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Comment on Mach cones and magnetic forces in Saturn's rings

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We clarify the underlying physics of long wavelength dust magnetoacoustic waves and short wavelength (in comparison with the ion gyroradius ρ_i) dust acoustic waves that are involved in the formation of Mach cones in a magnetized dusty plasma of Saturn's rings.

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About eight years ago, Havnes et al. [1] employed the dispersion relation of Rao, Shukla and Yu's [2] dust acoustic waves (DAWs) for an unmagnetized dusty plasma to predict the formation of dust acoustic Mach cones in Saturn's rings. The DAWs are low-frequency (in comparison with the dust plasma frequency), long wavelength (in comparison with the dusty plasma Debye radius [3]) electrostatic waves in which the restoring force comes from the pressures of inertialess electrons and ions, while the dust mass provides the inertia to maintain the waves. Thus, Havnes et al. [1] used the phase speed $V_p(=\omega/k)$ of unmagnetized DAWs to obtain the Mach cone opening angle $\theta=\sin^{-1}(V_b/V_p)$, where V_b $(>V_p)$ is the speed of a dust boulder that lies in the equatorial plane of Saturn's rings, which shall provide valuable information regarding the plasma and dust parameters once the Cassini spacecraft starts gathering data in July 2004. Since the dusty plasma in Saturn's rings is magnetized, it is very important to understand the properties of waves in a dusty magnetoplasma [3].

In a recent Letter, Mamun, Shukla and Bingham [4] predicted the formation of Mach cones involving low-frequency (in comparison with the dust gyrofrequency ω_{cd}), long wavelength (in comparison with the ion gyroradius ρ_i) slow dust magnetoacoustic waves in an ion-dust plasma. Mamun et al. [4] found that for the plasma parameters of Saturn's rings (viz. Saturn's magnetic field $B_0\sim 0.1$ G, the dust number density $n_d=10$ cm^{-3}, the dust charge number $Z_d\sim 10^3$, the dust material mass density 1 gm/cm^3, the dust radius 0.5 micron, the ion number density $n_i\sim 10^4$ cm^{-3}, the ion temperature $T_i\sim 10$ eV, the ion Debye radius $\lambda_{De}\simeq 23$ cm, and ion gyroradius $\rho_i\sim 45$ m), the dust Alfvén speed $V_{Ad}(=B_0/\sqrt{4\pi n_d m_d})$ is much larger than the DAW speed $C_D[=(Z_d^2 n_d T_i/n_i m_d)^{1/2}]$, since Saturn's plasma $\beta(=8\pi n_i T_i/B_0^2)$ is pretty low ($\sim 10^{-4}$), where m_d is the mass of the dust particle. Accordingly, they suggested that long wavelength slow dust magnetoacoustic waves propagating almost perpendicular to the external magnetic field direction are a viable candidate for the Mach cone formation in the equatorial region of Saturn's rings where a dust boulder is moving in a Keplerian orbit. Hartquist and Havnes [5] refute this scenario since they failed to derive dispersion relations (8) and (9) in Ref. [4], which have been obtained from our exact equations (4) and (5). It should be stressed that the forms of the low-frequency ($\ll\omega_{cd}=Z_d e B_0/m_d c$, where e is the magnitude of the electron charge and c is the speed of light in vacuum) dust shear Alfvén and dust magnetoacoustic wave dispersion relations in an ion-dust plasma are identical to those of the hydromagnetic waves in an electron-ion plasma [6], except that the role of electrons and ions is replaced by ions and dust, respectively. It turns out that in the phase speed of the dust magnetoacoustic waves, we have the dust Alfvén speed $(B_0/\sqrt{4\pi n_d m_d})$ and the ion skin depth (c/ω_{pi}), in contrast to the usual Alfvén speed $(B_0/\sqrt{4\pi n_i m_i})$ and the electron skin depth (c/ω_{pe}) in an electron-ion plasma, where ω_{pi} (ω_{pe}) is the ion (electron) plasma frequency and m_i is the ion mass. Thus, in slow dust magnetoacoustic waves the restoring force comes from the magnetic pressure and the dust mass provides the inertia. The wave dispersion is due to the ion inertial force. For Saturn's plasma parameters, the wavelengths of slow dust magnetoacoustic waves, which could be involved in the formation of Mach cones, are in the range of several hundred meters to a few kilometers, depending on the values of B_0 which vary between $0.025-0.2$ G in Saturn's rings where $c/\omega_{pi}\simeq 3.5$ km. As an illustration, we mention that for 2 km scale size slow dust magnetoacoustic waves we have $V_d/V_p\simeq 1.1$, $V_d/V_p\simeq 3.8$, and $V_d/V_p\simeq 3.8$ for $B_0=0.01$ G, $B_0=0.05$ G, and $B_0=0.1$ G, respectively.

