

# Coulomb effects in nanoscale SINIS junction

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We study the system of two superconductors connected by a small normal grain. We consider the modification of the Josephson effect by the Coulomb interaction on the grain. Coherent charge transport through the junction is suppressed by Coulomb repulsion. An optional gate electrode may relax the charge blocking and enhance the current leading to the single Cooper pair transistor effect. Temperature dependences of critical current and of the minigap induced in the normal grain by the proximity to superconductor are studied. Both temperature and Coulomb interaction suppress critical current and minigap but their interplay may lead to the nonmonotonous and even reentrant temperature dependence.

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Nanoscale SINIS junction consists of a small normal metallic grain connected to two superconductive leads by tunnel junctions. The Josephson effect in this system is provided by the Andreev reflection processes at the contacts: Cooper pairs enter the normal part of the junction and propagate as Cooperons. This allows the transport of Cooper pairs from one lead to another establishing a supercurrent. Another manifestation of the Andreev mechanism is the appearance of a minigap in the spectrum of the normal grain: the proximity effect. This minigap is a result of a non-zero Cooper pairs density come from superconductors. The mentioned effects rely on the phase conservation in the grain. Charging effects lead to fluctuations of the phase and break the coherent electron transport as well as the induced minigap. The interplay between proximity and charging effects in a normal grain connected to one superconductor was recently studied in [1]. Here we apply the same formalism to the system with two leads and consider the Josephson effect. We also study temperature dependence of the minigap and Josephson current, and find some unexpected reentrant behavior in a certain range of parameters.

We consider tunnel junctions between normal grain and left (L) and right (R) superconductors characterized by large [in units of  $e^2/\hbar$ ] normal-state conductances  $G_{L,R} \gg 1$  and (geometric) capacitances  $C_{L,R}$ . The gate electrode is coupled to the grain by the capacitance  $C_g$ . Mean level spacing in the grain  $\delta$  is the smallest energy scale of the system while the superconductive gap  $\Delta$  in the leads and Thouless energy  $E_{Th}$  of the grain are largest ones. We assume that Andreev conductances of both contacts are small,  $G_{L,R}^A \leq 1$  (together

with conditions  $G_{L,R} \gg 1$  it means that our junctions contain many weakly-transparent channels, quantitative estimates will be provided below). The proximity and charging effects in the grain are characterized by the bare minigap width  $E_{g0} = (G_L + G_R)\delta/4$  and Coulomb energy  $E_C = e^2/2C$  with  $C = C_L + C_R + C_g + \Delta C$  being the total capacitance of the grain. Here  $\Delta C = \frac{e^2}{2\Delta}(G_L + G_R)$  is the contribution to capacitance coming from virtual quasiparticle tunnelling [2]. We assume that  $\delta \ll (E_{g0}, E_C) \ll (\Delta, E_{Th})$ . With the help of the dynamical (in imaginary time) sigma-model in replica space [3], and the adiabatic approximation for charging effects developed in [1], we study the current-phase relation of SINIS junction as well as the dependence of the critical current on temperature and gate voltage  $V_g$ .

Electronic properties of the normal grain are characterized by its Green function. To capture proximity induced correlations one uses the matrix Green function in Nambu-Gor'kov representation. The sigma-model operates with the matrix field  $Q$  which apart from Nambu-Gor'kov structure carries two Matsubara energies and two replica indices. The standard Green function can be extracted from the diagonal in energies element of matrix  $Q$  by replica averaging. Charging effects are described by the fluctuating scalar field  $\phi$  corresponding to the electric potential and also carrying the replica index in the sigma-model formalism. The action for the two variables  $Q$  and  $\phi$  reads

$$S[Q, \phi] = -\frac{\pi}{\delta} \text{Tr} [(\varepsilon \hat{\tau}_3 + \phi)Q] - \frac{\pi}{4} \text{Tr} [(G_L Q_L + G_R Q_R)Q] + \sum_a \int_0^{1/T} d\tau \frac{(\phi^a - eV_g)^2}{4E_C}. \quad (1)$$

