exceeds the size of the plasma. In this regime the gluon yield is dominated by the $N = 1$ rescattering. Physically the anomalous quark mass dependence is due to oscillations of the LCWF for the in-medium $q \to gq$ transition. The dead cone model [7], which neglects the quantum FS effects, is not valid in this regime. The anomalous mass effect becomes more pronounced if one neglects the Coulomb effects and describes the parton rescatterings in terms of the transport coefficient. The neglect of the gluon quasiparticle mass also enhances the anomalous mass dependence.

Similarly to the gluon emission, the FS effects enhance the photon radiation from heavy quarks. For this reason the photon radiation from c-quark may be an important mechanism of the hard photon production at LHC.

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