Pis'ma v ZhETF, vol.60, iss.10, pp.678 - 681

THE DIRECT CALCULATION OF THE SLOPE OF THE QCD POMERON'S TRAJECTORY

N.N.Nikolaev[*+], *B.G.Zakharov*[+], *V.R.Zoller*[□]

[*] *IKP(Theorie), KFA Jülich*
5170 Jülich, Germany

[+] *L. D. Landau Institute for Theoretical Physics, RAS*
117334 Moscow, Russia

[□] *Institute for Theoretical and Experimental Physics*
117259 Moscow, Russia

Submitted 20 October, 1994

We demonstrate that the diffraction slope of the generalized BFKL pomeron amplitude has the conventional Regge growth $B(s) = B(0) + 2\alpha'_{\mathbf{P}} \log(s)$. This proves that the generalized BFKL pomeron is described by the moving j-plane singularity. We give an estimate for the slope $\alpha'_{\mathbf{P}}$ in terms of the correlation radius for the perturbative gluons.

1. Introduction

Whether the QCD pomeron is described by the fixed or moving singularity in the complex j-plane, remains one of the topical issues. The purpose of this communication is a proof that the generalized BFKL pomeron [1−4] is a moving cut. We present the first direct calculation of the slope $\alpha'_{\mathbf{P}}$ for the pomeron trajectory.

The early works on the BFKL (Balitskii−Fadin−Kuraev−Lipatov [5]) pomeron focused on the idealized scaling regime with fixed strong coupling $\alpha_S = \text{const}$ and infinite gluon correlation radius R_c. In this regime, the BFKL pomeron is described by a fixed cut in the complex angular momentum plane $-\infty < j \le \alpha_{\mathbf{P}}(0) = 1+\Delta_{\mathbf{P}}$. However, because of the diffusion property of the Green function of the scaling BFKL equation [5], the scaling regime is not self-consistent. Recently, there was much progress in understanding the BFKL pomeron in the framework of the dipole-cross section representation introduced in [6]. In previous papers of ours [1-4] we derived the generalized BFKL equation for the dipole cross section in a realistic model with the running (and freezing) strong coupling $\alpha_S(r)$ and with the finite corelation radius R_c of the perturbative gluons. While the property of the cut in the j-plane is retained, we found a profound impact of the running $\alpha_S(r)$ and of the finite R_c on the spectrum and solutions of our generalized BFKL equation. The crucial observation is that the intercept $\Delta_{\mathbf{P}}$ and the asymptotic behaviour of the dipole cross section are controlled by interactions at the dipole size $r \sim R_c$. Also, we noticed that the recovery of the conventional multiperipheral pattern is likely at asymptotic energies, which suggests the Regge growth of the diffraction cone. In this paper we confirm the latter observation and show that indeed the pomeron trajectory has the finite slope $\alpha'_{\mathbf{P}} \propto R_c^2$.

The starting point of our analysis is the generalization of our BFKL equation [1−4] to the profile function of the dipole cross section $\Gamma(r, \mathrm{b})$. Defining the impact parameter b with respect to the center of the parent q-$\bar{q}$ dipole, and repeating the derivation [1-4], we obtain

$$
\begin{aligned}
\frac{\partial\Gamma(\xi,r,\mathbf{b})}{\partial\xi} &= \mathcal{K}\otimes\Gamma(\xi,r,\mathbf{b}) = \\
&= \frac{3}{8\pi^3}\int d^2\vec{\rho}_1\,\mu_G^2\left|g_S(R_1)K_1(\mu_G\rho_1)\frac{\vec{\rho}_1}{\rho_1}-g_S(R_2)K_1(\mu_G\rho_2)\frac{\vec{\rho}_2}{\rho_2}\right|^2\times \\
&\times[\Gamma(\xi,\rho_2,\mathbf{b}+\tfrac{1}{2}\vec{\rho}_1)+\Gamma(\xi,\rho_1,\mathbf{b}+\tfrac{1}{2}\vec{\rho}_2)-\Gamma(\xi,r,\mathbf{b})]\,,
\end{aligned}
\tag{1}
$$

