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Solitons in a disordered anisotropic optical medium

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The radiation mediated interaction of solitons in a one-dimensional nonlinear medium (optical fiber) with birefringent disorder is shown to be independent of the separation between solitons. The effect produces a potentially dangerous contribution into the signal lost.

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The propagation of a pulse through an optical fiber with randomly varying anisotropy is usually addressed in the context of the Polarization Mode Dispersion (PMD). PMD is signal broadening caused by inhomogeneity of the medium birefringence. In the linear case, the study of PMD was pioneered by Poole [1], who showed that the pulse broadens as the two principal states of polarization split under the action of the random birefringence (see also [2]). Mollenauer and coauthors have numerically studied a nonlinear model of birefringent disorder in [3], where it was shown that a soliton, launched into the birefringent fiber, does not split but it does undergo spreading [3] (see also [4]). In this letter, we develop an analytical approach and confirm that a single soliton does degrade due to disorder in the birefringence. The degradation is observable once the soliton traverses the distance $z_{\rm degr} \sim D^{-1}$, where D stands for the strength of the noise in the birefringence, measured in units of the soliton width and period ($D \ll 1$ is assumed, the typical case for telecommunication fibers).

The *major* finding of this letter is a new phenomenon which occurs at scales much shorter than $z_{\rm degr}$. We report that the interaction between solitons induced by their combined radiation (generated by disorder) is an important factor affecting the soliton dynamics. Initially stationary solitons experience a relative acceleration, $\sim D$. The inter-soliton separation changes on the order of the soliton width at $z_{\rm int} \sim 1/\sqrt{D} \ll z_{\rm degr}$. We use and generalize here an approach developed previously to describe solitons interacting in an isotropic medium with fluctuating dispersion [5]. The soliton interaction, in the case of [5], decays algebraically. By contrast, in the anisotropic case discussed in this letter the interaction is separation-independent. The reason is that, in this case, a different type of waves scatters from the solitons. In the isotropic case, the scattering of the radiated waves, emitted by a soliton, on another soliton is not refracted. In the anisotropic case, radiation from one soliton pushes (literally) the other soliton, because the scattering potential is not transparent.

Let us briefly describe the problem setup. The electric field $\mathbf{E}$, corresponding to a carrying frequency ω wave packet, can be decomposed into complex components $\mathbf{E} = 2{\rm Re}\,[\mathbf{E}_\omega \exp(ik_0 z - i\omega t)]$, where z is the coordinate along the fiber. Concomitant averaging over fast oscillations and over the structure of fundamental mode (a mono-mode regime is assumed) constitutes the coarse-grained description for the signal envelope, described by the two-component complex field Ψ_α, $\overline{\mathbf{E}}_\omega = \Psi_1(z)\mathbf{e}_1 + \Psi_2(z)\mathbf{e}_2$, where $\mathbf{e}_{1,2}$ are unit vectors, orthogonal to each other and to the waveguide direction. The averaging results in the envelope equation [6, 7]

$$i\partial_z\Psi_\alpha - \Delta_{\alpha\beta}\Psi_\beta - im_{\alpha\beta}\partial_t\Psi_\beta + \partial_t^2\Psi_\alpha + \\ +\frac{4}{3}\left(|\Psi_1|^2 + |\Psi_2|^2\right)\Psi_\alpha + \frac{2}{3}\left(\Psi_1^2 + \Psi_2^2\right)\Psi_\alpha^* = 0. \quad (1)$$

Here, the wave packet is subjected to dispersion in retarded time t and to the Kerr nonlinearity, which is described by the last two terms on the lhs of (1). The matrix $\hat\Delta$ describes the differences in the wavevectors. The matrix $\hat m$ describes the anisotropy in the group velocity for the two distinct states of polarization (of the respective linear problem). The isotropy is broken in (1), because the core of any fiber is elliptic rather than circular in cross-section. It is assumed in (1) that the dispersion term and the nonlinear term are isotropic since in real fibers anisotropy of dispersion and nonlinearity is usually less important than the effects of anisotropy described by the matrices $\hat\Delta$ and $\hat m$. The coefficients of