Here  $\hat{\tau}_i$  are the Pauli matrices in Nambu-Gor'kov space. The trace operation implies summation over

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all possible variables including replica indices and integration over energies. The equilibrium superconductive matrices  $Q_{L,R}$  for the leads are diagonal in both energies and replicas and have the form  $Q_{L,R}(\varepsilon) = \hat{\tau}_1 \cos \varphi_{L,R} + \hat{\tau}_2 \sin \varphi_{L,R}$  in Nambu-Gor'kov space with  $\varphi_{L(R)}$  being the superconductive phase of the left (right) lead. This expression is valid at energies well below  $\Delta$ . The contribution from higher energies [2] has already been taken into account by renormalization of the capacitance:  $C \mapsto C + e^2(G_L + G_R)/(2\Delta)$ .

To exclude fast fluctuations due to shifts of the electron band by the potential  $\phi$  from the matrix  $Q$  we perform the following change of variables [4]

$$\phi^a(\tau) = \dot{K}^a(\tau), \quad Q_{\tau\tau'}^{ab} = e^{i\hat{\tau}_3 K^a(\tau)} \tilde{Q}_{\tau\tau'}^{ab} e^{-i\hat{\tau}_3 K^b(\tau')}. \quad (2)$$

The phase  $K$  is determined up to a constant, which will be fixed later to simplify further analysis. With new variables the action takes the form

$$S[\tilde{Q}, K] = -\frac{\pi}{\delta} \text{Tr}(\varepsilon \hat{\tau}_3 \tilde{Q}) + \int_0^{1/T} d\tau \left[ \frac{(\dot{K} - N)^2}{4E_C} - \frac{2\pi E_g(\varphi)}{\delta} \left( \tilde{Q}_{\tau\tau}^{(1)} \cos 2K + \tilde{Q}_{\tau\tau}^{(2)} \sin 2K \right) \right]. \quad (3)$$

In this formula we put  $\varphi = \varphi_L - \varphi_R$ , denote  $\tilde{Q}^{(i)} = \text{tr}(\hat{\tau}_i \tilde{Q})/2$ ,  $N = C_g V_g / e$  and introduce bare phase-dependent minigap

$$E_g(\varphi) = \frac{\delta}{4} \sqrt{G_L^2 + G_R^2 + 2G_L G_R \cos \varphi}. \quad (4)$$

The expression (3) is very similar to the action for an SIN system with one superconductive lead [1]. The only difference is the phase dependence of  $E_g$ . Further calculation will essentially follow the procedure of Ref. [1]. The key idea is the *adiabatic approximation* based on the separation of characteristic frequencies of matrix  $\tilde{Q}$  and phase  $K$  ensured by the inequality  $E_C \gg \delta$ . Characteristic timescale of the variable  $K$  fluctuations is much shorter than that of electronic degrees of freedom, thus we integrate the action over  $K(t)$  regarding  $\tilde{Q}$  as a time-independent matrix (it depends only on the difference of its two time arguments). Then we apply the *saddle point approximation* to the  $K$ -averaged action. The justification of this approximation will be provided below.

We parametrize the time-independent matrix  $\tilde{Q}$  by an angle  $\alpha$ :

$$\tilde{Q}(\varepsilon) = \hat{\tau}_3 \cos \alpha(\varepsilon) + \hat{\tau}_1 \sin \alpha(\varepsilon). \quad (5)$$

The  $\hat{\tau}_2$ -term here is eliminated by the proper choice of the constant in the definition of  $K$ . Inserting this ex-

pression into (3) we derive the Hamiltonian controlling the dynamics of the phase  $K$ :

$$\hat{H} = E_C [(-i\partial/\partial K - N)^2 - 2q(\varphi) \cos 2K]. \quad (6)$$

All physical quantities depend periodically on  $N$ . It is convenient to assume that  $|N| < 1/2$ . The parameter  $q(\varphi)$  is expressed in terms of the angle  $\alpha(\varepsilon)$

$$q(\varphi) = \frac{\pi E_g(\varphi) T}{E_C \delta} \sum_{\varepsilon_n} \sin \alpha(\varepsilon_n). \quad (7)$$

This sum diverges logarithmically and should be cut off at  $|\varepsilon| \sim \Delta$ . For large values of  $q$  the phase  $K$  is nearly localized in the minima of cosine potential and the fluctuations are weak. In the opposite case fluctuations of the phase get strong and proximity effect is mostly suppressed. Thus the parameter  $q$  quantifies the strength of proximity coupling competing with the charging effect.