in which $\vec{\rho}_2=\vec{\rho}_1-\mathbf{r}$, the arguments of the running QCD charge $g_S(r)=\sqrt{4\pi\alpha_S(r)}$ are $R_i=\min\{r,\rho_i\}$, $K_1(x)$ is the generalized Bessel function and $R_c=1/\mu_G$ is the correlation radius for the perturbative gluons. Here we use the standard definition of the profile function when

$$
A(s,t)=2is\int d^2\mathbf{b}\exp(-i\mathbf{q}\mathbf{b})\Gamma(\mathbf{b})
$$

and the dipole cross section equals $\sigma(\xi,r)=2\int d^2\mathbf{b}\,\Gamma(\xi,r,\mathbf{b})$. Hereafter we shall discuss the reduction of (2) to the equation for the diffraction slope

$$
B(\xi,r)=\frac{1}{2}\langle\mathbf{b}^2\rangle=\lambda(\xi,r)/\sigma(\xi,r),\quad \lambda(\xi,r)=\int d^2\mathbf{b}\,\mathbf{b}^2\ \Gamma(\xi,r,\mathbf{b}).
$$

Evidently, the diffraction slope for the dipole of size r contains the purely geometrical contribution $\frac{1}{8}r^2$ which comes from the elastic form factor of the dipole. Then, it is more convenient to consider $\eta(\xi,r)=\lambda(\xi,r)-\frac{1}{8}r^2\sigma(\xi,r)$, the equation for which takes the form

$$
\begin{aligned}
\frac{\partial\eta(\xi,r)}{\partial\xi} &= \frac{3}{8\pi^3}\int d^2\vec{\rho}_1\,\mu_G^2\left|g_S(R_1)K_1(\mu_G\rho_1)\frac{\vec{\rho}_1}{\rho_1}-g_S(R_2)K_1(\mu_G\rho_2)\frac{\vec{\rho}_2}{\rho_2}\right|^2\times \\
&\times\left\{\eta(\xi,\rho_1)+\eta(\xi,\rho_2)-\eta(\xi,r)+\frac{1}{8}(\rho_1^2+\rho_2^2-r^2)[\sigma(\rho_2,\xi)+\sigma(\rho_1,\xi)]\right\}= \\
&= \mathcal{K}\otimes\eta(\xi,r)+\beta(\xi,r)\,,
\end{aligned}
\tag{2}
$$

where the inhomegeneous term equals

$$
\begin{aligned}
\beta(\xi,r) &= \mathcal{L}\otimes\sigma(\xi,r)= \\
&= \frac{3}{64\pi^3}\int d^2\vec{\rho}_1\,\mu_G^2\left|g_S(R_1)K_1(\mu_G\rho_1)\frac{\vec{\rho}_1}{\rho_1}-g_S(R_2)K_1(\mu_G\rho_2)\frac{\vec{\rho}_2}{\rho_2}\right|^2\times \\
&\times(\rho_1^2+\rho_2^2-r^2)[\sigma(\rho_2,\xi)+\sigma(\rho_1,\xi)]\,.
\end{aligned}
\tag{3}
$$

Here the crucial point is that the homogeneous Eq. (2) is precisely our generalized BFKL equation for the dipole cross section

$$
\frac{d\sigma(\xi,r)}{d\xi}=\mathcal{K}\otimes\sigma(\xi,r)\,,
\tag{4}
$$

which enables us to prove on a very generic grounds that $\alpha'_{\mathbf{I\!P}}=\frac{1}{2}dB(\xi,r)/d\xi\neq 0$.

The proof goes as follows: In [3,4] we have shown that the generalized BFKL operator $\mathcal{K}$ has the continuous spectrum, which corresponds to the cut in the j-plane. Let $-\infty < \nu < \infty$ be the "wavenumber" which labels eigenfunctions $E(\nu,r)\exp[\Delta(\nu)\xi]$ of Eq. (4) with eigenvalue $\Delta(\nu)$. For the guidance, in the scaling limit of α_S = const and $R_c \to \infty$ $E(\nu,r) = r\exp[i\nu\log(r^2)] = \sigma_{\rm I\!P}(r)\exp[i\nu\log(r^2)]$ with the orthogonality condition [3-5]

$$\delta(\nu-\mu) = \frac{1}{2\pi}\int \frac{d\log(r^2)}{[\sigma_{\rm I\!P}(r)]^2}E^*(\nu,r)E(\mu,r)\,, \tag{5}$$

and ν is indeed the wavenumber of plane waves in the $\log(r^2)$ space. Properties of eigenfunctions $E(\nu,r)$ in the case of the running $\alpha_S(r)$ and the finite R_c are discussed in [3,4,7].