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nonlinearity and dispersion are re-scaled to unity, i.e. t and z are already dimensionless in (1). If the matrices, $\hat{m}$ and $\hat{\Delta}$, are zero the full problem is isotropic, and (1) supports the constant polarization solution, e.g. $\Psi_2 = 0$. Then the equation for Ψ_1 is the Scalar Nonlinear Shrödinger (SNLS) equation. The self-conjugated matrix $\hat{\Delta}$ is traceless, as the trace can be excluded by a simple phase transformation. The (also self-conjugated) matrix $\hat{m}$ is traceless as (1) is written in the reference frame moving with the mean group velocity. Both $\hat{\Delta}$ and $\hat{m}$ may contain regular and disordered parts. In a polarization maintaining fiber, at least one of the regular parts is nonzero. If the phase change between the two polarizations caused by a regular part (say $\hat{\Delta}_{\rm reg}$) becomes ~ 1 at a scale, $z_{\rm reg}$, an additional averaging over the distances larger than $z_{\rm reg}$ reduces (1) to [6, 7]

$$\left(i\partial_z+\partial_t^2+2\left|\Psi_1\right|^2+2\varepsilon\left|\Psi_2\right|^2\right)\Psi_1=(\Delta_{1\beta}+im_{1\beta}\partial_t)\,\Psi_\beta, \quad (2)$$

and analogously for Ψ_2. The quantities $\hat{m}$ and $\hat{\Delta}$ left in Eq. (2) represent random contributions. Generically, eigenvectors of $\hat{\Delta}_{\rm reg}$ correspond to elliptic polarizations, and the corresponding eigenvalues are complex. The quantity ε in Eq. (2) measures the degree of ellipticity, $2/3 \le \varepsilon \le 2$. In the degenerate limit of linear polarization (the eigenvectors are real) $\varepsilon = 2/3$. Subsequent analysis is devoted to Models (1) and (2) with $\hat{\Delta} = 0$ and random zero mean $\hat{m}$. The anisotropy matrix, $\hat{m}$, can be written in terms of Pauli matrices as follows, $\hat{m} = \sum_k h_k(z)\hat{\sigma}_k$ where $k = 1, 2, 3$ and the real field h_k is a function of z only because the disorder is frozen in the fiber. The correlation scale of the random field $h_j(z)$ is short. (It is typically constrained by the process of fiber pulling from a silica preform, cabling and spooling into a bobbin.) Therefore, according to the Central Limit Theorem, $h_j(z)$ at the greater scales can be treated as a Gaussian random process. The noise intensity is described by the matrix $\hat{D}$, $D_{ik} = \int dz\,\langle h_i(z)h_k(z')\rangle$. One assumes that the isotropy is restored in average, i.e. $D_{ik} \propto \delta_{ik}$. Then, the statistics of $\hat{m}$ is characterized by

$$\langle h_i(z_1)h_k(z_2)\rangle = D\delta_{ik}\delta(z_1 - z_2). \quad (3)$$

Similarly, one assumes that, $\hat{\Delta} = \sum_k b_k(z)\hat{\sigma}_k$, and $\langle b_i(z_1)b_k(z_2)\rangle = D_b\delta_{ik}\delta(z_1 - z_2)$.

We start from the single soliton story. One looks for a solution of (1) or (2) in the form

$$\Psi_\alpha = \delta_{1\alpha}\exp(iz)\cosh^{-1}t + v_\alpha. \quad (4)$$

For $v_{1,2} = 0$, (4) represents a single soliton solution of the ideal, $\hat{m} = 0$, problem. If the disorder is weak, one can substitute (4) into (1) or (2) and linearize with respect to $v_{1,2}$ to get

$$\left(\partial_z - i\hat{L}_{1,2}\right)\begin{pmatrix} v_{1,2} \\ v_{1,2}^* \end{pmatrix} = \begin{pmatrix} R_{1,2} \\ R_{1,2}^* \end{pmatrix}\frac{\tanh t}{\cosh t} + \begin{pmatrix} Q_{1,2} \\ Q_{1,2}^* \end{pmatrix}\frac{1}{\cosh t}, \quad (5)$$