Furthermore, we have critically examined the arguments of Hartquist and Havnes [5] regarding the neglect of the Lorentz force in the ion dynamics involved in *short* *wavelength, electrostatic DAWs* in a magnetized dusty plasma that we discussed on p. 645 of Ref. [4]. We note that a fluid ion response, given by Eq. (4) in Ref. [5], is incorrect for short wavelength ($b_i = k_\perp^2 \rho_i^2 \gg 1$, where $\mathbf{k}_\perp$ is the component of the wavevector $\mathbf{k}$ perpendicular to Saturn's magnetic field $\hat{\mathbf{z}} B_0$ and $\hat{\mathbf{z}}$ is the unit vector along the z axis) DAWs. For low-frequency ($\omega \ll \omega_{ci} = eB_0/m_i c$) arbitrary wavelength electrostatic modes, we must use, instead of Eq. (4) in Ref. [5], the ion density perturbation [7]

$$n_{i1} = -n_i \left[1 - \Gamma_0(b_i) + 2\Gamma_1(b_i) \frac{\omega^2}{\omega_{ci}^2} \right] \frac{e\phi}{T_i}, \qquad (1)$$

for two-dimensional ions in equatorial plane of Saturn's rings. Here, $\Gamma_{0,1}(b_i) = I_{0,1}(b_i) \exp(-b_i)$, I_0 (I_1) is the modified Bessel function of zero (first) order and ϕ is the DAW potential. For $b_i \gg 1$, we can approximate Eq. (1) as

$$n_{i1} = -\frac{n_i e\phi}{T_i}, \qquad (2)$$

which is a Boltzmann distribution associated with an ion susceptibility $\chi_i \approx 1/k^2 \lambda_{Di}^2$, where $\lambda_{Di} = (T_i/4\pi n_i e^2)^{1/2}$ is the ion Debye radius. Equation (2) physically dictates that in the potential of short wavelength DAWs, unmagnetized ions execute a straight-line orbit across $\hat{\mathbf{z}}$ and they charge neutralize negatively charged dust which have very slow motion. The dust number density perturbation n_{d1} for $\omega^2 \gg \omega_{cd}^2$ is [3]

$$n_{d1} = -\frac{n_d Z_d e}{m_d} k^2 \phi. \qquad (3)$$

For $\omega \ll k_z V_{Te}, \omega_{ce} k_z / k_\perp$, where k_z is the component of the wavevector along $\hat{\mathbf{z}}$, V_{Te} is the electron thermal speed and ω_{ce} is the electron gyrofrequency, the electrons rapidly thermalize along $\hat{\mathbf{z}}$ and establish a Boltzmann distribution. The corresponding electron number density perturbation is

$$n_{e1} = \frac{n_e e\phi}{T_e}, \qquad (4)$$

where T_e is the electron temperature and $n_e = n_i + Z_d n_d$. Substituting (2), (3), and (4) into Fourier transformed Poisson's equation we have the frequency of short wavelength dispersive DAWs

$$\omega = \frac{k \lambda_{Di} \omega_{pd}}{(1 + \sigma + k^2 \lambda_{Di}^2)^{1/2}} \equiv \frac{k C_D}{(1 + \sigma + k^2 \lambda_{Di}^2)^{1/2}}, \qquad (5)$$

which coincides with Rao, Shukla, and Yu's unmagnetized DAW frequency that Havnes et al. [1] employed to predict the DAW Mach cone in Saturn's ring. Here, $\omega_{pd} = (4\pi Z_d^2 e^2 n_d / m_d)^{1/2}$ is the dust plasma frequency and $\sigma = n_e T_i / n_i T_e \ll 1$. It turns out that for Saturn's plasma parameters the wavelengths of the DAWs should be $\sim$ 30 m for $\omega \simeq 1$ s^{-1}, T_i=10 eV, $T_e = 10\ T_i$, $n_i = 10^4$ cm^{-3}, $n_e = 0.1 n_i$, $\omega_{pd} \simeq 21$ s^{-1}, and $\lambda_{Di} \simeq 23$ cm.

In conclusion, we have discussed the drawbacks of the research carried out in Ref. [5], and have clarified the underlying physics of low-frequency ($\ll \omega_{cd}$), long wavelength ($k_\perp^2 \rho_i^2 \ll 1$) slow dust magnetoacoustic waves and short wavelength ($k_\perp^2 \rho_i^2 \gg 1$) intermediate frequency ($\omega_{cd} \ll \omega \ll \omega_{ci}$) DAWs that may participate in the formation of Mach cones in Saturn's rings. These transverse and longitudinal waves in magnetized dusty plasmas can be generated by ion temperature anisotropy and electron/ion beams. The resonance interaction between a dust boulder and short and long wavelength modes, as discussed here, can give rise to Mach cones in the equatorial plane of Saturn's rings, which should be detectable by on board instruments of the Cassini spacecraft.

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