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Reply to the comment "No robust phases in aerogel..."

I. A. Fomin

P. L. Kapitza Institute for Physical Problems RAS, 119334 Moscow, Russia

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The arguments of Volovik are refuted.

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1. The estimations made in the comment are based on the assumption that ABM order parameter (bulk A-phase) is the only relevant minimum of the Ginzburg and Landau (GL) free energy and its energy is smaller then that of other possible minima by the energy of the order of the full condensation energy. This situation is opposite to the situation considered in the criticized papers [C1], where competition of nearly degenerate states is assumed (in what follows references of the comment are prefixed by a capital C). Free energy of bulk (without aerogel) superfluid ^{3}He has 18 extrema [1] and the situation assumed in the comment does not seem to be very realistic.

For a present discussion relative energies of the states, corresponding to nonferromagnetic equal-spin pairing phases are of importance. Among the mentioned extrema there are four minima meeting this requirement [2]. Two of them – ABM and axiplanar state are so close in energy that identification of A-phase as ABM-state has been contested in the literature [3]. Axiplanar state unlike ABM contains in its vicinity robust states, as it was discussed earlier [4]. These states are also close in energy to the ABM. For a crude estimation of a relative difference of energies of competing states (to be referred as γ in what follows) weak coupling values of $\beta_1, ..., \beta_5$ parameters were used. With these values a relative difference of energies of the robust state and ABM corresponds to $\gamma \sim 1/20$. Strong coupling corrections to parameters β will change the difference, still $\gamma \sim 1/10$ is a fair estimation. Contribution of fluctuations to energy has to be compared not with the full condensation energy F_0 but with much smaller value γF_0. The regular part of this contribution, which comes from the gapped modes is of the order of αF_0 in agreement with and in the notations of the comment. A value of parameter $\alpha \sim (\eta^2/\sqrt{\tau})$ can be estimated from the measured width of the specific heat jump [5]. According to this data $\alpha \sim 1$ when $\tau \sim 1/30$. Because of the weak dependence on τ everywhere in the GL region parameter $\alpha \approx 1/5$ is at least comparable or greater then γ and even a regular contribution of fluctuations can mix-up relative energies of competing states in a contrast to the statement of the comment.

2. The main object of criticism in the comment is a contribution of fluctuations of Goldstone modes to the energy. According to the comment this contribution is of the order of $\alpha^2 F_0$ thus even smaller then the contribution of the gapped modes so that the free energy is a regular function of α and the original free energy $F_0(A^{(0)}_{\mu j})$ is a good starting point for expansion on a small α. This assertion is in a conflict with the Imry and Ma statement [C5] that the ordered state can be destroyed by "arbitrarily small random field". It indicates that new free energy $F(\bar{A}_{\mu j})$ which includes the contribution of fluctuations has to be a singular function of α and the argument based on continuity has to be taken with a great care.

The standard procedure [6] of finding of $F(\bar{A}_{\mu j})$ is based not on the direct averaging of the original free energy (or of its parts as it is done in the comment), but on a derivation of equation for the average order parameter, which in the present case has the following form:

$$\tau \bar{A}_{\mu j} + \frac{1}{2}\sum_{s=1}^{5}\beta_s\left[\frac{\partial I_s}{\partial A^*_{\mu j}} + \frac{1}{2}\left(\frac{\partial^3 I_s}{\partial A^*_{\mu j}\partial A_{\nu n}\partial A_{\beta l}}\langle a_{\nu n}a_{\beta l}\rangle + \right.\right.$$

$$\left.\left. + 2\frac{\partial^3 I_s}{\partial A^*_{\mu j}\partial A^*_{\nu n}\partial A_{\beta l}}\langle a^*_{\nu n}a_{\beta l}\rangle\right)\right] = -\langle a_{\mu l}\eta_{lj}\rangle. \qquad (1)$$

Corresponding free energy, if necessary, has to be constructed so that it generates the derived equation. The averages of fluctuations of the order parameter $\langle a_{\nu n}a_{\beta l}\rangle$ in "Goldstone" directions are proportional to a diverging integral, i.e. singular. It has been checked by a direct substitution that coefficients in front of the singular averages are not identical zeros. It means that GL equation contains singular terms. There is no reason for a cancellation of singular terms in the expression for free energy as well. It should be remarked though that free energy has not been used in the arguments of Refs. [C1].

Volovik in construction of the free energy followed "physical" argument, which does not take into account important features of the problem. In particular, he overlooks a fact that Goldstone directions depend on the average order parameter. As a result variation of his free energy will not contain terms which have to be present in the equation (1).

The singular terms in the Eq. (1) being proportional to the diverging integral are much greater then the regular terms. That determines a procedure of its solution. As a first step the principal terms are set to be equal to zero. This condition selects a degenerate class of robust order parameters. Remaining terms in the equation are treated as a perturbation, lifting this degeneracy. They have to be considered on a class of robust order parameters. So, robust order parameters are asymptotic solutions of GL equation in a limit $\gamma \to 0$, $\alpha \ll 1$ and the ABM order parameter is not solution of this equation in the considered limit in a contrast to the statement, made in the comment. Energies of two states were not compared directly. A problem of comparison of different states does not arise here because in the considered limit a family of robust phases is the only nontrivial extremum of the free energy.

Summing up one can say that the declared in the comment error in overestimation of fluctuations does not exist. The diverging terms are present in the GL equation and this is sufficient for selection of the robust phases. The robust phases are extrema of the proper free energy. The situation, considered in the comment and the one discussed in Refs. [C1] correspond to different regions of parameters: $\gamma \sim 1$, $\alpha \ll 1$, so that $\gamma \gg \alpha$ (comment), $\gamma \leq \alpha \ll 1$ (Refs. [C1]). For that reason a criticism presented in the first part of the comment has no relevance to the problem discussed in the criticized papers.

About the situation in the real ^{3}He it has to be remarked that the present knowledge of coefficients $\beta_1, ..., \beta_5$ is not sufficiently accurate for reliable reconstruction of "topography" of the GL free energy. Even though the given above estimations show that situation is favorable for realization of robust phases the competing situation [C7] can not be ruled out and it can realize in its range of parameters, for example when aerogel is very dilute if macroscopic description still applies.

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