where $R_1 = h_3$, $R_2 = h_1 + ih_2$, $Q_1 = ib_3$, $Q_2 = ib_1 - b_2$, and $\hat{L}_{1,2}$ are second order in t differential operators of the linear Schrödinger kind, with soliton-shaped ($\propto 1/\cosh$) potentials. It is convenient to expand $v_{1,2}$ in eigenfunctions of the operators $\hat{L}_{1,2}$. Spectra of the operators are separated into continuous and discrete parts, $v_{1,2} = v_{1,2}^{(0)} + \tilde{v}_{1,2}$. The four zero modes of $\hat{L}_1$, are related to variations of the soliton's amplitude, position, phase and phase velocity. There is also a localized eigen-mode of $\hat{L}_2$ identified with variations of the soliton polarization. In the case of Model (1) the polarization eigenmode becomes a zero-mode of $\hat{L}_2$ correspondent to the freedom in rotation of polarization axes, and also, an additional zero mode related to ellipticity appears. Some localized modes are subjected to the linear, first order in disorder, response. Thus the position of the soliton, y, varies randomly in z: $\langle y^2\rangle = Dz$. Second order effects in radiation lead to variations of the soliton amplitude, η. From the conservation law which accounts for the balance of "energy" between the soliton and the continuous radiation ($v_{1,2} = 0$ at $z = 0$ is assumed), one derives, $\eta\tilde{D}\int_0^z dz'\eta^2(z') = 1 - \eta$, where the lhs and the rhs represent radiative and soliton contributions, respectively, into the energy balance, and $\tilde{D} \sim D$. Solution of the integral equation, valid at any z, is

$$\eta = (1 + \tilde{D}z)^{-1/3}. \quad (6)$$

(Note that the single soliton radiation in the degenerate case of (2) with $\varepsilon = 1$ was studied in [8], where analogs of the aforementioned integral equation were derived. The equation was analyzed in [8] under assumption that $zd\eta/dz \ll 1$, which led to an answer for the soliton amplitude degradation valid at, $zD \ll 1$, only, where it coincides with (6).)

We now turn to the multi-soliton case. Only scales shorter than $z_{\rm degr} = 1/D$ are discussed, so the random walk of y and the degradation of the soliton amplitude can be neglected. The same argument applies to the polarization angle, ϕ, in the case of Model (2). In the isotropic model case (1), the jitter of ϕ becomes important at $z_\phi \sim 1/D^{1/3}$. The effect, however, is collective: polarizations of different solitons rotate to the same angle, so that the relative polarization angle is unchanged at $z \ll z_{\rm degr}$. We consider the N-soliton solution,

$$\Psi_\alpha = \sum_{j=1}^{N}\exp\left[i\alpha_j + i\beta_j(t - y_j)\right]\cosh^{-1}(t - y_j)\delta_{1\alpha} + v_\alpha$$

of (1), (2). One derives (and solves) generalization of (5), and equations for the slow variables, y_i, α_i, β_i, keeping in the later ones terms upto the second order in v. Direct averaging of the slow modes equations over the h-statistics is the next step. At $z \ll z_{\mathrm{degr}}$, the relative phases $\alpha_i - \alpha_j$ do not change while the soliton positions, y_j, and phase velocities, β_j, evolve according to,

$$\partial_z y_j = -2\beta_j, \qquad \partial_z \beta_j = F_j, \tag{7}$$
$$F_j = \int dt\, U(t) \tanh(t - y_j) \cosh^{-2}(t - y_j),$$

where $U(t)$ is a quadratic form of $\tilde{v}$, $U(t) = 4|\tilde{v}_1|^2 + \tilde{v}_1^2 + \tilde{v}_1^{*2} + 2\varepsilon|\tilde{v}_2|^2$ for Model 2. The force F_j acting on the soliton, is self-averaged at $z \gg 1$. Therefore, we come to a set of deterministic (like in classical mechanics) equations for the soliton positions and the phase velocities. (The latter play the role of classical momenta.) The general setting is familiar from [5]. However, the dependence of the inter-soliton forces on the separation between the solitons in the polarization problems is different: the force does not depend on the separation. The key feature of the polarization problems is the refractive nature of $\hat{L}_2$, which is closely related to the nonintegrability of the no-disorder ($\hat{m} = 0$) problem in both of our settings (1), (2). This is in contrast with the integrability of SNLS, which is the no-disorder limit of the scalar problem. Due to non-zero refraction, standing waves are formed in between the solitons, in such a way that the wave amplitude does not depend on the inter-soliton separation.

