Dirac lines in the parameter and momentum spaces

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Originally the topology of the points and lines of level crossing [1, 2] (diabolical points [3, 4]) has been investigated in a parameter space. In particular, while encircling a diabolical point in the space of two parameters, the wavefunction changes sign [3–5]. Typically this has been applied to electronic spectrum in molecular systems. Later the topological methods have been applied to the diabolical points in the spectrum of fermionic quasiparticles (Bogoliubov quasipartilces) in gapless superfluids and superconductors [6], where the parameter space is the space of linear momentum in superfluids and quasimomentum in superconductors, or the extended phase space $(\mathbf{p}, \mathbf{r})$ [7–9]. In particular, the topologically protected diabolical point in 3D momentum space – the Weyl point – gives rise to Weyl fermions and effective gauge and gravity fields emerging in the vicinity of the Weyl point [10–12]. This analog of relativistic quantum field allowed to experimentally verify the Adler–Bell–Jackiw [13, 14] equation for chiral anomaly in chiral superfluid ^{3}He-A [15]. Then this topological consideration has been extended to the spectrum of bosonic excitations, see, e.g., [16–19].

Recently the new trend is towards the topology in the extended space, which combines the momentum space and the parameter space, see, e.g. [20]. Here we show that the appropriate system, where the two spaces (momentum space and parameter space) are topologically connected, is the polar phase of superfluid ^{3}He discovered in nematically ordered aerogel [21].

In momentum space the polar phase contains the Dirac nodal line in the quasiparticle spectrum determined by the 2×2 Hamiltonian:

$$\mathcal{H}(\mathbf{p}) = v_{\rm F}(p - p_{\rm F})\tau^3 + \Delta_P \hat{\mathbf{m}} \cdot \hat{\mathbf{p}}\, \tau^1. \qquad (1)$$

Here τ^a are Pauli matrices; $p_{\rm F}$ and $v_{\rm F}$ are the Fermi momentum and Fermi velocity; Δ_P is the gap amplitude; $\hat{\mathbf{p}} = \mathbf{p}/p$; $\hat{\mathbf{m}}$ is the unit vector of anisotropy along aerogel strands ($\hat{\mathbf{m}} = \hat{\mathbf{z}}$). The nodal line is at $p_z = 0$ and $p = p_{\rm F}$, see Fig. 1 (*left*). In the vicinity of the Dirac line there emerges the peculiar type of quantum electrodynamics with the non-analytic action for the effective electromagnetic field, $(B^2 - E^2)^{3/4}$ [22].

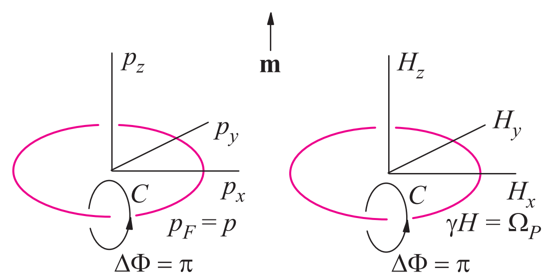

Fig. 1. (Color online) Exceptional lines of level crossing analyzed by von Neumann and Wigner [2] in the polar phase of superfluid ^{3}He. The geometric Berry phase around these lines changes by π. *Left*: Dirac line in the quasiparticle spectrum in space of the components of momentum (p_x, p_y, p_z). At this topologically protected line ($p_z = 0$, $p = p_{\rm F}$) the energy of the Bogoliubov quasiparticles in Eq. (1) is zero. *Right*: Dirac line in the space of parameters – components of magnetic field (H_x, H_y, H_z), which determine the frequency of magnons in Eq. (2). At this topologically protected line ($H_z = 0$, $\gamma H = \Omega_P$) two branches of magnons touch each other

The spectrum of magnons – the Goldstone modes of the polar phase – also experiences the Dirac line, but now in the parameter space, see Fig. 1 (*right*). This spectrum at different magnitudes and orientations of magnetic field has been measured in Ref. [23]. The equation for magnetization is $\omega^2\Psi = \mathcal{H}(\mathbf{H})\Psi$, where [23]:

$$\mathcal{H}(\mathbf{H}) = \frac{(\gamma H)^2 + \Omega_P^2}{2} + \left(\frac{(\gamma H)^2\Omega_P^2}{2} + \Omega_P^2 \cos^2\lambda\right)\tau^3 - \Omega_P^2 \sin\lambda\cos\lambda\, \tau^1. \qquad (2)$$

Here $\Psi = (M_\perp, M_\parallel - M)$, where $M_\perp$ and $M_\parallel$ are the transverse and longitudinal components of magnetization with respect to $\mathbf{H}$; $M = \chi_\perp H$ is an equilibrium magnetization; τ^a are the Pauli matrices connecting the two components of magnetization; Ω_P is the Leggett frequency (the frequency of the longitudinal NMR); γ

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is gyromagnetic ratio; λ is the angle of magnetic field with respect to $\hat{\mathbf{m}}$. For $\lambda = \pi/2$, the two branches are $\omega = \gamma H$ and $\omega = \Omega_P$. They do not interact with each other and cross each other at $\gamma H = \Omega_P$. This is the degeneracy point (Dirac point) of the level crossing. If one takes into account all three components of magnetic field $\mathbf{H}$, one obtains the Dirac line (circle) $H_z = 0$, $\gamma H = \Omega_P$ in the 3D space of magnetic field (H_x, H_y, H_z) in Fig. 1 (*right*). Close to the Dirac line, the Hamiltonian in Eq. (2) transforms to:

$$\mathcal{H}(\mathbf{H}) - \Omega_P^2 \approx \Omega_P(\gamma H - \Omega_P) + \\ + \Omega_P(\gamma H - \Omega_P)\tau^3 - \Omega_P^2 \hat{\mathbf{m}} \cdot \hat{\mathbf{h}}\, \tau^1, \qquad (3)$$

where $\hat{\mathbf{h}} = \mathbf{H}/H$. Eq. (3) is analogous to Eq. (1), with $\gamma\Omega_P$ playing the role of Fermi velocity. Since the analog of Fermi velocity equals the derivative of first term in the rhs, the Hamiltonians (2) and (3) describe the bosonic Dirac system on the border between the type-I and type-II [24, 25].

In both cases of fermionic and bosonic spectrum in Fig. 1, the Dirac nodal line has nontrivial topological charge $N_2 = 1$, see, e.g. [26, 27]:

$$N_2 = \frac{1}{4\pi i} \mathrm{Tr} \oint_C dl\, \tau_2 \tilde{\mathcal{H}}^{-1} \partial_l \tilde{\mathcal{H}}. \qquad (4)$$

Here $\tilde{\mathcal{H}}$ is the traceless part of the matrix $\mathcal{H}$, and the integral is along the loop C in momentum or parameter space enclosing the Dirac line. When $\mathbf{p}$ in Fig. 1 (*left*) or $\mathbf{H}$ in Fig. 1 (*right*) adiabatically evolves along C, the geometric Berry phase Φ changes by π.

The task for future is to combine effects of two Dirac lines, which form the 2D manifold in the 6D momentum + parameter space $(p_x, p_y, p_z, H_x, H_y, H_z)$. This will involve the effects related to dynamics of Bogoliubov quasiparticles near the fermionic Dirac line interacting with the spin waves in vicinity of the bosonic Dirac line, such as adiabatic Thouless pumping [28, 29].

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