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Дискуссия

Mach cones and magnetic forces in Saturn's rings

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Recently, Mamun, Shukla, and Bingham [2] have claimed that Havnes and his collaborators [1] mistakenly neglected magnetic fields in their work on Mach cones as potentially powerful diagnostics of properties in Saturn's rings. We show that the magnetic force on a charged particle is entirely negligible in comparison to the electric force on the particle in a wave with a wavenumber relevant to the Saturnian Mach cone problem. Havnes et al. [1] were not in error.

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Havnes and his collaborators [1] have pointed out that the detection and observation of Mach cones around larger solid bodies in Saturn's ring system would provide a valuable means of diagnosizing properties of the dusty ring plasma. They have based their analysis on the consideration of the properties of dust acoustic waves in a nonmagnetized plasma. Recently Mamun et al. [2] have argued that the analysis must be based on the study of hydromagnetic waves in a dusty plasma, e.g., [3]. Here we show that Havnes et al. [1] have been justified in the neglect of the magnetic field in their studies of Mach cones in Saturn's rings.

We consider a dust–ion plasma as in [2] and take ρ_d, ρ_i, $v_d\hat{z}$, $v_i\hat{z}$, $\rho_i c_i^2$, $Z_d e$, e, m_d m_i, and $E\hat{z}$ to be the mass density of dust, the mass density of ions, the dust velocity, the ion velocity, the thermal pressure of the ions, the charge carried by a dust grain, the charge of a proton, the mass of a dust grain, the mass of an ion, and the electric field, where $\hat{z}$ is the unit vector in the z direction. c_i is taken to be a constant. Waves are assumed to propagate in the z direction on a uniform static background medium. Background quantities are signified by subscript 0, and perturbation quantities are signified by subscript 1. Perturbation quantities are assumed to vary with time, t, and z as $\exp(i\omega t - ikz)$.

The linearized equations governing the propagation of a dust acoustic wave in the two fluid medium are

$$i\omega\rho_{d1} - ik\rho_{d0}v_{d1} = 0 \tag{1}$$

$$i\omega\rho_{d0}v_{d1} = \frac{\rho_{d0}}{m_d}Z_d eE \tag{2}$$

$$i\omega\rho_{i1} - ik\rho_{i0}v_{i1} = 0 \tag{3}$$

$$-ikc_i^2\rho_{i1} = \frac{\rho_{i0}}{m_i}eE \tag{4}$$

$$-ikE = 4\pi e\left(\frac{\rho_{i1}}{m_i} + \frac{Z_d\rho_{d1}}{m_d}\right) \tag{5}$$

We have taken the pressure of the dust fluid to be zero. The dust fluid is assumed to have a much greater density than the ion fluid leading to the neglect of the ion fluid's inertia in equation (4). We note that $\rho_{i0}/m_i = -(\rho_{d0}/m_d)Z_d$. As pointed out by Mamun et al. [1] for low frequency dust waves unaffected by the magnetic field $\omega/k = (Z_d^2\rho_{d0}k_B T_{i0}m_i/\rho_{i0}m_d^2)^{1/2}$ where k_B and T_{i0} are Boltzmann's constant and the ion temperature. $c_i = (k_B T_{i0}/m_i)^{1/2}$.

We take B_0 to be the strength of the background magnetic field. The maximum magnitude of the component of the ion velocity perpendicular to the direction of propagation is given by

$$|v_{i\perp}| = \left(\frac{e|v_{i1}|B_0}{c}\right)\left(\frac{1}{m_i\omega}\right) = \frac{\omega_{gi}}{\omega}\,|v_{i1}| \tag{6}$$

where c is the speed of light and $\omega_{gi} \equiv eB_0/m_i c$ is the ion gyrofrequency.

The magnetic force per unit volume in the z direction on ions has a maximum magnitude of

$$\begin{aligned} F_{BZ} &= \frac{\rho_{i0}}{m_i}\,e\,\frac{\omega_{gi}}{\omega}\,\frac{|v_{i1}|}{c}\,B_0 \\ &= \rho_{i0}\,\frac{\omega_{gi}^2}{\omega}\,|v_{i1}| \end{aligned} \tag{7}$$

The ratio of F_{BZ} to the magnitude of the pressure force per unit volume on the ions is

$$R \equiv \left(\rho_{i0}\,\frac{\omega_{gi}^2}{\omega}\,|v_{i1}|\right)\left(\frac{1}{k|\rho_i|c_i^2}\right) = \frac{\omega_{gi}^2}{\omega^2}\,\frac{\rho_{i0}}{\rho_{d0}} \tag{8}$$

The magnetic force is negligible if

$$R \ll 1 \tag{9}$$

For a magnetic field strength of 0.03 Gauss, at the co-rotation distance, and an ion mass of 16 a.m.u., $\omega_{gi} \approx 18\ \mathrm{s}^{-1}$. We assume that each grain is spherical with a radius of 0.25 μm and is composed of material with a density of 1 g cm^{-3}. Thus, $\rho_{i0}/\rho_{d0} = 5 \cdot 10^{-10}\ |Z_d|$. For a medium in which $T_{i0} = 10$ eV, the frequency of a wave unaffected by the magnetic field is given by $\omega = 2|Z_d|^{1/2}/\lambda$, where λ is the wavelength in meter.

Condition (9) shows that the neglect of the magnetic field in the work by Havnes et al. [1] on Mach cones in Saturn's rings is entirely justified and that $R \approx 0.01$ for $\lambda \simeq 500$ m. An examination of the results of Mamun et al. [2] also show that for wavelengths below several hundred meters, which will be generated by boulders, the effect of the magnetic field is negligible, and their dispersion relation reduces to that of the dust acoustic wave. For wavelengths of more than about 1 km the magnetic field becomes progressively more important and should be included. Li and Havnes [3] have done a study of such waves in Saturn's rings based on kinetic theory with magnetized ions and electrons but unmagnetized dust. However, such large wavelengths should not be important for Mach cones generated by large boulders.

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