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QED corrections in radiative return method at measurement of the $e^+e^- \to \pi^+\pi^-$ cross–section below 1 GeV

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We discuss radiative corrections at the measurement of the $e^+e^- \to \pi^+\pi^-$ cross–section by the radiative return method without photon tagging. The radiative corrections to the initial–state radiation process are computed for DAΦNE conditions, using the quasi–real electron approximation for both, the cross–section and the underlying kinematics. The efficiency of experimental rules for event selection by the restrictions on the lost invariant mass and so–called "track mass" is estimated. Some numerical calculations illustrate our analytical results.

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1. Testing the consistency of the SM requires a variety of measurements for which radiative corrections play a crucial role. Among such corrections effects caused by hadronic vacuum polarization in the photon propagator occupy a special place in electroweak precision physics. Because the interaction coupling between quarks and gluons increases at low energies, the corresponding contributions cannot be calculated by pQCD but may be computed via dispersion integrals over the experimental e^+e^-–annihilation data. Therefore, a precise determination of these effects in the running fine structure constant $\alpha^{\rm had}$ and in the muon anomalous magnetic moment a_μ depends on the precision of the low energy hadronic cross section $\sigma_h(e^+e^- \to \text{hadrons})$ [1 – 3]. The recent theoretical analysis of these quantities [4] shown that existing data is insufficient at least to solve two important problems: make more precise prediction about location of the light Higgs mass [5] and draw conclusion about possible "new physics" (beyond SM) which would contribute to a_μ [6].

The measurement of σ_h at e^+e^-–annihilation by radiative return using the initial state radiation (ISR) process has became objective reality last years [7, 8]. Theoretical aspects of such measurements accounting radiative corrections (RC) have been studied firstly in [9] for the case of collinear small–angle ISR events. Further different approaches including both analytical calculations [10] and Monte Carlo generators [11] have been developed to describe the small– and large–angle ISR photon events and satisfy experimental selection rules used at Φ– and b–fabrics where the respective experiments yet began.

It is the general opinion that the high–luminosity DAΦNE machine operating in Φ–resonance region is the ideal collider to scan $\sigma_h(q^2)$ with the center–of–mass energy $\sqrt{q^2}$ varying from the threshold up to 1 GeV just by ISR radiative events. In this region $\sigma_h(q^2)$ is mainly fulfilled by ρ–resonance that decays into pair $\pi^+\pi^-$. KLOE detector at DAΦNE allows to measure 3–momenta of charged pions and select with high efficiency events with fixed value the squared pion invariant mass $q^2 = (p_- + p_+)^2$ in ISR radiative process

$$e^-(p_1) + e^+(p_2) \to \pi^+(p_+) + \pi^-(p_-) + \gamma(k) \qquad (1)$$

without straightforward registration of ISR photon. This approach makes possible to use events with collinear photons which fly in so–called blind zone and cannot be detected by KLOE calorimeters. Such method has some advantages because the corresponding cross section becomes larger by a logarithm enhancement factor $L_0 = \ln(E^2\theta_0^2/m^2)$, where E is the beam energy, θ_0 is the maximum angle of collinear photon and m is the electron mass.

In Born approximation one can apply the quasi–real electron method (QRE) [12] to write the cross section of the process (1) with small–angle ISR photons in terms of the cross section $\sigma(q^2)$ of the process $e^+ + e^- \to \pi^+ + \pi^-$

$$\frac{d\sigma^{B}}{dq^2} = \frac{\sigma(q^2)}{4E^2}\frac{\alpha}{2\pi}P(z, L_0),$$
$$P(z, L_0) = \frac{1+z^2}{1-z}L_0 - \frac{2z}{1-z}, \qquad (2)$$
$$z = \frac{q^2}{4E^2},\ L_0 = \ln\frac{E^2\theta_0^2}{m^2}.$$

However, one must apply an additional restriction on event selection to exclude background due to possible strong 3–pion decays: $\omega \to \pi^+\pi^-\pi^0$ and $\Phi \to \pi^+\pi^-\pi^0$.