With the derived Hamiltonian we are able to calculate the free energy of the  $K$  degree of freedom  $F(q, T)$  and then extract the total free energy of the system from the action (3)

$$\mathcal{F} = -\frac{2\pi T}{\delta} \sum_{\varepsilon_n} \varepsilon_n \cos \alpha(\varepsilon_n) + F(q, T). \quad (8)$$

The equilibrium value of  $\alpha(\varepsilon)$  is determined by the minimum of this free energy functional (saddle-point approximation). This gives  $\tan \alpha(\varepsilon) = \tilde{E}_g / \varepsilon$  with  $\tilde{E}_g$  obeying the self-consistency equation

$$\frac{\tilde{E}_g}{E_g(\varphi)} = -\frac{1}{2E_C} \frac{\partial F}{\partial q}. \quad (9)$$

This  $\tilde{E}_g$  is the minigap appearing in the spectrum of the normal grain. The estimation of matrix  $tQ$  fluctuations justifies the saddle-point approximation provided  $\tilde{E}_g \gg \delta$ . The system of two equations (7) and (9) determines two parameters,  $q$  and  $\tilde{E}_g$ . A trivial solution  $q = \tilde{E}_g = 0$  always exists. It is analogous to the normal state which is the stationary point (local maximum) of the superconductor free energy. We are looking for a non-trivial solution leading to nonzero value of the minigap  $\tilde{E}_g$ . Once this solution exists it has lower energy than the trivial solution.

After solving the equations we can calculate the free energy (8) and all physical properties of the junction. We are interested in the current-phase relation given by the identity  $I(\varphi) = (2e/\hbar)(d\mathcal{F}/d\varphi)$ . Using the self-consistency relation (9) and the identity (7) we express the current as

$$I(\varphi) = \frac{e\delta^2 E_C \tilde{E}_g q}{4\hbar E_g^3(\varphi)} G_L G_R \sin \varphi. \quad (10)$$

Below we consider analytically two limiting cases of weak ( $q \gg 1$ ) and strong ( $q \ll 1$ ) charging effect and then discuss the numerical results for arbitrary  $q$ . The spectrum of the Hamiltonian (6) is given by the characteristic values of the Mathieu equation which is elementary solved in these two limits. We first calculate the current at zero temperature taking the ground state of (6) for the free energy  $F$ .

*Weak Coulomb blockade.* When charging effects are weak and the parameter  $q$  is large the phase  $K$  is localized near 0 or  $\pi$  in the minima of the cosine potential. The applied gate voltage is ineffective in this case. Expanding the potential to the second order near its minimum we find the ground state energy  $E_0 = E_C(-2q + 2\sqrt{q})$ . The pair of equations (7), (9) can be solved iteratively. In the considered regime the minigap is slightly suppressed in comparison with its bare value  $E_g(\varphi)$ . First, we estimate  $q$  substituting  $E_g(\varphi)$  in the r.h.s. of (7)

$$q_0 = \frac{E_g^2(\varphi)}{E_C \delta} \log(\Delta/E_{g0}). \quad (11)$$

Here we neglect the  $\varphi$ -dependence of  $E_g$  in the argument of logarithm. At the next iteration we put  $q_0$  in the r.h.s. of (9) and then refine the value of  $q$  inserting the calculated  $\tilde{E}_g$  into (7):

$$\tilde{E}_g = E_g(\varphi)(1 - 1/2\sqrt{q_0}), \quad q = q_0(1 - 1/2\sqrt{q_0}). \quad (12)$$