Now we proceed with solution of the inhomogeneous equation (2). If $G(\nu,r) = \mathcal{L}\otimes E(\nu,r) = \int d\omega g(\nu,\omega)E(\omega,r)$, then the inhomogeneous term (3) can be written as

$$\beta(\xi,r) = \mathcal{L}\otimes\sigma(\xi,r) = \int d\nu E(\nu,r)\int d\omega f(\omega)g(\omega,\nu)\exp[\Delta(\omega)\xi]\,. \tag{6}$$

We search for a solution of the form $\eta(\xi,r) = \int d\nu\tau(\xi,\nu)E(\nu,r)\exp[\Delta(\nu)\xi]$. Making use of the property of eigenfunctions $\mathcal{K}\otimes E(\nu,r) = \Delta(\nu)E(\nu,r)$ we find

$$\frac{\partial\tau(\xi,\nu)}{\partial\xi}\exp[\Delta(\nu)\xi] = \int d\omega f(\omega)g(\omega,\nu)\exp[\Delta(\omega)\xi] \tag{7}$$

and

$$\eta(\xi,r) = \int d\nu\tau(\xi=0,\nu)E(\nu,r)\exp[\Delta(\nu)\xi] +$$
$$+\int_0^\xi d\xi'\int d\nu E(\nu,r)\exp[\Delta(\nu)(\xi-\xi')]\int d\omega f(\omega)g(\omega,\nu)\exp[\Delta(\omega)\xi']\,. \tag{8}$$

Here $\tau(\xi=0,\nu)$ describes a solution of the homogeneous equation (2) and is determined by the initial condition $\eta(\xi=0,r)$.

The singularity structure of $g(\omega,\nu)$ can be found considering the large-r behaviour of $G(\nu,r) = \mathcal{L}\otimes E(\nu,r)$. Because of the exponential decrease of the Bessel function $K_1(x)\propto\exp(-x)$, the integration in (3) will be dominated by the two contributions from $\rho_1 \lesssim R_c$, $\rho_2\approx r$ and $\rho_2\lesssim R_c$, $\rho_1\approx r$. For the sake of definiteness, consider the former case. Notice that, in this regime, $(\rho_1^2+\rho_2^2-r^2)\approx\rho_1^2$ and $E(\nu,\rho_2)\approx E(\nu,r)$, which gives the contribution of the form $2G_1E(\nu,r)$ to $\mathcal{L}\otimes E(\nu,r)$. Evidently, it corresponds to the singular term $g_1(\omega,\nu) = 2G_1\delta(\omega-\nu)$. The contribution from the term $\propto\rho_1^2E(\nu,\rho_1)$ to $\mathcal{L}\otimes E(\nu,r)$ does not depend on r at large r and corresponds to $g_2(\omega,\nu) = G_2(\nu)\delta(\omega)$. Apart from these singular terms, $g(\omega,\nu)$ will also have the smooth component $g_3(\omega,\nu)$.

Evidently, the $2G_1\delta(\omega-\nu)$ component of $g(\omega,\nu)$ gives a contribution to $\eta(\xi,r)$ of the form

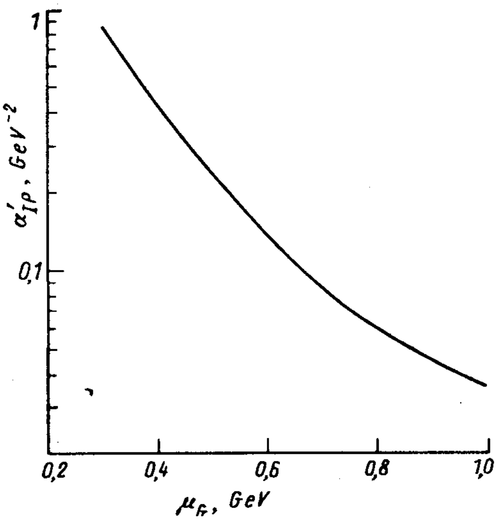
$$\eta_1(\xi,r) = 2G_1\int_0^\xi d\xi'\int d\nu f(\nu)E(\nu,r)\exp[\Delta(\nu)\xi] = 2G_1\xi\sigma(\xi,r)\,, \tag{9}$$