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The reason is that in this case photon is not detected and these decays in general can imitate events like in reaction (1) because the neutral pion remains also invisible. It is obvious that to get rid such 3–pion background one may to select events with the small lost invariant mass $M_{\rm lost}$ which is smaller than pion mass

$$M^2_{\rm lost} = (p_1 + p_2 - p_+ - p_-)^2 < m^2_\pi. \qquad (3)$$

At the Born level this restriction always satisfied because in this case $M^2_{\rm lost} = k^2 = 0$. Thus, it has to be taken into account only when calculating contribution in RC to cross section (2) caused by additional hard photon emission. Such additional photon, in principle, can be radiated from initial as well from final states. But the collinear (with respect to the electron beam) final state radiation (FSR) is suppressed by a factor θ_0^2, and even for $\theta_0 = 10^o$ the approximation in which terms of the order θ_0^2 are neglected has very high accuracy on the level of RC. Therefore, one may consider only contribution into RC due two hard ISR photons with their invariant mass less than m_π.

2. Instead of restriction (3) in [13] was suggested to select events with a small difference between the lost energy and the lost 3–momentum modulus

$$\Omega - |\mathbf{K}| < \eta E, \ \Omega = 2E - E_+ - E_-, \\ |\mathbf{K}| = |\mathbf{p}_+ + \mathbf{p}_-|, \ \eta = 0.02, \qquad (4)$$

where $E_\pm$ ($\mathbf{p}_\pm$) are the energies (3–momenta) of $\pi^\pm$. For reaction $e^+ + e^- \to \pi^+ + \pi^- + 2\gamma$ the quantity Ω ($\mathbf{K}$) is the total energy (3–momentum) of two photons. In any event the lost squared invariant mass cannot be more than $2E(\Omega - |\mathbf{K}|)$, therefore choice $\eta \le m_\pi^2/2E^2 \simeq 0.039$ provides an exclusion of the 3–pion background. Thus, inequality (4), in fact, is equivalent to (3) one.

The restriction (3) defines the upper limit of Ω variation as

$$\Omega_{\max} = E(1-z)\big(1 + \frac{\eta}{2}\big). \qquad (5)$$

The corresponding RC were calculated in [14] neglecting terms of the order η.

In real experiments at DAΦNE some other rule for event selection named as "track mass" selection was used [7]

$$M_{tr} - m_\pi < \Delta M_{tr} \simeq 10 \text{ MeV}. \qquad (6)$$

The track mass M_{tr} for multiphoton annihilation process $e^+ + e^- \to \pi^+ + \pi^- + n\gamma$ is defined in such a way that $E_\pm = \sqrt{\mathbf{p}^2_\pm + M^2_{tr}}$ provided that for the lost energy one must use $2E - E_+ - E_- = |\mathbf{K}|$ but not Ω as it is given in (4). It is clear that in Born approximation $\Omega = |\mathbf{K}|$, $M_{tr} = m_\pi$ and for $n > 1$ always $M_{tr} > m_\pi$. At measured 3–momenta $\mathbf{p}_+$ and $\mathbf{p}_-$

$$M^2_{tr} = \frac{1}{4}\Big[(2E - |\mathbf{K}|)^2 - 2(\mathbf{p}^2_+ + \mathbf{p}^2_-) + \frac{(\mathbf{p}^2_+ - \mathbf{p}^2_-)^2}{(2E - |\mathbf{K}|)^2}\Big]. \qquad (7)$$

To express the squared pion mass we have to substitute Ω instead of $|\mathbf{K}|$ in the right–hand side of Eq.(7). It is clear from physical reason that track mass selection has to lead to some constraint like (4) and (5) because there is not other possibility to avoid 3–pion final states (at measured 3–momenta of charged pions) except to forbid large lost invariant mass.