With this  $q$  and  $\tilde{E}_g$  we calculate the Coulomb correction to the current using (10)

$$I(\varphi) = I_0(\varphi) \left[ 1 - \frac{1}{\sqrt{q_0}} \right], \quad (13)$$

$$I_0(\varphi) = \frac{e\delta}{4\hbar} G_L G_R \log(\Delta/E_{g0}) \sin \varphi. \quad (14)$$

Here we denote current in the absence of Coulomb interaction by  $I_0(\varphi)$ . Suppression of the current by the Coulomb interaction becomes stronger as the phase on the junction increases. Qualitatively, the bare minigap  $E_g(\varphi)$  decreases and the charging effects win the competition with proximity further suppressing the current. In the symmetric junction ( $G_L = G_R$ ) the proximity effect vanishes completely as  $\varphi$  approaches  $\pi$ . The weak interaction approximation becomes invalid in this limit even if it is correct for small  $\varphi$ .

*Strong Coulomb blockade.* For small values of the parameter  $q$  we calculate the ground state of (6) pertur-

batively:  $E_0 = -E_C q^2/2(1 - N^2)$ . The equations (7), (9) give

$$\tilde{E}_g = 2\Delta \exp \left[ -\frac{2E_C \delta}{E_g^2(\varphi)} (1 - N^2) \right], \quad q = \frac{2\tilde{E}_g(1 - N^2)}{E_g(\varphi)}. \quad (15)$$

Exponentially small minigap survives at  $T = 0$  when Coulomb blockade is strong. Josephson current is exponentially small as well:

$$I(\varphi) = \frac{2e\delta^2 E_C \Delta^2}{\hbar E_g^4(\varphi)} G_L G_R (1 - N^2) \times \exp \left[ -\frac{4E_C \delta}{E_g^2(\varphi)} (1 - N^2) \right] \sin \varphi. \quad (16)$$

We solve numerically the system of equations (7), (9) and plot the dependence of the current on the phase difference in Fig.1. Critical current  $I_c = \max_\varphi I(\varphi)$  as

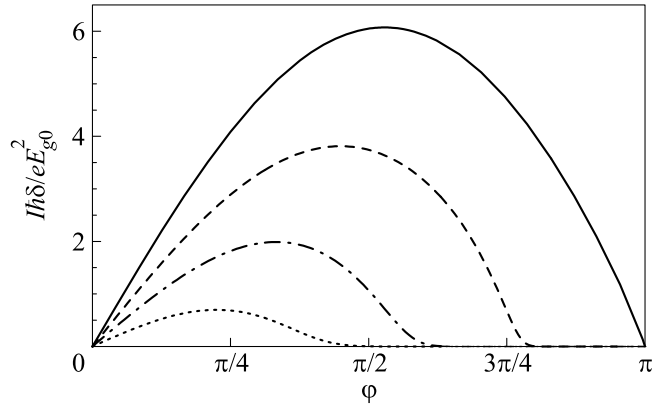


Fig.1. Current vs. phase for the symmetric junction. Solid curve illustrates the case  $E_C = 0$ . It is given by the expression (14). Other curves correspond to  $E_C\delta/E_g^2 = 0.5, 1.5, 2.5$ . At small values of this parameter and at small  $\varphi$  the current is given by (13). At larger  $\varphi$  charging effects are always strong and the current is exponentially suppressed (16). We assume  $G_L = G_R = 20$  and  $\log(\Delta/E_{g0}) = 5$

function of charging energy is shown in Fig.2.

The gate voltage enhances both the minigap and the current [see Fig.3]. Large Coulomb energy makes the charge of the grain to be

current disappear. As temperature approaches its critical value  $T_c$  the parameter  $q$  becomes arbitrarily small. This allows to expand the free energy of the  $K$  degree of freedom:  $F(q, T) = F(0, T) - E_C \beta(N, T) q^2$ . The coefficient  $\beta(N, T)$  may be found with the help of perturbation theory for the Hamiltonian (6):