We present here quantitative results for Model (2), obtained by numerical evaluation of the integral in (7) (with $U_{1,2}$ found, via analytical integration of the generalized version of (5), and averaged over (3)). Description of the calculation details are to be published elsewhere. The $O(D)$, y-independent, contribution to the inter-soliton force for the two-soliton pattern is shown in Fig.1. The force is independent of the phase mismatch, $\alpha_1 - \alpha_2$. It is always negative (the solitons repel). The minimum value of the force is achieved at the boundary value, $\varepsilon = 2/3$. The separation-independent contribution is zero at $\varepsilon = 1$. This corresponds to transparent scattering, as the no-disorder limit is integrable in this case [9]. The independence of the force on the overall size of the soliton pattern persists into the multi-soliton case, although a new feature, sensitivity to the phase-mismatches, emerges here. The dependence of the forces in the three-soliton pattern on the phase mismatch, in the special case, $\alpha_2 = 0$, $\alpha_3 = -\alpha_1 = \alpha$ and $\epsilon = 2/3$, for various values of the relative separation, $y = (y_3 - y_2)/(y_2 - y_1)$, is shown in Fig.2. In the "symmetric" case, $y = 1$, $F_2 = 0$ while $F_3 = -F_1$ and the value is twice larger than the force acting on the second particle in the two-soliton case. In all other, $y \neq 1$ situations the forces do depend on α. The values of the forces oscillate about the symmetric ($y = 1$) values.

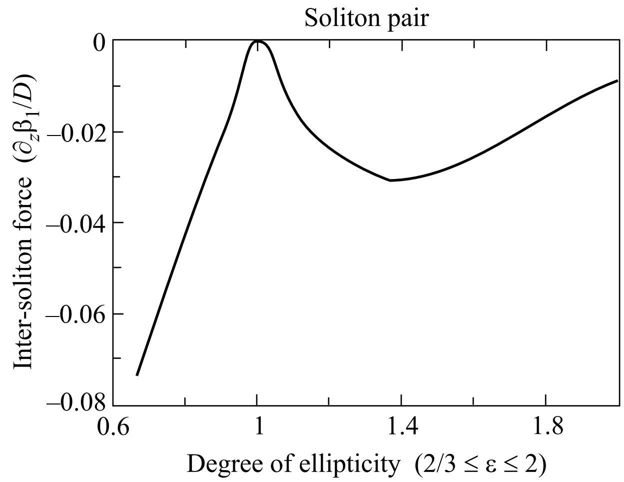

Fig.1. Two solitons. Inter-soliton force vs degree of ellipticity

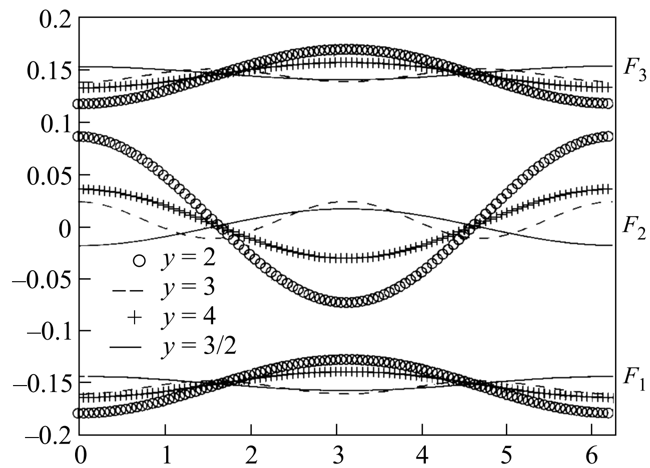

Fig.2. Three solitons. Forces vs inter-soliton phase mismatch

To conclude, we have shown that the major destructive factor for a set of well separated pulses in random birefrengent fibers is due to soliton-soliton interaction mediated by radiation. Note that the analytical method described in this paper can be easily generalized to a variety of more complicated sources of anisotropy in optical fibers.

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