By expanding the difference $M^2_{tr} - m^2_\pi$ with respect to small quantities $\Delta M_{tr}/E$ and $(\Omega - |\mathbf{K}|)/E$ and using inequality (6) we arrive at

$$\Omega < E(1-z)\big[1 + \frac{\eta(x)}{1+z}\big], \\ \eta(x) = \frac{2m_\pi \Delta M_{tr}}{E^2} f(x), \\ f(x) = \frac{4x}{(1+x)^2}, \quad x = \frac{E_-}{E_+}. \qquad (8)$$

At fixed the squared dipion invariant mass q^2 the energies of pions can be expressed via the scattering angle of pion with respect to the electron beam direction and the pion mass by the following relation

$$E_- = \frac{2z[1 + z - (1-z)cK]}{(1+z)^2 - (1-z)^2c^2}, \\ K = \sqrt{1 - \frac{\delta^2}{z^2}[(1+z)^2 - (1-z)^2c^2]}, \\ E_+ + E_- = E(1+z), \ c = \cos\theta, \ \delta = \frac{m_\pi}{2E}, \qquad (9)$$

where θ is the π^- polar angle.

In limiting cases $z = 1$ (very soft the ISR photon) and $z = m^2_\pi/E^2$ (at threshold) $E_+ = E_-$ and $f(x) = 1$. In the most interest region of ρ–resonance ratio E_-/E_+ changes between 2 and 0.5 that gives

$$\frac{8}{9} < f(x) < 1.$$

The minimum and maximum values of the ratio E_-/E_+ corresponds to back–to–back events when pion with the smaller energy flies in the direction of the ISR photon ($c = 1$) and pion with the larger energy in opposite direction ($c = -1$). If the angle θ increases the maximum (minimum) value of this ratio decreases (increases) and the most events trend to concentrate near $f(x) \simeq 1$. Thus, with good enough accuracy one can calculate RC in the case of the track mass event selection by simple substitution

$$\eta \to \frac{\bar{\eta}}{1+z}, \quad \bar{\eta} = \frac{4m_\pi \Delta M_{tr}}{E^2} = 0.0225 \qquad (10)$$

in analytical formulae of work [14]. It follows from comparison inequalities (5) and (8) at $f(x) = 1$.

On Fig.1a we compare the total RC for event selection with restriction (6) (curve *1*) and (4) (curve *2*) in

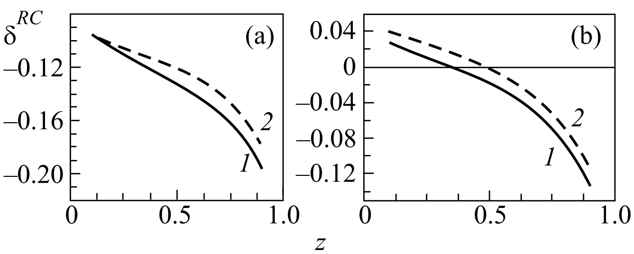

Fig.1. z – dependence of the quantity $\delta^{\rm RC}$ as given by Eq.(11) for different restrictions on event selection. Curves on Fig.1a derived at $\theta_0 = 5^o$ by the help of formulae (63) in [14] and rule (10). Curve *1* takes into account restriction (6) and curve *2* restriction (4). Fig.1b illustrates RC without any restriction on the lost invariant mass for $\theta_0 = 5^o$ (curve *1*) and $\theta_0 = 10^o$ (curve *2*) derived at this work using Eqs.(22) and (23)

term of quantity $\delta^{\rm RC}$ defined as

$$\frac{d\sigma_{\rm obs}}{dq^2} = \frac{d\sigma^B}{dq^2}(1+\delta^{\rm RC}), \qquad (11)$$

where $d\sigma_{\rm obs}$ is the observed cross section which includes effects of radiative corrections and $d\sigma^B$ is defined by Eq.(2). One can see that use of the track mass selection leads to slight loss (for about 1.5–2 per cent) in events in region of ρ–resonance ($0.5 < z < 0.7$) as compared with selection defined by rule (4). They say that efficiency of selection (4) is slightly higher than selection (6). As we see it depends on RC caused by an additional hard photon emission.

3. To estimate the absolute efficiency in event selection caused by restrictions (4) or (6) one needs to know the number of events without any such restriction. The rest of this paper dedicated to calculation RC in this case provided that at least one collinear ISR photon flies in direction of the electron beam. For the such type events the inequality

$$\mathbf{Kp_1} > |\mathbf{K}|Ec_0\ , \quad c_0 = \cos\theta_0, \qquad (12)$$

takes place, and to describe radiation of this photon we use QRE approximation for both, the cross section form and underlying kinematics.