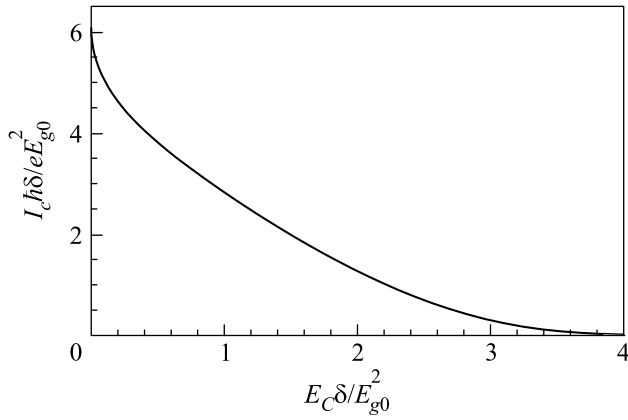


Fig.2. The dependence of the critical current on  $E_C$  for the symmetric junction. The parameters are  $G_L = G_R = 20$ ,  $\log(\Delta/E_{g0}) = 5$

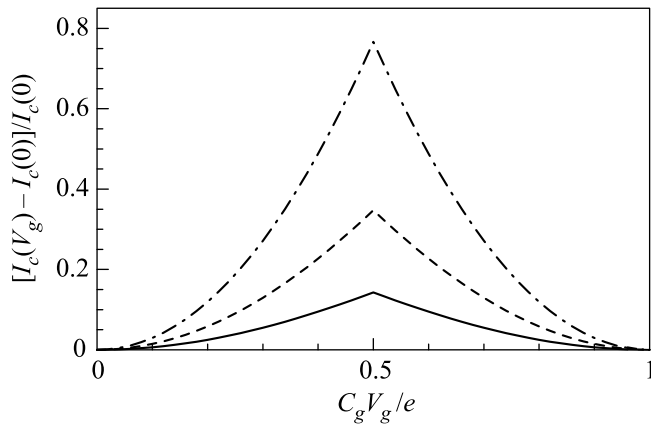


Fig.3. Relative increase of critical current by gate voltage for  $E_C\delta/E_{g0}^2$  equal to 1.5, 2, and 2.5. This effect gets stronger at larger charging energies. Other parameters of the junction are  $G_L = G_R = 20$ ,  $\log(\Delta/E_{g0}) = 5$

$$\beta(N, T) = \frac{\sum_{-\infty}^{\infty} e^{-(n-N)^2 E_C/T} / (1 - (n-N)^2)}{2 \sum_{-\infty}^{\infty} e^{-(n-N)^2 E_C/T}} = \begin{cases} 1/[2(1-N^2)], & T \ll E_C; \\ E_C/T - (2/3)(E_C/T)^2, & T \gg E_C. \end{cases} \quad (17)$$

Note that  $\beta$  is non-monotonous function of temperature. At large temperature highly excited levels of the Hamiltonian insensitive to the  $q$ -perturbation play the main role. Thus  $\beta$  goes to zero in this limit. At small temperature the phase  $K$  is almost frozen at the ground state. The  $q$ -term mixes two lowest excited states. As temperature grows these two states begin to contribute to the free energy increasing its dependence on  $q$ . When  $N$  approaches  $1/2$  the ground state becomes degenerate and  $\beta$  falls monotonously with temperature.

nearly conserving quantity. Ground state corresponds to zero charge and is separated by the gap  $4E_C$  from the excited states with charge  $\pm 2e$ . States with odd charge are ineffective because electrons tunnel from leads by pairs. This situation changes when the gate voltage approaches  $e/2C$ . The gap between ground state and excited state gets twice smaller assisting tunneling of Cooper pairs. At higher gate voltage the ground state carries odd charge and the critical current starts to diminish. The increase of current with gate voltage is analogous to that studied in [5] where the similar setup with the superconductive grain was considered.