which gives precisely the Regge growth of the diffraction slope $B(\xi,r)$ with $\alpha'_{\rm I\!P} = G_1$. We have an explicit estimate for the slope of the pomeron trajectory

$$\alpha'_{\rm I\!P} \sim \frac{3}{16\pi^2}\int d^2\mathbf{r}\,\alpha_S(r)\mu_G^2r^2K_1^2(\mu_Gr) \propto \frac{3}{64\pi}R_c^2\alpha_S(R_c)\,. \tag{10}$$

The effect of $g_2(\omega,\nu) = G_2(\nu,\omega)$ can be evaluated making use of the explicit form of $E(\nu,r)$ [3,4,7]. It also contributes to the slope of the pomeron trajectory $\alpha'_{\mathbb{P}} \sim G_2(0) \sim G_1$. The smooth part of $g(\omega,\nu)$ does not contribute to the slope of the pomeron trajectory,

In the numerical calculation of the slope $\alpha'_{\mathbb{P}}$ we start with the dipole-dipole cross section and the corresponding diffraction slope as described in [5,6]. We calculate the ξ dependence of the dipole cross section $\sigma(\xi,r)$ and of the diffraction slope $B(\xi,r)$ and verify that at $\xi \to \infty$ the effective intercept $\Delta_{eff}(\xi,r) = \partial \log\sigma(\xi,r)/\partial\xi$ and the effective slope $\alpha'_{eff}(\xi,r) = \partial B(\xi,r)/\partial\xi$ tend to the limiting values $\Delta_{\mathbb{P}}$ and $\alpha'_{\mathbb{P}}$, respectively, which are independent of the size of the projectile and target colour dipoles. We take the running coupling with the infrared freezing $\alpha_S(r) \leq \alpha_S^{(fr)} = 0.82$ [3,4]. The dependence of the slope $\alpha'_{\mathbb{P}}$ on $\mu_G = 1/R_c$ is shown in Figure. The simple estimate (10) is close to these numerical results.



The slope of the pomeron trajectory $\alpha'_{\mathbb{P}}$ as a function of the inverse correlation radius $\mu_G = 1/R_c$ for the perturbative gluons

To summarize, we have shown that the generalized BFKL pomeron [1-4] is the moving cut in the complex angular momentum plane. We derived a simple analytical estimate (10) for the slope $\alpha'_{\mathbb{P}}$ of the pomeron trajectory and found the dependence of the slope on the gluon correlation radius by an accurate numerical solution of our generalized BFKL equation (2) for the diffraction slope.

This work was supported by the INTAS Grant 93-239. V.R.Z. acknowledged the partial support by the Grant NMT5000 of the G.Soros ISF.

---

1. N.N.Nikolaev and B.G.Zakharov, ZhETF **105**, 1117 (1994) (English translation: JETP **78**, 598 (1994)); Z. Phys. C (1994) in press.
2. N.N.Nikolaev, B.G.Zakharov, and V.R.Zoller, JETP Lett. **59**, 8 (1994).
3. N.N.Nikolaev, B.G.Zakharov, and V.R.Zoller, Phys. Lett. **B328**, 486 (1994).
4. N.N.Nikolaev, B.G.Zakharov, and V.R.Zoller, ZhETF **105**, 1498 (1994).
5. E.A.Kuraev, L.N.Lipatov, and V.S.Fadin, Sov. Phys. JETP **44**, 443 (1976); **45**, 199 (1977); Ya.Ya.Balitskii and L.N.Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978); L.N.Lipatov, Sov. Phys. JETP **63**, 904 (1986); L.N.Lipatov. Pomeron in Quantum Chromodynamics. In: Perturbative Quantum Chromodynamics, ed. A.H.Mueller, World Scientific, 1989.
6. N.N.Nikolaev and B.G.Zakharov, Z. Phys. **C49**, 607 (1991); Z. Phys. **C53**, 331 (1992).
7. N.N.Nikolaev and B.G.Zakharov, Phys. Lett. **B327**, 157 (1994).