The total RC includes contributions due to additional virtual, soft (with the energy less than ΔE, $\Delta \ll \ll 1$) and hard (with the energy more than ΔE) photon emission. Virtual and soft corrections do not depend on restrictions on quantity $\Omega - |\mathbf{K}|$ and must be taken as given in [14]. It is convenient to divide the correction caused by hard photon by three parts. The first one describes events when both photons fly inside the narrow cone with the opening angle $2\theta_0$ along the electron beam direction. The corresponding contribution into cross section at used accuracy also does not depend on above mentioned restrictions and found in [14].

The second part corresponds to events when one photon flies in the electron beam direction whereas the second one along the positron beam inside the narrow cone with the opening angle $2\theta_0'$, $\theta_0' \ll 1$. In the frame of QRE approximation the respective cross section has the form

$$\frac{d\sigma_2^H}{dq^2} = \frac{\sigma(q^2)}{4E^2}\left(\frac{\alpha}{2\pi}\right)^2 \int\limits_{\sqrt{z}}^{1-\Delta} P\Big(\frac{z}{y}, L_0\Big) P(y, L_0')\frac{dy}{y},$$
$$L_0' = \ln\frac{E^2\theta_0'^2}{m^2}, \qquad (13)$$

where the energy fraction of photon in the electron beam direction is $1 - z/y$ and in the positron beam $1 - y$. To determine the lower limit of integration we have to bear in mind that event must be looked as radiation along the electron beam (due to inequality (12)), therefore energy fraction $1 - z/y$ must be larger than $1 - y$ or $y > \sqrt{z}$.

It is convenient to represent the result of integration in the right–hand side of Eq.(13) as

$$\frac{d\sigma_2^H}{dq^2} = \frac{\sigma(q^2)}{4E^2}\left(\frac{\alpha}{2\pi}\right)^2 \Big[2P(z,L)\ln(1-z)(L_0-1) - \\ - \frac{1+3z^2}{2(1-z)}\ln z L L_0 + \Big(\frac{z(1+z)}{1-z}\ln z - \\ - 1 + z\Big)(L + L_0) - 2(1-z)\ln(1-\sqrt{z})l_\theta - \\ - \frac{2z}{1-z}\ln z - 2P(z, L_0)(L-1)\ln\Delta - G_4 l_\theta'\Big], \qquad (14)$$

$$l_\theta' = \ln\frac{\theta_0'}{4},$$

$$G_4 = \Big(\frac{1+3z^2}{2(1-z)}\ln z + 1 - z\Big)L_0 - \\ - \frac{z(1+z)}{1-z}\ln z + 2(1-z)\ln(1-\sqrt{z}),\ L = \frac{4E^2}{m^2}.$$

Note that parameter θ_0 defines rule (12) for event selection and is the physical one, while infrared parameter Δ and angular collinear parameter θ_0' are auxiliary and have to disappear in final result for the total RC. The auxiliary angular parameter vanish when the third part of RC due to hard photon emission will be added. This part corresponds to events when one hard photon with 4–momentum $k_1 = (\omega_1, \mathbf{k}_1)$ belongs to the forward narrow cone (with respect to the electron beam) and the other one with 4–momentum $k_2 = (\omega_2, \mathbf{k}_2)$ escapes both (forward and backward) narrow cones, but 3–momentum $\mathbf{K} = \mathbf{k_1} + \mathbf{k_2}$ lies inside the forward cone as defined by condition (12).

In accordance with QRE approximation the starting point for our calculations of the respective differential cross section, suitable for the unrestricted pion phase space, is the following

$$d\sigma_3^H = \frac{\sigma(q^2)}{4E^2}\frac{\alpha}{2\pi}P(x,L_0)L^{\gamma}_{\mu\nu}(xp_1,p_2,k_2)\tilde{g}_{\mu\nu}\frac{\alpha}{4\pi^2}\frac{dxd^3k_2}{x\omega_2}, \quad (15)$$

$$\frac{d^3k_2}{\omega_2} = 2\pi\omega_2 d\omega_2 dc_2, \quad c_2 = \cos\theta_2\ , \quad x = 1 - \frac{\omega_1}{E},$$