Now we turn to the thermodynamic properties of the junction. The temperature dependence of the critical current is found numerically and depicted in Fig.4. At some temperature both the minigap and the Josephson

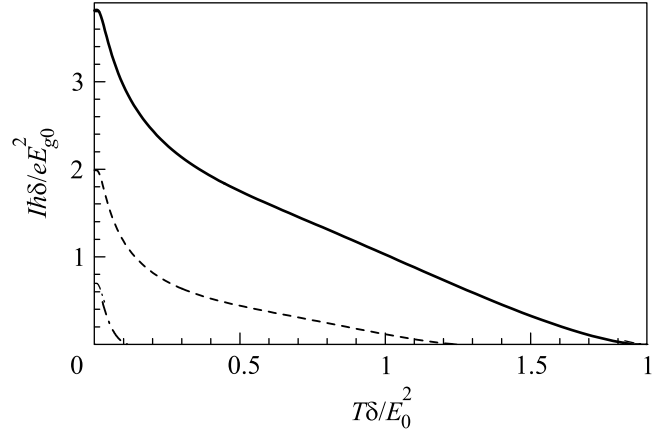


Fig.4. Critical current vs. temperature for  $E_C\delta/E_{g0}^2 = 0.5, 1.5, 2.5$ . Other parameters of the junction are  $G_L = G_R = 20$ ,  $\log(\Delta/E_{g0}) = 5$

The expression (7) in the limit  $\tilde{E}_g \ll T$  gives

$$q = \frac{E_g \tilde{E}_g}{E_C \delta} \log \frac{2\gamma \Delta}{\pi T}. \quad (18)$$

Here  $\log \gamma \approx 0.577$  is the Euler constant. The self-consistency condition (9) in the limit of small  $q$  takes the form  $\tilde{E}_g = E_g(\varphi)\beta q$ . Substituting this equation into Eq. (18) we find for the critical temperature:

$$T_c = \frac{2\gamma}{\pi} \Delta \exp \left[ -\frac{E_C \delta}{E_g^2(\varphi)\beta(N, T_c)} \right]. \quad (19)$$

The same equation may be obtained by the expansion of (8) in powers of  $\tilde{E}_g$  and setting the coefficient of  $\tilde{E}_g^2$  to zero. It can be checked that the forth-order term of this expansion always remains positive. This justifies our assumption that  $\tilde{E}_g$  vanishes continuously at the critical temperature.

In the regime of strong Coulomb interaction critical temperature appears to be much less than  $E_C$ . Tak-

ing low temperature asymptotic of  $\beta(N, T)$  we find that  $T_c = (\gamma/\pi)\tilde{E}_g(T=0)$ , where  $\tilde{E}_g(T=0)$  is given by Eq. (15). This is the BCS relation between the gap and the critical temperature. The phase of the grain strongly fluctuates and is mainly independent of the phase in the leads. The only effect of superconductive leads is a weak effective attraction in the Cooper channel which leads to formation of very weak BCS-like state.

In the opposite limit of weak Coulomb interaction we employ high temperature asymptotic of  $\beta$  and find

$$T_c = \frac{E_g^2(\varphi)}{\delta} \log \frac{2\gamma\Delta\delta}{\pi E_g^2(\varphi)} - \frac{2}{3}E_C. \quad (20)$$

The whole dependence of the critical temperature on parameters is shown in Fig.5. It is possible that the equation (19) has more than one solution at a given value

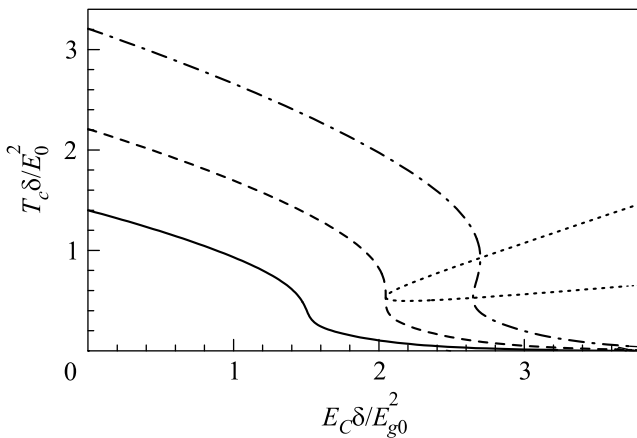


Fig.5. The critical line demonstrating the dependence of critical temperature on  $E_C$ . The three curves are plotted for  $\Delta\delta/E_g^2 = 5, 17.7, 70$ . At large enough values of  $\Delta$  this dependence may not be single-valued. Dotted line shows the turning points of critical temperature. Small disk denotes the critical point where the dotted line touches the  $T_c$  vs.  $E_C$  curve. This critical point gives the following critical values:  $T_c^*\delta/E_g^2 = 0.54$ ,  $E_C^*\delta/E_g^2 = 2.05$ ,  $\Delta^*\delta/E_g^2 = 17.7$