where θ_2 is the polar angle of the non–collinear photon and for $L^{\gamma}_{\mu\nu}(p_1,p_2,k)$ see, for example [15]. Since our aim is to derive the differential distribution in the squared pion invariant mass q^2, it is convenient to use the relation between q^2 and c_2 to avoid the integration over c_2 in the right–hand side of Eq. (15). In addition, it is convenient to use the total photon energy Ω instead of ω_2

$$q^2 = 4E(E-\Omega) + 2\omega_1\omega_2(1-c_2),\ \omega_2 = \Omega - \omega_1,$$
$$dc_2 \to \frac{dq^2}{2\omega_1\omega_2},\ d\omega_2 = d\Omega. \quad (16)$$

Because the photon with 4–momentum k_2 is non–collinear one, we can neglect the electron mass in expression for $L^{\gamma}_{\mu\nu}$ in the right–hand side of Eq.(15) and write the distribution over the dipion squared invariant mass in the following form

$$\frac{d\sigma_3^H}{dq^2} = \frac{\sigma(q^2)}{4E^2}\Big(\frac{\alpha}{2\pi}\Big)^2 \times \Big\{ -L_0 - \frac{4(L_0-1)E\Omega_z}{\omega_1^2} - \frac{zE^2L_0}{(E-\omega_1)^2} + \frac{2zE - [(1+z)E+\Omega_z]L_0}{E-\omega_1} + \quad (17)$$
$$+\frac{[2(1+z)^2-4z-(1-z)^2]E^2 + (1+z)(1-2L_0)E\Omega_z + \Omega_z^2L_0 - (1-z)(\Omega-2\omega_1)E}{Z}\Big\}\frac{d\omega_1 d\Omega}{E\Omega_z},$$
$$\Omega_z = \Omega - E(1-z)\ , \quad Z = \omega_1(\Omega-\omega_1) - E\Omega_z.$$

This distribution differs from given in [14] where restriction (4) was also applied and approximation $\Omega = E(1-z)$ used for all non–singular at this point terms. It is no case for considered situation because the upper limit of integration with respect to Ω now differs considerably from $E(1-z)$. To find the integration region in (17) we have to take into account together with condition (12) also

$$-c_0' < c_2 < c_0, \quad \Delta E < \omega_1 < \Omega - \Delta E, \quad c_0' = \cos\theta_0'\ . \quad (18)$$

The system of inequalities (12) and (18) defines the integration region with respect to ω_1 and Ω as given by relations (47) in [14] with the only but very essential change: $\Omega_{\max} = 2E(1-\sqrt{z})$. This region is shown on Fig.2.

The list of all necessary integrals which contribute in the limiting case $1-c_0 \ll 1,\ 1-c_0' \ll 1$ is

$$\frac{1}{E\Omega_z} = 2(1-z), \quad \frac{1}{\Omega_z(E-\omega_1)} = \ln^2 z + 2Li_2(1-z), \quad \frac{E}{\Omega_z(E-\omega_1)^2} = -\frac{2}{z}\ln z\ ,$$
$$\frac{1}{Z} = -\frac{1}{2}\ln z(l_\theta - l_\theta') + \frac{\pi^2}{6} - \frac{1}{4}\ln^2 z + 2Li_2(-\sqrt{z}),\ l_\theta = \ln\frac{\theta_0^2}{4},$$
$$\frac{\Omega_z}{EZ} = \big[z - 1 - \frac{1+z}{2}\ln z\big](l_\theta - l_\theta') + 2\sqrt{z}(1-\sqrt{z}) + z\ln z - (1-z)\ln(1-\sqrt{z}) +$$
$$+ (1+z)\big[\frac{\pi^2}{6} - \frac{1}{4}\ln^2 z + 2Li_2(-\sqrt{z})\big],$$
$$\frac{E}{Z\Omega_z} = \frac{1}{1-z}\big[-\frac{1}{2}l_\theta^2 - (l_\theta + l_\theta')\ln\frac{1-z}{\Delta} - \frac{1}{2}\ln z(l_\theta + l_\theta') + \frac{\pi^2}{3} - \frac{1}{4}\ln^2 z + 2Li_2(-\sqrt{z})\big],$$
$$\frac{\Omega - 2\omega_1}{Z\Omega_z} = \frac{1}{2}l_\theta^2 + (l_\theta + l_\theta')\ln\frac{1-z}{\Delta} + 2(l_\theta - l_\theta')\ln(1+\sqrt{z}) + 2\ln^2(1+\sqrt{z}) -$$
$$-4Li_2(1-\sqrt{z}) + 4Li_2\Big(\frac{1-\sqrt{z}}{1+\sqrt{z}}\Big) + \frac{\pi^2}{2}, \quad (19)$$

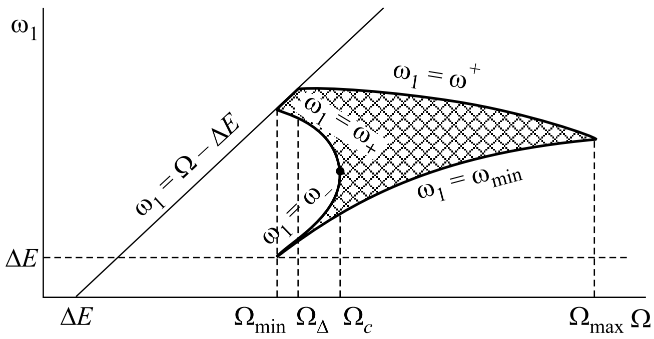

Fig.2. The integration region with respect to ω_1 and Ω in cross section (17), as defined by inequalities (47) in [14] (where the all notation also given) but with $\Omega_{\max} = 2E(1-\sqrt{z})$

where we omit for a brevity symbol of integral and differentials $d\omega_1$ and $d\Omega$ in left sides of these relations.

The corresponding cross section reads

$$\frac{d\sigma_3^H}{dq^2} = \frac{\sigma(q^2)}{4E^2}\left(\frac{\alpha}{2\pi}\right)^2 \times \times \left[2P(z,L_0)G_1 + L_0G_2 + G_3 + G_4 l'_\theta\right], \qquad (20)$$

$$G_1 = -\frac{1}{2}l_\theta^2 - \ln\frac{(1-z)\sqrt{z}}{\Delta}l_\theta + \frac{\pi^2}{3} - \frac{1}{4}\ln^2 z + 2Li_2(-\sqrt{z}),$$

$$G_2 = \left[z - 1 + \frac{1+z}{2}\ln z\right]l_\theta - 2(1-\sqrt{z}) + +(2+z)\ln z - 2(1-z)\ln(1+\sqrt{z}) - -(1+z)\left[\frac{\pi^2}{6} + \frac{3}{4}\ln^2 z + Li_2(1-z) + Li_2(-\sqrt{z})\right],$$

$$G_3 = -[2(1-z)\ln(1+\sqrt{z}) + z\ln z] + +(3z-2)\frac{\pi^2}{3} + z\left[\frac{3}{2}\ln^2 z + 4(Li_2(1-z) + Li_2(-\sqrt{z}))\right] + +2(1-z)\left[-\ln^2(1+\sqrt{z}) + 2\left(Li_2(1-\sqrt{z}) - Li_2\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)\right)\right].$$

As one can see terms containing l'_θ enter with opposite signs in (14) and (20) and vanish in their sum.

To eliminate the infrared auxiliary parameter Δ and write the total RC to the Born cross section (2) of radiative process (1) at considered conditions for event selection we have to sum all possible contributions

$$\frac{d\sigma^{RC}}{dq^2} = \frac{d\sigma^{S+V}}{dq^2} + \frac{d\sigma_1^H}{dq^2} + \frac{d\sigma_2^H}{dq^2} + \frac{d\sigma_3^H}{dq^2}. \qquad (21)$$