of  $E_C$ . This implies reentrant behavior of the minigap and critical current as functions of temperature. Mathematically, it is due to nonmonotonic behavior of the function  $\beta(T)$ . Physical explanation is most simple in charge rather than phase representation of the Hamiltonian (6). Two excited states with charge  $\pm e$  have equal charging energies. Tunneling of Cooper pair, that switches between these two states, is not blocked by Coulomb interaction. Finite temperature may excite the system to one of this states leading to the temperature-assisted proximity effect and the enhancement of the minigap. The Hamiltonian (6) conserves the parity of electron

number. Thus thermalization of the system implies some parity-breaking processes (e.g. single electron tunneling with energy above  $\Delta$ ), that may take a long time.

At large enough values of  $\Delta > \Delta^* = 17.7E_g^2/\delta$ , reentrant behavior of the minigap with temperature was found in some (dependent on  $\Delta/\Delta^*$  ratio) interval of the Coulomb parameter  $E_C\delta/E_g^2$ , cf. Fig.6. Fine tuning of the parameter  $E_C\delta/E_g^2$  can be achieved by an appropriate phase bias, cf. Eq. (4).

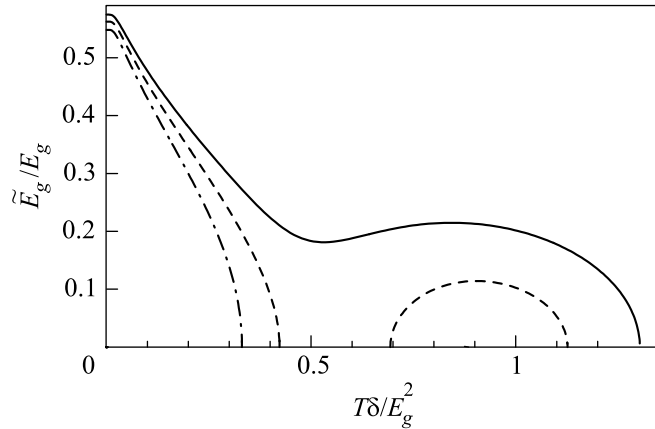


Fig.6. The minigap in the normal dot as a function of temperature. Solid line is plotted for  $E_C\delta/E_g^2 = 2.6$ . Dashed line illustrates the case  $E_C\delta/E_g^2 = 2.67$ . Chain line is for  $E_C\delta/E_g^2 = 2.75$ . All curves are plotted for  $\Delta = 70E_g^2/\delta$ . The critical temperature as a function of  $E_C$  for the same value of  $\Delta$  is shown by the chain line in Fig.5

To conclude, we have described the Josephson effect in a nanoscale SINIS junction modified by the Coulomb interaction. The most important feature of SINIS structure (in comparison with usual SIS structure) is that it can demonstrate both good metallic conductance in the normal state and Coulomb blockade of Josephson current at very low temperatures, since both the conditions  $G_{L,R} \gg 1$  and  $E_J = \frac{\hbar}{2e}I_c \leq E_C$  can be fulfilled simultaneously. We calculated the current-phase characteristic of the junction in both weak and strong Coulomb blockade limit. The enhancement of the current by the gate voltage is predicted. The temperature dependence of the critical current and of the minigap induced in the normal part of the junction was found. A grain of noble metal with size about 50 nm connected to Nb superconductive electrodes by tunnel oxide barriers with transparency per channel of the order of  $10^{-5}$  could present an example of the studied system with  $E_J \sim E_C \sim 1K$ . We are grateful to T. Kontos and Ya. Fominov for useful discussions. This research was supported by the Program "Quantum Macrophysics" of the Russian Academy of Sciences, Russian Ministry of Science and RFBR under

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