The expressions for $d\sigma^{S+V}$ and $d\sigma_1^H$ are given by Eqs.(30), (31) and (35)–(37) in [14], respectively. Using these expressions as well (14) and (20) after some algebraic exercise we arrive at

$$\frac{d\sigma^{RC}}{dq^2} = \frac{\sigma(q^2)}{4E^2}\left(\frac{\alpha}{2\pi}\right)^2 \times \left[P_{2\theta}(z)L_0^2 + P(z,L_0)H_1(l_\theta,z) + L_0H_2(z) + H_3(z)\right], \qquad (22)$$

where $P_{2\theta}(z)$ is the well known θ–term of the second order electron structure function in nonsinglet channel caused by photonic corrections

$$P_{2\theta}(z) = 2\frac{1+z^2}{1-z}\left(\ln\frac{(1-z)^2}{z} + \frac{3}{2}\right) + (1+z)\ln z - 2 + 2z$$

and

$$H_1(l_\theta,z) = -l_\theta^2 - l_\theta\left(3 + 2\ln(1-z)^2 z\right) - \frac{5}{2} - -4\ln\frac{1-z}{z} - \frac{1}{2}\ln^2 z + \frac{5\pi^2}{3} + 4Li_2(-\sqrt{z}) - 2Li_2(z),$$

$$H_2(z) = 2\sqrt{z} - \frac{1+z}{2} - \frac{9z}{1-z} + \frac{1+5z}{1-z}\ln z - -\frac{1+z}{4}\ln^2 z - 2(1-z)\ln(1+\sqrt{z}) - -(1+z)\left(\frac{\pi^2}{6} + 2Li_2(1-z) + 2Li_2(-\sqrt{z})\right), \qquad (23)$$

$$H_3(z) = -\frac{8}{3} + \frac{5}{3(1-z)} + \frac{3-18z+7z^2}{3(1-z)^2} + +\frac{3-3z+3z^2-9z^3-2z^4}{6(1-z)^3}\ln^2 z + 2z\ln^2(1-z) - -2(1-z)\ln^2(1+\sqrt{z}) + \left(\frac{26}{3z} - 2 - \frac{1}{1-z}\right)\frac{\pi^2}{6} + +\left(4z - 2 + \frac{1}{1-z}\right)Li_2(z) + \frac{3+5z}{3}Li_2(1-z) - -4(1-z)Li_2\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right) + +4(1-z)Li_2(1-\sqrt{z}) + 4zLi_2(-\sqrt{z}) + J(z).$$

The quantity $J(z)$ in H_3 is defined as

$$J = \int\limits_0^{1-z}\left\{\frac{z^2+(1-x)^4}{x\lambda(1-x)^2}\left[\frac{1}{2}L_2^2(x,z) + Li_2\left(\frac{2xz}{h}\right) + +Li_2\left(-\frac{2x^2\lambda}{h}\right) + Li_2\left(-\frac{\lambda}{xz}\right)\right] + +\frac{z+x}{2(1-x)}L_1(x,z) + \frac{x\lambda - 3z}{2(1-x)^2}L_2(x,z)\right\}dx,$$

where

$$h = \lambda(1-x)^2 + x(z - x\lambda) + (1-x)\sqrt{F(x,z)},$$

$$L_1(x,z) = \ln\frac{(x+z)\sqrt{F(x,z)} + \lambda(z - x\lambda) + x(x+z)^2}{2z\lambda},$$

$$L_2(x,z) = \ln \frac{(1-x)\sqrt{F(x,z)} + x(z - x\lambda) + \lambda(1-x)^2}{2\lambda(1-x)^2} ,$$

$$F(x,z) = \lambda^2(1-x)^2 + 2x\lambda(z - x\lambda) + 2x^2(x+z)^2 .$$

On Fig.1b we show quantity $\delta^{\rm RC}$ (at two values of θ_0) defined in the same way as in Eq.(11), where $\delta^{\rm RC} = d\sigma^{\rm RC}/d\sigma^B$ and now $d\sigma^{\rm RC}$ is given by Eq.(22). Change of the limiting angle from 5^o up to 10^o increases RC for about 1.5 per cent. On the other hand, the curves on Figs.1a and 1b at fixed angle θ_0 indicates that the use of restrictions (4) or (6) decreases the number of events for about 10 per cent as compared with considered case without constraint on the lost invariant mass.

Both these effects can be understood very easy on the quality level because they caused by an expansion of the real photon phase space, which provides additional positive contribution into RC. Note that the loss in events due to use of restrictions (4) or (6) is small enough and is very modest payment for the 3–pion background that can be removed by them.

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